ASSIGNMENT 2

Due on: September 30 at 8:00 in the morning (submit before class begins). Be sure to write your name and UBC ID on your assignment.

Questions:

- (1) Compute the following limits:
- (a) $\lim_{x \to -2} \frac{x^2 + 4x + 4}{x^2 + 3x + 2}$ (b) $\lim_{x \to -2} \frac{x^2 + 2x + 2}{x^2 + x + 2}$ (c) $\lim_{x \to -2} \frac{x^3 + 8}{x^2 4}$ (Hint: $a^3 + b^3 = (a + b)(a^2 ab + b^2)$) (d) $\lim_{x \to 1} \frac{\sqrt{x^2+3}-2}{x-1}$ (e) $\lim_{x \to 0} \frac{x(x+2)}{\sqrt{x^2+x+1}-\sqrt{x^2+1}}.$ (2) Fact: $1 + x \le e^x \le 1 + x + x^2$ for all $0 \le x \le 1.$ Use the fact and the squeeze theorem to prove that $\lim_{x \to 0^+} \frac{e^x-1}{x} = 1.$
- (3) **Fact:** $\lim_{x \to 0} \frac{\sin x}{x} = 1.$

Use the fact and limit rules to compute $\lim_{x\to 0} \frac{x^2+x^3}{(\sin x)^2}$.

- (4) A ball is thrown vertically into the air. It is given that the height of the ball after t seconds is $20t - 5t^2$ meters.
 - (a) When will the ball hit the ground? (The ground is height 0.)
 - (b) Find the average speed of the ball on the interval $1 \le t \le 2$.
 - (c) Compute (according to the definition) the instantaneous speed of the ball at t = 0 and t = 1.
- (5) Let

$$f(x) = \begin{cases} a \cdot \sqrt{x} & x > 1\\ \frac{x+b}{x^2+1} & x \le 1 \end{cases}$$

(a and b are parameters).

- (a) Is f(x) continuous on the left at x = 1? Explain.
- (b) What equation should a and b satisfy for f(x) to be continuous at x = 1?
- (c) Give values of a and b for which f(x) is continuous at x = 1.
- (d) Give values of a and b for which f(x) is not continuous at x = 1.
- (6) Use the Intermediate Value Theorem to show that the equation $3^x x = 3$ has at least two solutions, one greater than 0 and one smaller than 0.