Basic yoga: chain complexes are a good model for understanding spectra.

Ch_*	$\mathcal{S}p$
seq. of R-mod + ∂ maps	seq of spaces + structure maps
Have a chain complex of maps	have a space/spectrum of maps
finite sums = finite products	finite wedges $\simeq$ finite products (connectivity of cofiber $\to \infty$)

Mirror all of this w/ G-action: Big addition G-indexed sums)

We have 2 choices for the morphisms: G-maps or all maps, & $F_G(X,Y) = F(X,Y)^G$
(so we lose data)

This gives 2 category structures, with the former being the fixed points of the latter.

$H \leq G$, $i_H^*: \mathcal{S}_G \to \mathcal{S}_H$ is just "forget all but H-actions". This has both adjoints:

$G_+ \wedge_H (-) =$ Induction = left adjoint & $\bigvee_{\alpha \in G/H} X_\alpha$

$F_H(G_+, -) =$ Coinduction = right adjoint $\prod_{\alpha \in G/H} X_\alpha$ G acts on index too!

Now we get a second choice: we have a natural map $\bigvee_{G/H} X \to \prod_{G/H} X$.

A property of spectra we usually like is "sums are products". We can include this as data or not.

3 Equiv Results: ① $G_+ \wedge_H (-) \xrightarrow{\simeq} F_H(G_+, -)$

② If V is a rep, S^V is invertible

③ $D(G/H) = G/H$ (Atiyah duality for G-manifolds)

Now, up to $\simeq$, every spectrum can be built out of spheres. In the equiv context, induct. tells us we need more: $G/H_+ \wedge S^n$, $n \in \mathbb{Z}$. These, by the adjunction property, exactly have
$F_G(G/H_+ \wedge S^n, X) = F_H(S^n, X) = F(S^n, X^H)$.

Our assumption gives something weirder: transfers. $[S^0, G/H_+]_G = [S^0, DG/H_+]_G = [G/H_+, S^0]_G = [S^0, S^0]_H$

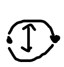
This is decidedly not 0, and the element 1 is the "transfer".

As you've seen, this makes everything Mackey functor valued. If $\underline{M}$ is a Mackey functor, then
$H_*(X; \underline{M})$ is the Bredon homology & $H^*(X; \underline{M}) = H_{-*}(DX; \underline{M})$ is the cohomology.

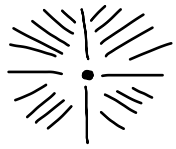
$G = C_{2^n}$, $\underline{M} = \underline{\mathbb{Z}}$. We can write down explicit chain complexes to compute these!

① Find a convenient cell-structure on S^V
② Write down the cellular chains.

Ex: σ the sign rep. S^σ has a cell structure

 $= C(C_{2+} \to S^0)$. So what chain complex do we get?

$\underset{0}{\mathbb{Z}} \xleftarrow{\nabla} \underset{1}{\mathbb{Z}^2}$ & map is determined by the requirement that $i_e^* S^\sigma = S^1$.

Ex: S^λ $\lambda : C_{2^n} \hookrightarrow S^1$.  $= S^0 \cup C_{2^n_+} \wedge e^1 \cup C_{2^n_+} \wedge e^2$. $\rightsquigarrow$ $\mathbb{Z} \xleftarrow{\nabla} \mathbb{Z}^{2^n} \xleftarrow{1-\gamma} \mathbb{Z}^{2^n}$

What about bigger ones?

Method:

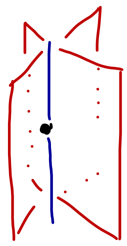
Let $k_i = \dim(V^{C_{2^{n-i}}})$. Then $k_0 \le k_1 \le \dots \le k_n = \dim V$. Build a chain complex $C_m(S^V)$:

$C_m(S^V) = \mathbb{Z}[C_{2^n}/C_{2^{n-i}}]$ $\quad k_{i-1} < m \le k_i$. The maps are forced by the requirement that $H_*(C_\cdot(S^V)) = \tilde{H}_*(S^{k_n})$.

Ex: $V = \rho - 1 = \bar{\rho}$, C_4 : $\quad k_0 = 0 \le k_1 = 1 \le k_2 = 3$:

$= \sigma + \lambda$

$$\underset{0}{\mathbb{Z}} \xleftarrow{\nabla} \underset{1}{\mathbb{Z}^2} \xleftarrow{1-\gamma} \underset{2}{\mathbb{Z}^4} \xleftarrow{1+\gamma} \underset{3}{\mathbb{Z}^4}$$



$2\rho - 1$: $\quad 1 \le 3 \le 7$

$$\underset{1}{\mathbb{Z}} - \underset{2}{\mathbb{Z}^2} - \underset{3}{\mathbb{Z}^2} - \mathbb{Z}^4 - \mathbb{Z}^4 - \mathbb{Z}^4 - \mathbb{Z}^4$$

This can always be done. Why? Linearly ordered s.g.

For S^{-V}, just take the dual complex.

Thm: $H_{-1,-2,-3}(S^{\pm k\rho}; \mathbb{Z}) = 0$.