DIFFERENTIATION RULES

This document is a concise summary of all differentiation formulas and rules that you are expected to know.

1. Derivatives of Common Functions

1.1. Constant Functions. Let c be a number.

$$(c)' = 0$$

1.2. Powers of x.

$$(x^n)' = nx^{n-1}$$
$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$
$$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$$

Remarks:

- The rule $(x^n)' = nx^{n-1}$ works even when n is not an integer and also when n is negative, e.g. $(x^{-1.7})' = -1.7x^{-2.7}$.
- In order to differentiate $\sqrt[n]{x}$, use $\sqrt[n]{x} = x^{\frac{1}{n}}$.

1.3. Exponentials.

$$(e^x)' = e^x$$
$$(a^x)' = a^x \ln a$$

1.4. Logarithms.

$$(\ln x)' = \frac{1}{x}$$
$$(\log_a x)' = \frac{1}{x \ln a}$$

1.5. Trigonometric Functions.

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = \frac{1}{(\cos x)^2}$$

Remark: These formulas assume that x is measured in radians (180 degrees equal π radians).

1.6. **Inverse Trigonometric Functions.** [1]

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$
$$(\arctan x)' = \frac{1}{1+x^2}$$

2. Differentiation Rules

Let $f(x)$ and $g(x)$ be differentiable functions and let c be a constant number.

- Scalar multiple rule:
$$(cf(x))' = cf'(x)$$
- Addition/subtraction rule:
$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$
- Product rule
$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$
- Quotient rule:
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
- Chain rule:
$$(g(f(x)))' = g'(f(x))f'(x)\ .$$

3. How to Differentiate The Power of Two Functions

There are two ways to differentiate expressions of the form

$$f(x)^{g(x)}$$

3.1. **First way.** Rewrite $f(x)^{g(x)}$ as follows:

$$f(x)^{g(x)} = \left(e^{\ln f(x)}\right)^{g(x)} = e^{g(x)\ln f(x)}$$

Now use all previous rules to differentiate the right hand side. Namely,

$$\left(e^{g(x)\ln f(x)}\right)' = e^{g(x)\ln f(x)}\left(g(x)\ln f(x)\right)' = f(x)^{g(x)}\left(g(x)\ln f(x)\right)' = \dots$$

[1] We will learn these later in the course.

3.2. **Second way.** Use logarithmic differentiation: Write $y = f(x)^{g(x)}$ and apply $\ln(\dots)$ to both sides in order to get

$$\ln y = g(x) \ln f(x) \ .$$

Now use implicit differentiation. This will give

$$\frac{y'}{y} = (g(x) \ln f(x))'$$
$$y' = y(g(x) \ln f(x))'$$

Now substitute $y = f(x)^{g(x)}$ to get

$$y' = f(x)^{g(x)} \left(g(x) \ln f(x)\right)' = \dots$$

Remark: This also works well with functions of the form $y = f(x)^{g(x)} h(x)^{k(x)}$.