MATH 317

Midterm 1

21 July 2015

TIME: 75 MINUTES

LAST NAME: _____ FIRST NAME: _____

STUDENT # : _____ SIGNATURE: _____

This Examination paper consists of 8 pages (including this one). Make sure you have all 8.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.

MARKING:

Q1	/16
Q2	/5
Q3	/8
Q4	/9
Q5	/12
TOTAL	/50

NAME OF INSTRUCTOR: Uriya First

Q1 [16 marks]

A helium balloon weighing 0.02kg is released from ground level on a windy day. It is given that the balloon's position after t seconds $(0 \le t \le 5)$ is

$$\vec{\mathbf{r}}(t) = \frac{\sqrt{2}}{3}t^3 \,\vec{\mathbf{i}} + (2t - t^2)\,\vec{\mathbf{j}} + (2t + t^2)\,\vec{\mathbf{k}}\ .$$

(the coordinates are measured in meters).

(a) At what time is the balloon's velocity parallel to the plane $2y + z = 2$?

(b) Find the force $\vec{F}(t)$ that the balloon feels at time t.

(c) Find the curvature of the balloon's path at $t = 1$.

(d) Find the distance that the balloon has traveled after t seconds.

Q2   [5 marks]

Let C be the intersection of the surfaces $3x + xy = z^2$ and $y^2 + 4z^2 = 4$. Find a parameterization of C.

Q3   [8 marks]

Below are 4 plots of (possibly scaled) vector fields. Clearly mark each plot with the letter corresponding to the vector field it represents. Note that 2 letters will not be used.
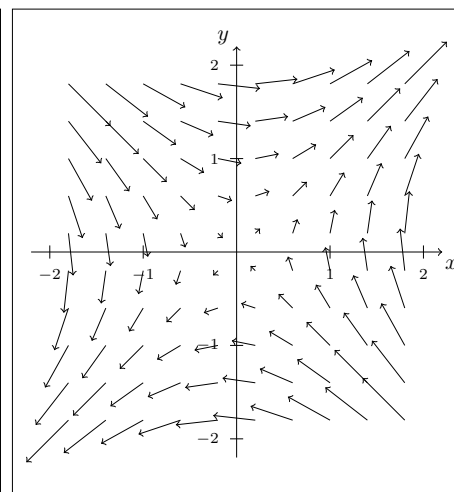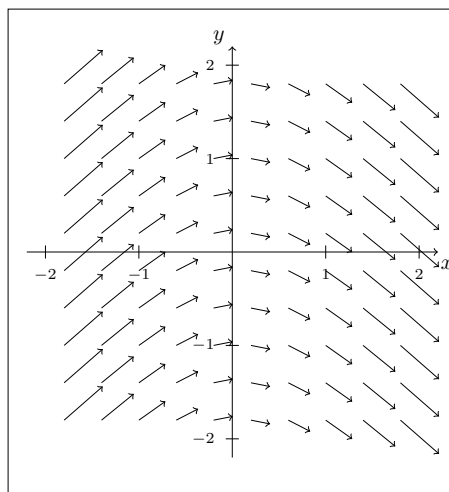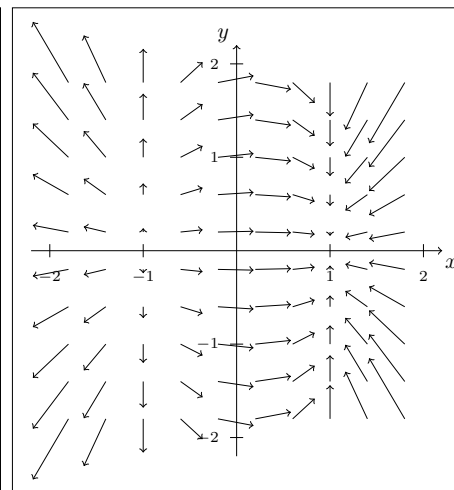
A: $\vec{F}(x,y) = y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$
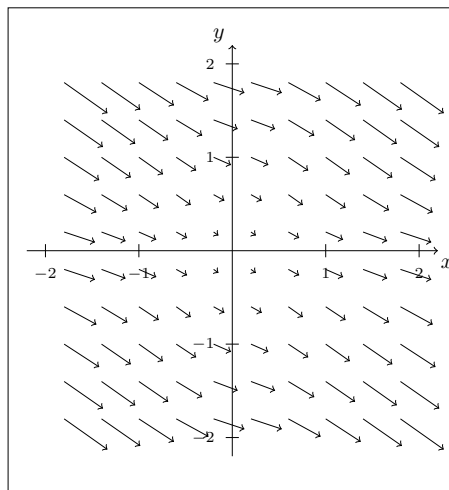
B: $\vec{F}(x,y) = \sqrt{x^2+1}\,\vec{\mathbf{i}} - x\vec{\mathbf{j}}$

C: $\vec{F}(x,y) = (y+1)\vec{\mathbf{i}} + (x+1)\vec{\mathbf{j}}$

D: $\vec{F}(x,y) = (x-y)\vec{\mathbf{i}} + e^y\vec{\mathbf{j}}$

E: $\vec{F}(x,y) = 2\cos(\frac{\pi}{2}x)\vec{\mathbf{i}} - xy\vec{\mathbf{j}}$

F: $\vec{F}(x,y) = \sqrt{x^2+y^2}\,\vec{\mathbf{i}} - \sqrt{|xy|}\,\vec{\mathbf{j}}$

Q4 [9 marks]

Let

$$\vec{F}(x, y) = (e^x + e^{x+2y})\,\vec{\mathbf{i}} + (2e^{2y} + 2e^{x+2y})\,\vec{\mathbf{j}}$$

(a) Show that $\vec{F}$ is conservative and find a potential function.

(b) Find $\int_C \vec{F}(x, y) \cdot d\vec{\mathbf{r}}$ where C is the the parameterized curve $\vec{\mathbf{r}}(t) = \langle \sin(\pi t), \tan(\pi t^2) \rangle$ $(-\frac{1}{2} \leq t \leq \frac{1}{2})$ with direction increasing with t.

Q5   [12 marks]

Let C be the closed plane curve consisting of the line segment from $(0, 1)$ to $(0, 0)$, the line segment from $(0, 0)$ to $(1, 0)$, and the quarter of the unit circle $x^2 + y^2 = 1$ starting at $(1, 0)$ and ending at $(0, 1)$. Find

$$\int_C x e^y \, ds \ .$$

Extra space for work.