Mathematical Methods in Biomedical Imaging and Intensity-Modulated Radiation Therapy (IMRT)

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Preface

This book brings together 24 state-of-the-art refereed research and review papers, by leading experts and practitioners in mathematical methods in biomedical imaging and intensity-modulated radiation therapy (IMRT). The emphasis is on trying to discover relations and connections between these two fields that will enhance progress in each of them. As this volume shows, applicable mathematical work in these fields goes hand-in-hand with real-world applications and the mutual "technology transfers" between them leads to further advances.

The topics covered here include mathematical aspects and practical problems in current major and emerging technologies in diagnostic and therapeutic medicine and biology research. The contributed work signifies the interdisciplinary cooperation between mathematicians and scientists from medical physics, engineering, clinical medicine and biology, that leads to mathematically-based solutions of practical problems in biomedical imaging and IMRT.

The Centro di Ricerca Matematica (CRM) Ennio De Giorgi in Pisa, Italy, headed by Professor Mariano Giaquinta, recognizing the importance of the field and the need for interaction between theoreticians and practitioners, provided us with a special grant to organize an *Interdisciplinary Workshop on Mathematical Methods in Biomedical Imaging and Intensity-Modulated Radiation Therapy (IMRT)*, that took place in October 15-19, 2007, at the Aula Dini, Via del Castelleto, in the picturesque old town of Pisa, just a few blocks away from the famous Leaning Tower of Pisa.

Experts from around the world were invited and participated. They came from Finland, France, Germany, Greece, Iran, Israel, Italy, P. R. China, Poland, Spain, Sweden, the United Kingdom and the United States of America. Most of the papers in this volume originated from the lectures presented at this Workshop while others were written in the wake of discussions held during the Workshop.

We thank Professor Mariano Giaquinta and the team of the CRM, especially Dr. Ilaria Gabbani and Dr. Antonella Gregorace for their cooperation and extraordinary support in organizing and conducting the Workshop. Many thanks are due to the referees whose work enhanced the final versions of the papers which appear here. Last but not least, we thank the participants of the Research Workshop and the authors who contributed their work to this volume. We gratefully acknowledge the help of Dr. Luisa Ferrini from the Edizioni della Normale publisher's office for her work on the production of this volume.

We hope that researchers in applied mathematics, medical physics, biomedical imaging and intensity-modulated radiation therapy will find this book a useful tool in their current research and development efforts.

April 30, 2008

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