The University of British Columbia

MATH 317

Practice Midterm 2

August 6, 2015

Time: 75 minutes

LAST NAME: ______________________ FIRST NAME: ______________________

STUDENT # : ______________________ SIGNATURE: ______________________

This Examination paper consists of 9 pages (including this one). Make sure you have all 9.

instructions:

No memory aids allowed. No calculators allowed. No communication devices allowed.

marking:

Q1	/5
Q2	/7
Q3	/12
Q4	/16
Q5	/10
TOTAL	/0

Name of Instructor: Uriya First

Q1 [5 marks]

Let C be the closed curve

$$\vec{\mathbf{r}}(t) = \left\langle t - t^2, t^2 - t^3 \right\rangle, \qquad 0 \le t \le 1$$

Find the area of the region bounded by C using Green's Theorem. You do not have to check that C is simple, closed, and positively oriented. (Hint: Find $P(x, y)$ and $Q(x, y)$ satisfying $Q_x - P_y = 1$.)

Q2 [7 marks]

"Free Style Integration": Evaluate

$$\int_C (y^2 + 2\arctan(x^{-1}y))\,\mathrm{d}x + \ln(x^2 + y^2)\mathrm{d}y$$

where C is the triangle with vertices $(1,1)$, $(1,2)$, $(3,1)$, oriented clockwise.

Q3 [12 = 2+6+4 marks]

Let

$$\vec{F} = \frac{yz}{x^2y^2+z^2}\,\vec{\mathbf{i}} + \frac{xz}{x^2y^2+z^2}\,\vec{\mathbf{j}} - \frac{xy}{x^2y^2+z^2}\,\vec{\mathbf{k}}$$

(a) Simplify $\operatorname{div}\vec{\mathbf{F}}$.

(b) Simplify $\operatorname{curl}\vec{\mathbf{F}}$.

(c) Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where C is the circle $\vec{\mathbf{r}}(t) = \langle 2\cos t, 1, 2\sin t\rangle$, $0 \le t \le 2\pi$ with direction increasing with t.

Q4 [16 = 2+6+8 marks]

The graph of the function $y = \sin x$ revolves around the x axis. Let S be the part of the resulting surface lying between the planes $x = 0$ and $x = \frac{\pi}{2}$.

(a) Find a parameterization of S.

(b) Find the equation of the tangent plane to S at $(\frac{\pi}{3}, -\frac{3}{4}, -\frac{\sqrt{3}}{4})$.

(c) It is given that the temperature of a substance with conductivity $K = \frac{1}{2}$ at the point (x, y, z) is $u(x, y, z) = 2y^2 + 3z^2$. Find the flow of heat toward the inside of S.

Q5 [10 marks]

Complete the following parameterizations:

(a) The part of the hyperboloid $z = \sqrt{1 + 2x^2 + 2y^2}$ lying under the plane $z = y + 3$ can be parameterized as $\vec{\mathbf{r}}\,(u, v) = u\,\vec{\mathbf{i}} + v\,\vec{\mathbf{j}} + \sqrt{1 + 2u^2 + 2v^2}$ where $A(u - B)^2 + C(v - D)^2 \leq 1$.

$A =$ ☐

$B =$ ☐

$C =$ ☐

$D =$ ☐

(b) The part of the cylinder $x^2 + y^2 = 4$ lying inside the ellipsoid $\frac{x^2}{40} + \frac{(y-1)^2}{20} + \frac{(z-1)^2}{80} = 1$ can be parameterized as $\vec{\mathbf{r}}\,(\theta, r) = A\cos\theta\,\vec{\mathbf{i}} + A\sin\theta\,\vec{\mathbf{j}} + r\,\vec{\mathbf{k}}$ where $0 \leq \theta \leq 2\pi$ and $f(\theta) \leq r \leq g(\theta)$.

$A =$ ☐ (assume $A \geq 0$)

$f(\theta) =$ ☐

$g(\theta) =$ ☐

(c) The part of the ellipsoid $x^2 + y^2 + \frac{1}{6}z^2 = 1$ lying between the planes $z = \sqrt{2}$ and $z = \sqrt{3}$ can be parameterized as $\vec{\mathbf{r}}(\theta, \phi) = (\sin\phi\cos\theta, \sin\phi\sin\theta, A\cos\phi)$ where $0 \le \theta \le 2\pi$ and $B \le \phi \le C$.

$A =$ ☐ (assume $A \ge 0$)

$B =$ ☐

$C =$ ☐