HOMEWORK 9

Due on: November 18 at 2:00pm (submit via Crowdmark).

Remark: We will learn about asymptotes and limits at ∞ on Nov 7. Homework 8 and Homework 9 are both due on Nov 18. You can use them to prepare for the midterm.

Question 1. Evaluate the following limits.

(1) $\lim\limits_{x\to\infty} \frac{4+3x^2}{1-6x-x^2}$

(2) $\lim\limits_{x\to 2^-} \frac{x^2+3}{x^2-4}$

(3) $\lim\limits_{x\to 0^+} \left(\frac{1}{1+\ln x} + e^x\right)$

(4) $\lim\limits_{x\to 0^+} \frac{100}{x} - \frac{1}{x^2}$

(5) $\lim\limits_{x\to -\infty} \frac{x}{\sqrt{2x^2+x+1}}$

Question 2. Let $f(x)$ be a continuous function defined for all $x \neq 0$.
It is given that:

- $f(x)$ is increasing on $(-1, 0)$, $(2, \infty)$ and decreasing on $(-\infty, -1)$, $(0, 2)$.
- $(-1, -2)$ and $(2, 0)$ are local minimum points.
- $f(x)$ is concave up on $(-3, 0)$, $(0, 1.5)$ and concave down on $(-\infty, -3)$, $(1.5, 2)$, $(2, \infty)$.
- $(-3, 0)$ and $(1.5, 1)$ are inflection points.
- $y = 2$ is a horizontal asymptote.
  More precisely, $\lim\limits_{x\to\infty} f(x) = 2$ and $\lim\limits_{x\to-\infty} f(x) = \infty$.
- $x = 0$ is a vertical asymptote.
  More precisely, $\lim\limits_{x\to 0^+} f(x) = \infty$ and $\lim\limits_{x\to 0^-} f(x) = 0$.

Draw the graph of $f(x)$ based on the previous information. Indicate extremal points, inflection points and asymptotes.

Question 3. Let $f(x) = \dfrac{1}{2x} + \arctan x$.

(a) Find all extremal points and the intervals on which $f(x)$ is increasing/decreasing.

(b) Find the x-coordinate of all inflection points and the intervals on which $f(x)$ is concave up/down.

(c) Find all asymptotes. [Suggestion: In order to determine $\lim\limits_{x\to\infty} \arctan x$, consider the behavior of $y = \tan x$ to the left of $\frac{\pi}{2}$.]

(d) Draw a graph of $f(x)$ indicating all previous information.