Exercise. Let $f(x)$ be an everywhere differentiable function and let

$$g(x) = \frac{x}{f(x)+1}$$

(a) Express $g'(x)$ using $f(x)$ and $f'(x)$.
(b) It is given that $f(2) = 0$ and $f'(2) = 2$. Find the line tangent to $y = f(x)$ at $x = 2$.

Solution. (a)

$$\begin{aligned} g'(x) &= \frac{(x)'(f(x)+1) - x(f(x)+1)'}{(f(x)+1)^2} \\ &= \frac{1 \cdot (f(x)+1) - xf'(x)}{(f(x)+1)^2} = \boxed{\frac{f(x)+1-xf'(x)}{(f(x)+1)^2}} \end{aligned}$$

(b)

$$\begin{aligned} g(2) &= \frac{2}{f(2)+1} = \frac{2}{0+1} = 2 \\ g'(2) &= \frac{f(2)+1-2f'(2)}{(f(2)+1)^2} = \frac{0+1-2\cdot 2}{(0+1)^2} = -3 \end{aligned}$$

The line tangent to $y = g(x)$ at $x = 2$ is

$$\begin{aligned} y &= g'(2)(x-2) + g(2) \\ y &= -3(x-2) + 2 \\ y &= -3x + 6 + 2 \\ &\boxed{y = -3x + 8} \end{aligned}$$