ASSIGNMENT 3

Due on: October 9 at 8:00 in the morning (submit before class begins).

Be sure to write your name and student ID on your assignment.

Questions:
(1) Differentiate the following functions:
   (a) $5x^5 - 7x^3 + 2x - 9$
   (b) $\frac{x^2}{x^3+1}$
   (c) $x^2 \cdot \ln x \cdot \cos x$
   (d) $\sqrt[3]{x} + \sqrt{e^x + 1}$
   (e) $\sqrt{\ln(e^x + \sin x)}$.
(2) Find the derivative of the the following functions <u>according to the limit definition of the derivative</u> (no credit will be given for other methods).
   (a) $f(x) = \frac{1}{x^2+1}$
   (b) $f(x) = \sqrt{x^2 + x}$.
(3) Let
$$f(x) = \begin{cases} x^3 - x & x > -1 \\ 2x + 2 & x \leq -1 \end{cases}$$
   (a) Prove that f is differentiable at $x = -1$ and find $f'(-1)$. (Hint: Compute the left and right limits of $\frac{f(-1+h)-f(-1)}{h}$ as h approaches 0 separately.)
   (b) Find a formula for $f'(x)$ when $x \neq -1$, and draw the graph of $f'(x)$ on the interval $-3 \leq x \leq 1$.
(4) Let f and g be differentiable functions.
   (a) Express the derivatives of the following functions using f, g and their derivatives:
$$x^2 f(x) - g(x), \qquad f(x^2 - g(x)).$$
   (b) It is given that
$$f(2) = 2 \qquad\qquad\qquad g(2) = 3$$
$$f'(2) = -2 \qquad\qquad\qquad g'(2) = 1.$$
   Find the equation of the tangent line to the graph of $y = \frac{f(x)+1}{g(x)+1}$ at $x = 2$. (Recall: The equation of a line with slope m passing through a point (a, b) is $y - b = m(x - a)$.)
(5) Peter Pan sells flying powder. Denote by q the amount of power produced (in grams) and p the price (in dollars) of one gram of power. It is given that:
   (i) p and q are related via $p^2 + q^2 = 5000$.
   (ii) The cost of producing q grams of powder is $C(q) = 1000 + 10q$.
   Answer the following:
   (a) Find the revenue (R) and profit (P). Express them as functions of q.
   (b) Find the marginal cost and marginal revenue. Express them as functions of q.

(c) Suppose $q = 50$. What is the marginal revenue and marginal cost? Does increasing q increases the profit?