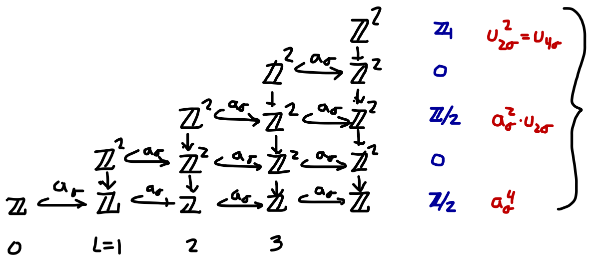
We have a filtration of $MU^{((G))}$ (& $\Omega_\mathbb{O}$) such that $Gr = A \wedge H\underline{\mathbb{Z}} = \bigvee \left(C_{8+} \underset{H_p}{\wedge} S^{\frac{|p|}{|H_p|}\rho_{H_p}}\right) \wedge H\underline{\mathbb{Z}}$. To

All that's left is to show π_* is 256 periodic & the fixed & homotopy fixed points agree.

$K_{\mathbb{R}}$ is again a good toy model: $Gr(K_{\mathbb{R}}) = \bigvee_{\substack{p \in \text{ monic} \\ \text{monomials} \\ \text{in } \mathbb{Z}[\bar v_1^{\pm 1}]}} S^{\frac{|p|}{2}\rho_2} \wedge H\underline{\mathbb{Z}}$, and have maps $\bar v_1^{\pm 1}: S^{\pm\rho_2} \to K_{\mathbb{R}}$ ($\bar v_1$ represents $\mathbb{CP}^1$).

So the set-up is the same!

Look at $S^{4\rho_2} \wedge H\underline{\mathbb{Z}}$:



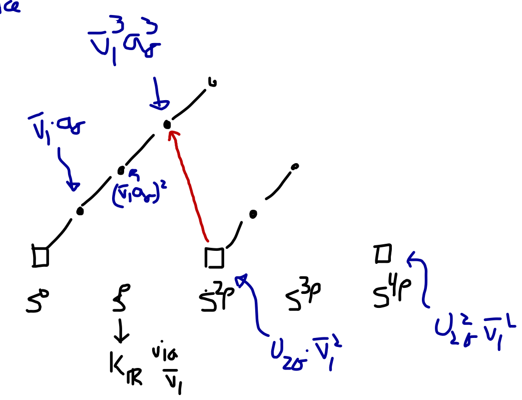
If V is an orientable G-rep, then we have a fundamental class $u_V \in H_{\dim V}(S^V; \underline{\mathbb{Z}})$.

Properties: ① $u_1 = 1$
② $\operatorname{res} u_V = u_{\operatorname{res} V}$
③ $u_{V \oplus W} = u_V \cdot u_W$
④ $(N_H^G u_V) \cdot u_{\operatorname{Ind}_H^G d} = u_{\operatorname{Ind} V}$.

$\left(S^d \to S^V \wedge H\underline{\mathbb{Z}} \rightsquigarrow S^{\operatorname{Ind}_H^G d} \wedge H\underline{\mathbb{Z}} \to S^{\operatorname{Ind} V} \wedge H\underline{\mathbb{Z}} \right)$, with $S^{d[G:H]} \to S^{\operatorname{Ind}_H^G d} \wedge H\underline{\mathbb{Z}}$ and $S^{d[G:H]} \to S^{\operatorname{Ind} V} \wedge H\underline{\mathbb{Z}}$

For $G = C_{p^n}$, any element in $H_*(S^V; \underline{\mathbb{Z}})$ can be written as $a_W \cdot u_{V-W}$.

Here is the slice E_2-term in the range computed



This is a SS of algebras:

① a_σ is in the Hurewicz image $\Rightarrow$ perm cycle
② $\bar v_1$ is a map $S^\rho \to K_{\mathbb{R}}$ $\Rightarrow$ perm cycle
③ $u_{2\sigma}$ comes from diff quotient $\Rightarrow$ not nec a perm cycle.

So only one choice! $u_{2\sigma}$ determines all diffs!

$d_3(u_{2\sigma} \cdot \bar v_1^2) = (\bar v_1 a_\sigma)^3$
$\Rightarrow d_3(u_{2\sigma}) = \bar v_1 a_\sigma^3$.

OK. So why does this help us? If $\bar v_1$ is a unit, then we get $d_3(u_{2\sigma}/\bar v_1) = a_\sigma^3$ & no way for $a_\sigma^3 \neq 0$.

So Cor For $K_{\mathbb{R}}$, $K_{\mathbb{R}}^{\Phi C_2} \simeq *$, and $K_{\mathbb{R}}^{C_2} \simeq K_{\mathbb{R}}^{hC_2}$. This was Atiyah's result!

We could determine this differential as well if we knew the geom. fixed points. The same is true for $MU_{\mathbb{R}}$, where we know $\Phi^G MU_{\mathbb{R}} = MO$. This determines everything.

For $MU_{\mathbb{R}}$, we just have more book keeping. We've all of the classes $\bar x_i$, a_σ, and $u_{2\sigma}$, and the E_2-page is freely generated by them. The $\bar x_i$ are God-given homotopy classes, as is a_σ. So the only way to get MO from $MU_{\mathbb{R}}$ is to have differentials on the powers of $u_{2\sigma}$

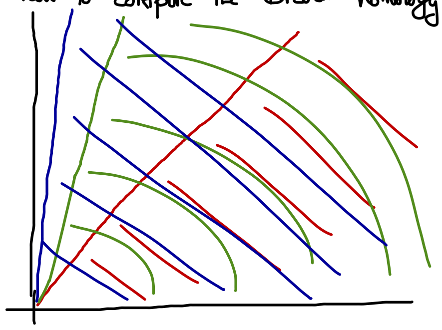
$u_{2\sigma}^{2^i} \longmapsto \bar x_{2^{i+1}-1} \, a_\sigma^{2^{i+2}-1}$. ← This is the best kind of SS argument. Classes have to die & there is only one way to do it. No choices!

For $MU^{(G)}$, it's a little more cumbersome, with all the classes. But we can still use the same basic results:

① $\phi^G(X) = \left(X[a_\sigma^{-1}]\right)^G$
② $\phi^G(MU^{(G)}) = MO$
} Exactly like for C_2

Now recall the slice filtration on $MU^{(G)}$: $Gr(\ ) = \bigvee_{\pi_*^u MU^{(G)}} C_{8+} \underset{H_p}{\wedge} S^{\frac{|H|}{|H|}\rho_H} \wedge H\underline{\mathbb{Z}}$.

We know how to compute the Bredon homology of all of the pieces. Now we just put it together:



$C_{8+} \underset{C_2}{\wedge} (-)$: slope 1 vanishing line
$C_{8+} \underset{C_4}{\wedge} (-)$: slope 3 vanishing line
} Killed by ϕ^G, since induced
$C_{8+} \underset{C_8}{\wedge} (-)$: slope 7 vanishing line ← parametrized by $\mathbb{Z}[N_{C_2}^{C_8} \bar{r}_1, \dots] \cong \pi_*^u MU$

So even though there is much more noise, the parts seen by ϕ^G are just as before (but sheared up to between the slope 3 & 7 lines)

Thm: ① There are differentials: $u_{2\sigma}^{2^i} \longmapsto N(\bar{r}_{2^{i+1}-1})\, a_- a_\sigma^{2^{i+1}}$.

② If the target of this differential is zero, then $u_{2\sigma}^{2^i}$ is a perm cycle.

The first part is the only way for ϕ^G to yield MO, the second is because this is the last possible target.

Cor: If $N(\bar{r}_{2^i-1})$ is a unit, then

ⓐ $u_{2\sigma}^{2^i}$ is a perm cycle &

ⓑ $\phi^G(\ ) \simeq *$

This explains ways we can force $(\ )^G \simeq (\ )^{hG}$: invert $N(\bar{r}_{2^i-1})$ for various subgroups.