

## ASSIGNMENT 5

**Due on:** October 30 at 8:00 in the morning (submit before class begins).

Be sure to write your name and student ID on your assignment.

### Questions:

- (1) A farmer sells wheat at  $p$  dollars per kilogram. The demand  $q$  (in kilograms per week) is related to  $p$  via  $q^2 + (100p)^2 = 650000$ .
  - (a) Find the elasticity when  $p = 1\$$ .
  - (b) Assuming  $p = 1\$$ , estimate the percentage by which the demand will change if  $p$  is decreased by 3%.
  - (c) Find all values of  $p$  for which the demand is elastic (notice that  $p$  cannot be arbitrarily large).
  - (d) Find the maximal revenue.
  - (e) The cost function for producing  $q$  kilograms of wheat is  $C(q) = aq + b$ . Find  $a$  and  $b$  if the maximal profit is  $1500_{\$/\text{week}}$  and it is attained when  $q = 500_{\text{kg}/\text{week}}$ .
- (2) In the year 1990, a woman started a saving account with annual interest  $r$  (the interest is compounded continuously), and on 2006, the money in the saving account has been doubled.
  - (a) Find  $r$ .
  - (b) How many years (from opening the account), did it take for the money in the account to increase by 50%?
  - (c) In 2008, the woman deposited additional 2000\$ into the account, and in 2014, the account contained 30000\$. Find the initial amount of money in the account (i.e. in 1990).
  - (d) Find the *rate of change* of the money in the account in the year 2000.
- (3) The interior of a certain wine cup has the shape of a circular conic (the tip is pointing down, the cone's circular base is parallel to the ground). The cone's radius equals one third of its height. Wine is poured into the glass at a constant rate of 0.5 cubic inches per second. How fast does the wine level in the cup changes when it equals 3 inches?
- (4) Let  $ABC$  be a triangle such that  $\angle A = 90^\circ$ . It is given that  $AB$  is increased at a rate of  $a_{\text{cm}/\text{sec}}$  and  $AC$  is increased at a rate of  $b_{\text{cm}/\text{sec}}$ .
  - (a) How fast does the area of the triangle changes when  $AB = 3_{\text{cm}}$  and  $AC = 4_{\text{cm}}$ ? Express your answer using  $a$  and  $b$ .
  - (b) How fast does the length of  $BC$  changes when  $AB = 3_{\text{cm}}$  and  $AC = 4_{\text{cm}}$ ? Express your answer using  $a$  and  $b$ .
  - (c) How fast does the angle  $\angle B$  changes (in radians per second) when  $AB = 3_{\text{cm}}$  and  $AC = 4_{\text{cm}}$ ? Express your answer using  $a$  and  $b$ .
  - (d) Assume that answer to (a) is  $5_{\text{cm}^2/\text{sec}}$  and the answer to (b) is  $5_{\text{cm}/\text{sec}}$ . Find  $a$  and  $b$ .