

HOMEWORK 11 — NOT FOR SUBMISSION!

These homework are not for submission. Use them for practice.

Question 1. It is given that the 5-th Taylor polynomial of $f(x)$ near $x = -1$ is

$$p_5(x) = \frac{1}{2} - \frac{1}{3}(x+1) + \frac{1}{5}(x+1)^2 + \frac{1}{7}(x+1)^3 - \frac{1}{11}(x+1)^4 - \frac{1}{10}(x+1)^5$$

What is $f''(-1)$?

What is $f^{(5)}(-1)$?

What is the 3-rd Taylor polynomial of $f(x)$ near $x = -1$?

Question 2. Find the 3rd Taylor polynomial of $f(x) = \sqrt{x}$ near $x = 4$. Use it to estimate $\sqrt{3}$. You may leave your estimation in calculator ready form.

Question 3. Let $p_n(x)$ be the degree n approximation of $\ln(1-x)$ near $x = 0$.

- (a) Find $p_1(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$. Do you notice a pattern? Find $p_7(x)$ based on the pattern.
- (b) Estimate $\ln 0.7$ using $p_1(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, and find the true value of $\ln 0.7$ using a calculator. Are the approximations getting better and better?
- (c) Use Taylor's Theorem to bound $|p_4(0.3) - \ln 0.7|$. Give the smallest possible upper bound guaranteed by the theorem.
- (d) Estimate $\ln 3$ using $p_1(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, and find the true value of $\ln 3$ using a calculator. Are the approximations getting better and better? Why is this situation different from the situation in (b)?