HOMEWORK 4

Due on: October 7 at 1:00pm (submit at the end or at the beginning of class).

Question 1. The following question appeared in a calculus exam:

> *Let* $f(x) = \frac{1}{x} + x + x^3$. *Does the equation* $f(x) = 0$ *has a solution in the interval* $[-2, 2]$*? Why?*

One of the students wrote:

> `Yes!`
> `The function f(x) is continuous.`
> `We have` $\mathtt{f}(-2) = -\frac{1}{2} - 2 - 2^3 < 0$ `and` $\mathtt{f}(2) = \frac{1}{2} + 2 + 2^3 > 0$`,`
> `so by the Intermediate Value Theorem, there exists`
> $-2 < \mathtt{c} < 2$ `such that` $\mathtt{f}(\mathtt{c}) = 0$`.`

Answer the following:

(a) Explain the mistake in the student's answer.
(b) Show that $f(x) = 0$ has no solutions.

Question 2.

(a) Find the derivative of $f(x) = \frac{1}{x^2}$ using the limit definition of the derivative (no credit will be given for other method).
(b) Find the line tangent to $y = \frac{1}{x^2}$ at the point $(2, \frac{1}{4})$.

Question 3. Consider the function:

$$f(x) := \begin{cases} x^2 & x < 1 \\ 2 - x & x \geq 1 \end{cases}$$

(a) Draw the graph of $f(x)$ in the interval $[-1, 3]$. Indicate the cusp ("corner") in the graph.
(b) Find $\lim_{x \to 1^-} \dfrac{f(x) - f(1)}{x - 1}$ (the left derivative at $x = 1$)
and $\lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$ (the right derivative at $x = 1$).
(c) Based on your answer to (2), find $f'(1)$ or explain why it does not exist.
(d) Find $f'(x)$ when $x < 1$. Find $f'(x)$ when $x > 1$.
You may use differentiation rules.
(e) Using your previous answers, draw the graph of $f'(x)$ in the interval $[-1, 3]$.