HOMEWORK 3

Due on: September 30 at 1:00pm (submit at the end or at the beginning of class). Question 1.

(a) It is given that $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3} g(x) = 4$. Use limit laws to evaluate $\lim_{x\to 3} \sqrt{f(x)^4 - f(x)g(x) + 1}$. (b) Fact: $\lim_{x \to 0} \frac{x}{\sin x} = 1$. Use the fact and limit laws to find $\lim_{x \to 0} \frac{x^3 - 2x^2}{(\sin x)^2}$.

Question 2. Consider the function:

$$f(x) := \begin{cases} ax^2 - x & x < 2\\ 6 & x = 2\\ 2^{-bx} & x > 2 \end{cases}$$

where a and b are parameters.

- (1) Find lim f(x) and lim f(x). Express your answer in terms of a and b.
 (2) Find all values of a and b for which f(x) is everywhere continuous.
 (3) Give specific values of a and b for which lim f(x) exists, but f(x) is not everywhere continuous.

Question 3. Use the Intermediate Value Theorem (and your calculator) to show that the equation

 $e^{x} = 5 - x$

has a solution in the interval [1,2]. Find the solution's first two decimal digits after the dot (you must justify your answer using the Intermediate Value Theorem).