Related Rates

Question 1. In the countryside, a jogger begins his jog 500 meters south of an old barn. He heads west at a constant speed of $10_{\text{km/h}}$. How fast does the distance between the jogger and the barn increase when the jogger passed 1.2 kilometers?

Question 2. A kid blows air into a spherical balloon at a constant rate of 5 cubic inches per second.

- (a) How fast does the radius of the balloon grow when the radius of the balloon is 5_{in}.
- (b) How fast does the radius of the balloon grow when the balloon contains 36π cubic inches of air?
- (c) Continuing (a), at what rate does the surface area of the balloon grow?

Question 3. The length of rectangle is increased at a rate of $3_{\text{cm/sec}}$ while at the same time its width is decreased at a rate of $2_{\text{cm/sec}}$. At what rate does the area of the rectangle change when its length and width both equal 10_{cm}?

Question 4. [1] A spotlight is placed on the ground 6 meters from a wall. A woman 2 meters tall is walking at a speed of $2_{\text{m/sec}}$ from the spotlight straight toward the wall, casting a shadow on the wall. At what rate does the height of the shadow change when the woman is 2 meters from the wall?

Question 5. Water are poured at a rate of $2_{\text{cm}^3/\text{sec}}$ into a test tube shaped as cone 20 centimeters wide and 30 centimeters tall (the tip of the cone points upward, the base is parallel to the ground). What is the rate of change of the water level in the test tube when the water level is 15 centimeters?

Question 6. A clock has two hands, indicating the hours and the minutes respectively. The hour hand is 10_{cm} long while the minutes hand is 20_{cm} long. Suppose that the hour is 3:00. What is the rate of change of the distance between the tips of the clock's hands? Specify your answer in centimeters per hour.
(Suggestion: Use the law of cosines.)

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[1]Taken from a past midterm.

<div style="text-align:center">Derivatives as Rates of Change</div>

Question 1. Water are pouring out through a tap located at the bottom of a cylindrical barrel 20 inches tall and 20 inches wide. It is given that the water level in the barrel at time t (measured in seconds, $0 \leq t \leq 20$) is $\frac{1}{20}(t-20)^2$ inches.[2]

  (a) How much water are in the tank at time t?
  (b) What is the average rate (in cubic inches per second) at which water are coming out of the barrel from $t = 0$ to $t = 10$?
  (c) What is the rate at which water are coming out when $t = 15_{\text{sec}}$?

Question 2. Jane from Homework 1 opens a saving account with an annual interest rate of 2%, compounded continuously. She opens the account at time $t = 0_{\text{years}}$ with a starting balance of $1000.

  (a) How much money is in the account at time t?
  (b) What is the average yearly change in the balance (in dollars per year) from $t = 0$ to $t = 3$?
  (c) What is the rate at which the money in the account is increasing when $t = 2.5_{\text{years}}$?

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[2] This is very close to reality, in fact.