ASSIGNMENT 4

Due on: October 21 at 8:00 in the morning (submit before class begins).

Be sure to write your name and student ID on your assignment.

Questions:

(1) The following question appeared in a calculus exam:

Let $f(x) = \frac{1}{x} + x + x^3$. Does the equation $f(x) = 0$ has a solution in the interval $[-2, 2]$? Why?

One of the students wrote:

`Yes!`
`We have` $f(-2) = -\frac{1}{2} - 2 - 2^3 < 0$ `and` $f(2) = \frac{1}{2} + 2 + 2^3 > 0$`,`
`so by the Intermediate Value Theorem, there exists`
$-2 < c < 2$ `such that` $f(c) = 0$`.`

Answer the following:

(a) Find the mistake in the student's answer.
(b) Show that $f(x) = 0$ has no solutions at all.

(2) Differentiate the following functions:

(a) $2^x + \log_3 x - 2x^\pi$
(b) $(5^x - x)^{1.4}$
(c) $x^{(e^x)}$
(d) $(\ln x)^{\ln x}$

(3) Use implicit differentiation to express $\frac{dy}{dx}$ as a function of x and y in the following cases:

(a) $x^3 + xy + y^3 = 1$
(b) $e^x + e^y = xy + 1$

(4) Find the tangent line to the curve $x + \cos x = y^5 + y^4 - 1$ at the point $(0, 1)$.

(5) Find all values of a for which the tangent line to the curve $x^2 - axy + y^2 = 1$ at the point $(1, 0)$ passes through the point $(2, 5)$.

(6) Suppose a function f has an inverse function f^{-1}. Show that $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ by using implicit differentiation of $x = f(y)$.

(7) A rocket is launched into space. It is given that t seconds after the launching ($t \leq 20$), the rocket has traveled $f(t) = 0.1t^3 + 2t^2 + 20t$ meters.

(a) Find the velocity and acceleration of the rocket at time t. (The acceleration is the rate of change of the velocity as a function of the time.)
(b) Find the average acceleration from $t = 2$ to $t = 4$.
(c) Find all t-s for which the acceleration equals $13_{\mathrm{m/sec^2}}$.