The University of British Columbia

## MATH 317

# Practice Midterm 2

August 6, 2015

TIME: 75 MINUTES

LAST NAME: _	FIRST NAME:
STUDENT # :	SIGNATURE:

This Examination paper consists of 9 pages (including this one). Make sure you have all 9.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.

#### MARKING:

Q1	/5
Q2	/7
Q3	/12
Q4	/16
Q5	/10
TOTAL	/0

#### Q1 [5 marks]

Let C be the closed curve

$$\vec{\mathbf{r}}(t) = \left\langle t - t^2, t^2 - t^3 \right\rangle, \qquad 0 \le t \le 1$$

Find the area of the region bounded by C using Green's Theorem. You do not have to check that C is simple, closed, and positively oriented. (Hint: Find P(x, y) and Q(x, y) satisfying  $Q_x - P_y = 1$ .)

### $\mathbf{Q2}$ [7 marks]

"Free Style Integration": Evaluate

$$\int_{C} (y^{2} + 2 \arctan(x^{-1}y)) \, \mathrm{d}x + \ln(x^{2} + y^{2}) \, \mathrm{d}y$$

where C is the triangle with vertices (1, 1), (1, 2), (3, 1), oriented clockwise.

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 $\mathbf{Q3} \quad [12=2{+}6{+}4 \text{ marks}]$ 

Let

$$\vec{F} = \frac{yz}{x^2y^2 + z^2} \, \vec{\mathbf{i}} + \frac{xz}{x^2y^2 + z^2} \, \vec{\mathbf{j}} - \frac{xy}{x^2y^2 + z^2} \, \vec{\mathbf{k}}$$

(a) Simplify div  $\vec{\mathbf{F}}$ .

(b) Simplify  $\operatorname{curl} \vec{\mathbf{F}}$ .

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(c) Evaluate  $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ , where C is the circle  $\vec{\mathbf{r}}(t) = \langle 2\cos t, 1, 2\sin t \rangle, 0 \le t \le 2\pi$  with direction increasing with t.

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 $\mathbf{Q4} \quad [\mathbf{16} = \mathbf{2}{+}\mathbf{6}{+}\mathbf{8} \text{ marks}]$ 

The graph of the function  $y = \sin x$  revolves around the x axis. Let S be the part of the resulting surface lying between the planes x = 0 and  $x = \frac{\pi}{2}$ .

(a) Find a parameterization of S.

(b) Find the equation of the tangent plane to S at  $(\frac{\pi}{3}, -\frac{3}{4}, -\frac{\sqrt{3}}{4})$ .

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(c) It is given that the temperature of a substance with conductivity  $K = \frac{1}{2}$  at the point (x, y, z) is  $u(x, y, z) = 2y^2 + 3z^2$ . Find the flow of heat toward the inside of S.

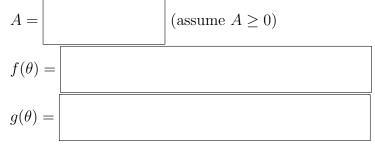
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#### Q5 [10 marks]

Complete the following parameterizations:

(a) The part of the hyperboloid  $z = \sqrt{1 + 2x^2 + 2y^2}$  lying under the plane z = y + 3 can be parameterized as  $\vec{\mathbf{r}}(u, v) = u \vec{\mathbf{i}} + v \vec{\mathbf{j}} + \sqrt{1 + 2u^2 + 2v^2}$  where  $A(u - B)^2 + C(v - D)^2 \le 1$ . A = B = C = D =

(b) The part of the cylinder  $x^2 + y^2 = 4$  lying inside the ellipsoid  $\frac{x^2}{40} + \frac{(y-1)^2}{20} + \frac{(z-1)^2}{80} = 1$  can be parameterized as  $\vec{\mathbf{r}}(\theta, r) = A \cos \theta \, \vec{\mathbf{i}} + A \sin \theta \, \vec{\mathbf{j}} + r \, \vec{\mathbf{k}}$  where  $0 \le \theta \le 2\pi$  and  $f(\theta) \le r \le g(\theta)$ .



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(c) The part of the ellipsoid  $x^2 + y^2 + \frac{1}{6}z^2 = 1$  lying between the planes  $z = \sqrt{2}$  and  $z = \sqrt{3}$  can be parameterized as  $\vec{\mathbf{r}}(\theta, \phi) = (\sin \phi \cos \theta, \sin \phi \sin \theta, A \cos \phi)$  where  $0 \le \theta \le 2\pi$  and  $B \le \phi \le C$ .

A =	(assume $A \ge 0$ )
B =	
C =	