A Bijection Between Permutations and Floorplans, and its Applications

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Abstract

A *floorplan* represents the relative relations between modules on an integrated circuit. Floorplans are commonly classified as *slicing*, *mosaic*, or general. *Separable* and *Baxter* permutations are classes of permutations that can be defined in terms of forbidden subsequences. It is known that the number of slicing floorplans equals the number of separable permutations and that the number of mosaic floorplans equals the number of Baxter permutations [17]. We present a simple and efficient bijection between Baxter permutations and mosaic floorplans with applications to integrated circuits design. Moreover, this bijection has two additional merits: (1) It also maps between separable permutations and slicing floorplans; and (2) It suggests enumerations of mosaic floorplans according to various structural parameters.

Keywords: Baxter permutations, separable permutations, mosaic floorplans, slicing floorplans.

1 Introduction

During the physical design process of an integrated circuit, one determines the shape, size, and position on chip of every module. The shape of the chip and that of each of the modules (blocks) is usually a rectangle. A *floorplan* describes the relative positions of the blocks, thus it is often represented by a partition (dissection) of a rectangle by non-intersecting segments into m rectangles (rooms) such that there is a one-to-one mapping from the n ($\leq m$) blocks to the rooms. In a *mosaic* floorplan there are no empty rooms, that is, $n = m$. A special kind of mosaic floorplans are *slicing* floorplans (here we follow the definition in [14]) in which the subdivision to rectangles can be obtained by recursively cutting either vertically or horizontally a rectangle into two smaller rectangles. Slicing floorplans can also be characterized as mosaic floorplans that do not contain a 'pin-wheel' structure. See Figure 1 for examples of general, mosaic, and slicing floorplans.

Separable and *Baxter* permutations are classes of permutations that can be defined in terms of forbidden subsequences. A separable permutation can be defined as a permutation that does not contain a subsequence of four elements with the same pairwise comparison as 2413 or 3142 (an alternative definition is given in Section 2.2). A Baxter permutation has a similar forbidden condition, but it can contain such a subsequence if the absolute difference

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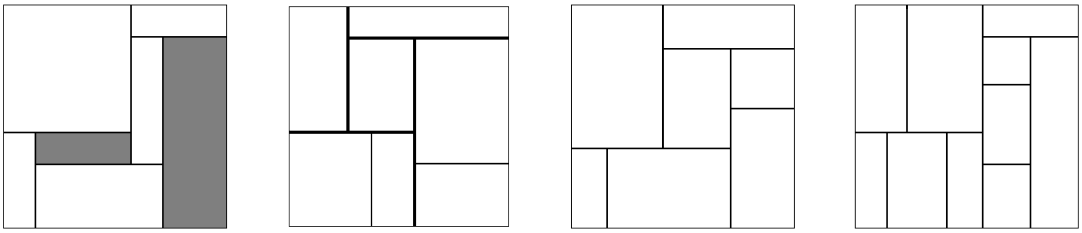


(a) General (empty rooms are shaded)

(b) Mosaic (a clockwise pin-wheel structure is shown in bold)

(c) Mosaic

(d) Slicing

Figure 1: Floorplans (b and c are equivalent).

between the first and last element in the subsequence is greater than one (a more formal definition appears in Section 2.1). Thus, separable permutations are a subclass of Baxter permutations.

Sakanushi et al. [12] were the first to consider the number of distinct mosaic floorplans. They found a recursive formula for this number, but did not recognize it to be the same formula suggested by Chung et al. [5] in their analysis of the number of Baxter permutations. Yao et al. [17] showed a bijection between mosaic floorplans and *twin binary trees* whose number is known [6] to be the number of Baxter permutations. They have also shown that the number of slicing floorplans containing n blocks is the nth Schröder number.

A connection between floorplans and permutations was first presented by Murata et al. [9], who suggested representing floorplans as a pair of permutations (*sequence-pair*). In a later work, Murata et al. [10] described a mapping from sequence-pairs to floorplans. From this mapping one can deduce a mapping from Baxter permutation to mosaic floorplans. Recently and independently, Kajitani [8] has suggested representing a floorplan by a permutation (*single-sequence* in his terminology) and explored, along with others, the properties and advantages of this simple representation [19, 20, 21, 22]. Among other things they showed mappings between (mosaic) floorplans and (Baxter) permutations.

In this work we present another bijection between Baxter permutations and mosaic floorplans. This bijection is direct, as opposed to the bijection that can be deduced from the work of Yao et al. [17] and the work of Dulucq and Guibert [6]. The mapping from permutations to floorplans we suggest is much simpler and more efficient than the mapping described in [9]. Comparing with the mappings suggested recently in [19] and [20], our mapping is as efficient (has a linear time and space complexity) and at least as simple. Furthermore, the mapping algorithm can easily find the direct neighbors of every block, with performances matching that of the algorithm suggested in [22]. This information is useful for the actual placement of the blocks. The bijection we describe has the following additional merits: First, it maps separable permutations to slicing floorplans. Second, by combining it with known results about Baxter permutations, we obtain enumerations of mosaic floorplans according to various structural parameters, such as the number of vertical segments in the partition and the number of blocks on the boundary of the floorplan. Some of our results appeared in a preliminary form in [1].

The paper is organized as follows. In Section 2 we give some background on Baxter and separable permutations and define an equivalence relation on (mosaic) floorplans. Then we show in Section 3 the bijection between Baxter permutations and (equivalence classes of) mosaic floorplans and discuss its applications. In Section 4 we explore the enumeration of mosaic floorplans according to various parameters. Finally, we discuss our results in Section 5.

2 Preliminaries

In order to distinguish between different (mosaic) floorplans we must first define when two floorplans are considered equivalent. Here we follow the definition of Sakanushi et al. [12]. Given a floorplan f a segment s *supports* a room r in f if s contains one of the edges of r. We say that s and r hold a *top*-, *left*-, *right*-, or *bottom-seg-room* relation if s supports r from the respective direction. Two floorplans are equivalent if there is a labeling of their rectangles and segments such that they hold the same seg-room relations. Thus, for example, the floorplans in Figures 1(b) and 1(c) are equivalent.

2.1 Baxter Permutations

A Baxter permutation on $[n] = 1, 2, \ldots, n$ is a permutation $\pi = (\sigma_1\sigma_2 \ldots \sigma_n)$ for which there are no four indices $1 \le i < j < k < l \le n$ such that

1. $\sigma_k < \sigma_i + 1 = \sigma_l < \sigma_j$; or
2. $\sigma_j < \sigma_l + 1 = \sigma_i < \sigma_k$.

For example, for $n = 4$, 3142 and 2413 are the only non-Baxter permutations. This class of permutations was introduced by Baxter [3] in the context of fixed points of the composite of commuting functions. The nth Baxter number, $\mathrm{B}(n)$, is the number of Baxter permutations on $[n]$. Chung et al. [5] showed that

$$\mathrm{B}(n) = \sum_{r=0}^{n-1} \frac{\binom{n+1}{r}\binom{n+1}{r+1}\binom{n+1}{r+2}}{\binom{n+1}{1}\binom{n+1}{2}}$$

Dulucq and Guibert [6] showed one-to-one correspondences between Baxter permutations, *twin binary trees*, and some type of three non-intersecting paths on a grid. Shen et al. [14] analyzed the asymptotic behavior of the Baxter numbers and proved that $\mathrm{B}(n) = \Theta(8^n/n^4)$. The first Baxter numbers (starting from $n = 0$) are $\{0, 1, 2, 6, 22, 92, 422, 2074, \ldots\}$.

2.2 Separable Permutations

Let $\pi_1 = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\pi_2 = (\beta_1, \beta_2, \ldots, \beta_m)$ be two permutations on $[n]$ and $[m]$, respectively. We say that $\pi = (\sigma_1, \sigma_2, \ldots, \sigma_{n+m})$ is the result of concatenating π_2 *above* π_1 if $\pi_i = \alpha_i$ for $1 \le i \le n$ and $\pi_{n+i} = n + \beta_i$ for $1 \le i \le m$. Likewise, we say that $\pi = (\sigma_1, \sigma_2, \ldots, \sigma_{n+m})$ is the result of concatenating π_2 *below* π_1 if $\pi_i = m + \alpha_i$ for $1 \le i \le n$ and $\pi_{n+i} = \beta_i$ for $1 \le i \le m$.

A permutation π is *separable* if either

1. $\pi = (1)$; or
2. There are two separable permutations π_1 and π_2 such that π is the the concatenation of π_2 above or below π_1.

Bose et al. [4] coined the term separable permutations and showed a polynomial-time algorithm for finding a given sub-permutation P within a permutation T, where P is separable. A similar definition was suggested by Shapiro and Stephens [13] in their analysis of permutation-matrices that eventually fill up under *bootstrap percolation*. They have also shown that the number of separable permutations on $[n]$ is the $(n-1)$st Schröder number[1]. Avis and Newborn [2] showed that separable permutations are exactly the permutations that can be sorted by an unbounded sequence of pop-stacks (in a pop-stack the pop operation unloads the entire stack).

Another characterization of separable permutations is in terms of *forbidden subsequences*. A permutation $\pi = (\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n) \in S_n$ *avoids* a certain sub-permutation $\tau \in S_k$ (for $k \leq n$) if it does not contain a subsequence $(\sigma_{i_1}, \sigma_{i_2}, \ldots, \sigma_{i_k})$ with the same pairwise comparisons as τ. The set of permutations on $[n]$ avoiding τ is denoted by $S_n(\tau)$. It can be shown [4] that the set of separable permutations is equal to $S_n(3142, 2413)$, suggesting an alternative proof [16] that their number is the $(n-1)$st Schröder number.

3 The Bijection

In this section we show a direct and simple bijection between Baxter permutations and mosaic floorplans. In Section 3.1 we describe a mapping from mosaic floorplans to Baxter permutations, while in Section 3.2 we present a mapping in the other direction and thus show that these two mappings define a bijection.

3.1 Mapping Mosaic Floorplans to Baxter Permutations

In this section we describe a mapping from mosaic floorplans to permutations. It is essentially the same mapping presented implicitly in [12] and explicitly in [19], however, we describe it here for completeness and prove that it always produces a Baxter permutation.

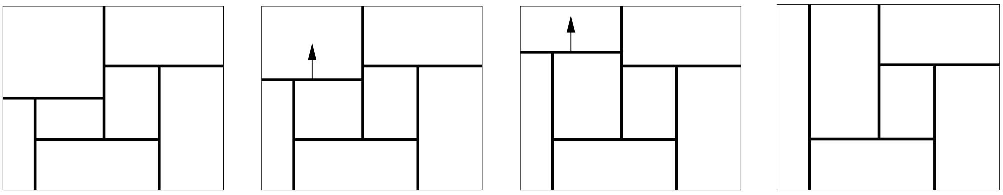
Given a mosaic floorplan of n blocks (rectangles) we can obtain a mosaic floorplan of $n-1$ blocks by using the *block deletion* operation introduced by Hong et al. [7].

Definition 3.1 (block deletion) *Let f be a mosaic floorplan with $n > 1$ blocks and let b be the top-left block in f. If the bottom-right corner of b is a '$\dashv$'- (resp., '$\perp$'-) junction, then one can delete b from f by shifting its bottom (resp., right) edge upwards (resp., leftwards), while pulling the T-junctions attached to it until the edge hits the bounding rectangle.*

See Figure 2 for an example of the block-deletion operation. Note that we can delete in a similar manner a block from any corner of a floorplan. Using the block-deletion operation we now define a mapping from mosaic floorplans to Baxter permutations.

For example, the permutation that corresponds to the floorplan in Figure 3 is 521463. Before we show that the output of Algorithm `FP2BP` is always a Baxter permutation, we need

[1]The Schröder numbers arise in numerous other enumerative combinatorial problems [15, pp. 239–240]. One example is the number of paths on an orthogonal grid from $(0,0)$ to (n,n) that do not go above the line $y = x$ and use only the steps $(1,0)$, $(0,1)$, and $(1,1)$. When denoting by r_n the nth Schröder number, we have $r_n = \sum_{k=0}^{n} \binom{2n-k}{k} C_{n-k}$, where C_n is the nth Catalan number. It can be shown (see, e.g., [14]) that $r_n = \Theta\left((3+\sqrt{8})^n / n^{1.5}\right)$. The first Schröder numbers (starting from $n = 0$) are $\{0, 1, 2, 6, 22, 90, 394, 1086, 8558, \ldots\}$.



(a) (b) (c) (d)

Figure 2: Block deletion

Input: A mosaic floorplan f with n blocks.
Output: A (Baxter) permutation on $[n]$.

1: Label the blocks of f according to their deletion order from the top-left corner;
2: Return the permutation of labels obtained by deleting the blocks of f from the bottom-left corner.

Algorithm 1: Algorithm `FP2BP`

the following definition and observations.

Definition 3.2 *Let f be a mosaic floorplan and let b_1 and b_2 be two blocks in f. We say that b_1 is* left of *(resp.,* above*) b_2 if there is either 1. a segment which contains the right (resp., lower) edge of b_1 and the left (resp., upper) edge of b_2; or 2. a block b' such that b_1 is left of (resp., above) b' and b' is left of (resp., above) b_2. If a block b_1 is left-of (resp., above) block b_2 by the first rule, then b_1 is* directly *left-of (resp., above) b_2.*

Observation 3.3 ([10, Property 5]) *Let f be a mosaic floorplan and let b_1 and b_2 be two blocks in f. Then exactly one of the following relations holds: b_1 is left of b_2, b_1 is above b_2, b_2 is left of b_1, or b_2 is above b_1.*

Observation 3.4 *If a block b_1 precedes a block b_2 according to the top-left corner-deletion order and b_2 precedes b_1 according to the bottom-left corner-deletion order, then b_1 is above b_2. Similarly, if b_1 precedes b_2 according to both orders, then b_1 is left of b_2.*

Proof: Notice that when a block is deleted from the top-left corner it is to the left or above any other block in the floorplan. Additionally, the relation between any two blocks remains

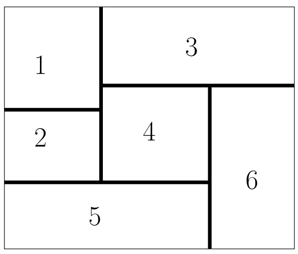


Figure 3: Applying Algorithm `FP2BP` on this floorplan yields the permutation 521463.

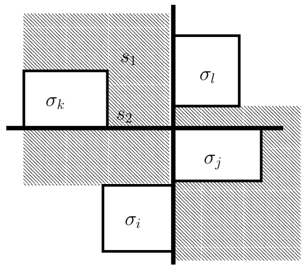


Figure 4: An illustration for the proof of Lemma 3.6.

the same after applying the deletion operation. Therefore, if a block b_1 precedes a block b_2 according to the top-left (resp., bottom-left) corner-deletion order, then b_1 is to the left of or above (resp., below) b_2. Hence the claim follows. $\square$

The following observation is easy.

Observation 3.5 *If a block b_1 follows immediately a block b_2 according to one of the orders, then there is a segment that contains edges of both b_1 and b_2.*

Next, we prove that Algorithm FP2BP always produces a Baxter permutation.

Lemma 3.6 *Given a mosaic floorplan f with n blocks, the permutation π obtained by applying Algorithm FP2BP on f is a Baxter permutation on $[n]$. Moreover, if f is a slicing floorplan, then π is a separable permutation.*

Proof: Suppose $\pi = (\sigma_1\sigma_2 \ldots \sigma_n)$ is not a Baxter permutation. Then there are four indices $1 \le i < j < k < l \le n$ such that either 1. $\sigma_k < \sigma_i + 1 = \sigma_l < \sigma_j$; or 2. $\sigma_j < \sigma_l + 1 = \sigma_i < \sigma_k$. Assume that the first case holds, and choose j and k such that $k = j + 1$. According to Observations 3.4 and 3.5, block σ_i is left of block σ_l, and some segment s_1 supports both blocks. Similarly, block σ_j is below block σ_k, and some segment s_2 supports both blocks. According to Observation 3.4, block σ_k is to the left of block σ_l and above block σ_i. Similarly, block σ_j is to the right of block σ_i and below block σ_l. Thus, s_1 and s_2 must intersect (see Figure 4). The proof in the second case is similar and is thus omitted.

Now suppose f is a slicing floorplan, and let s be the segment that cuts the bounding rectangle of f into two. Suppose s is horizontal, and denote by f_1 the m blocks above s and by f_2 the $n - m$ blocks below s. Then, the blocks in f_1 precede the blocks of f_2 according to the top-left deletion order, and follow them according to the bottom-left deletion order. By induction, the blocks in f_1 form a separable permutation on $1, \ldots, m$, and the blocks in f_2 form a separable permutation on $m + 1, \ldots, n$. Thus, by definition, π is a separable permutation. The proof for the case in which s is vertical is similar. $\square$

Next we show that the mapping defined by Algorithm FP2BP is one-to-one.

Lemma 3.7 *Let f_1 and f_2 be two mosaic floorplans, each containing n blocks, and let π_1 and π_2 be the permutations produced by Algorithm FP2BP when applied to f_1 and f_2, respectively. Then, if $f_1 \neq f_2$ then $\pi_1 \neq \pi_2$.*

Proof: We prove by induction on n that if $\pi_1 = \pi_2$ then $f_1 = f_2$. Let b_1 (resp., b_2) be the first block which is removed from the top-left corner of f_1 (resp., f_2), and let s_1 (resp., s_2)be the segment that is shifted in the course of this action. Then s_1 and s_2 must have the same orientation, otherwise the numbers 1 and 2 would have different orders in π_1 and π_2. Let f'_1 and f'_2 be the resulting floorplans after the deletion. The permutation that corresponds to f'_1 and f'_2 is the permutation obtained from π_1 by deleting the number 1 and decreasing every remaining number by 1. Thus, by the induction hypothesis, $f'_1 = f'_2$. It remains to verify that when we reverse the deletion operation, then the same number of blocks are "pushed" by s_1 and s_2. Indeed, if this number is different then there is a block x which is pushed in one floorplan, say f_1, but not on the other floorplan. Thus, x is to the left of block 1 in f_1 while it is below block 1 in f_2. It follows that 1 and x will have different orders in π_1 and π_2. □

3.2 Mapping Baxter Permutations to Mosaic Floorplans

Input: A Baxter permutation $\pi = (\sigma_1\sigma_2 \dots \sigma_n)$.
Output: A mosaic floorplan with n blocks.

Draw a block and name it σ_1;
Construct an $n \times n$ grid within the block;
for $i = 2$ to n **do**
  if $\sigma_i < \sigma_{i-1}$ **then**
    Slice the top-right block by a horizontal segment at the ith level of the grid;
    Name the new block σ_i;
    while the block σ' to the left of σ_i has a label smaller than σ_i **do**
      Extend block σ_i leftwards (at the expense of σ');
    end while
  else
    Slice the top-right block by a vertical segment at the ith level of the grid;
    Name the new block σ_i;
    while the block σ' below σ_i has a label greater than σ_i **do**
      Extend block σ_i downwards (at the expense of σ');
    end while
  end if
end for

Algorithm 2: Algorithm BP2FP

Given a Baxter permutation on $[n]$ Algorithm BP2FP constructs a mosaic floorplan with n blocks. See Figure 5 for an example. The algorithm simply inserts blocks one by one into the top-right corner of the floorplan. The current block is created by slicing the previous block into two, and is labeled according to the current element in the permutation. If the previous element is smaller (resp., greater) than the current element, then we slice the block vertically (resp., horizontally). The horizontal (resp., vertical) slicing segment is extended leftwards (resp., downwards) as long as the block to the left of it (resp., below) has a smaller (resp., greater) label than the current block.

The output of Algorithm BP2FP is clearly a mosaic floorplan. We show next that Algorithms BP2FP and FP2BP define a one-to-one correspondence (bijection) between Baxter permutations and mosaic floorplans.

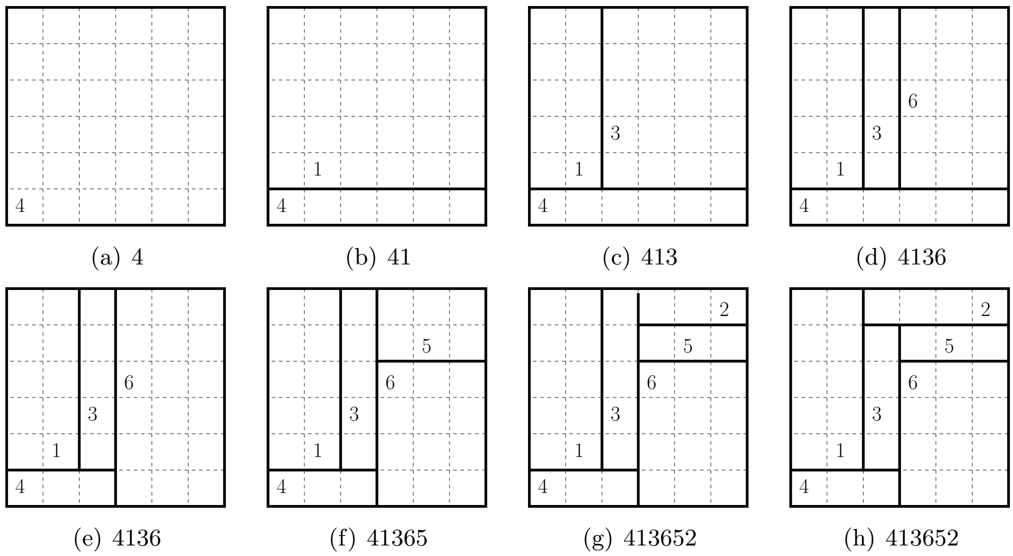


Figure 5: Applying Algorithm BP2FP to the permutation 413652

Theorem 1 *There is a bijection between Baxter permutations on $[n]$ and mosaic floorplans with n blocks. Moreover, it remains a bijection when restricted to separable permutations on $[n]$ and slicing floorplans with n blocks.*

Proof: Let π be a Baxter permutation on $[n]$, and let f be the output of Algorithm BP2FP when it is applied on π. Clearly, f is a valid mosaic floorplan containing n blocks. Let π' be the output of Algorithm FP2BP applied to f. To prove the theorem it is enough to show that $\pi' = \pi$. It is easy to see that during the computation of π', the blocks are deleted from the bottom-left corner of f in the same order they were inserted to the top-right corner of f in the course of Algorithm BP2FP. Therefore, it is sufficient to prove that the order in which the blocks of f are deleted from the top-left corner is $1, 2, \ldots, n$. It is clear that the block labeled 1 is the first removed block (no other block is above or to the left of it). Assume that for every $1 \leq i \leq k$ the block labeled i is the ith removed block from the top-left corner. We now show that the next deleted block is the one labeled $k+1$. Suppose that $k+1$ precedes k in π, that is, $\pi = (\ldots, k+1, A, B, k, \ldots)$, where A is a (possibly empty) sequence of integers that are greater than $k+1$ and B is a (possibly empty) sequence of integers that are smaller than k. (There are no other options since π is a Baxter permutation.) Figure 6(a) shows the floorplan after k was inserted in the course of Algorithm BP2FP. According to the induction hypothesis, all the blocks in B are removed before block k, so when k is removed (from the top-left corner) the left edge of the block labeled $k+1$ is also on the boundary. The bottom-right corner of k is either a '$\vdash$' or '$\perp$' junction. In the first case $k+1$ is clearly the next block to be deleted. For the second case, note that a '$\perp$' junction can be formed only when the first block with a label greater than k and to the right of k in π (denote this block by c) has a smaller label than the block below k and sharing the same segment (as c) as a right edge (denote this block by a). Figures 6(b,c) illustrate the situation before and after the insertion of c. Note that a is the last of the elements of A and $k < c < a$. If A is empty, then $a = k+1$; thus, there cannot be such a block c. Otherwise, there must be an integer i such that $k+1 \leq i \leq c-1$, i is to the left of a in π, and $i+1$ is either c or to the right of c. Therefore, $i, a, k, i+1$ form a

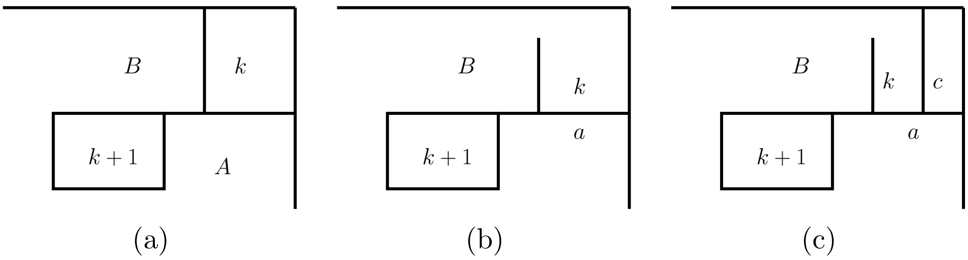


Figure 6: Illustration for the proof of Theorem 1.

forbidden subsequence, and π is not a Baxter permutation. The proof for the case k precedes $k+1$ in π is similar and is thus omitted. $\square$

Definition 3.8 *Given a floorplan f and a block b in f, the* direct relation set *(DRS) of b is the set of blocks that are directly left of, right of, above, or below b.*

The DRS is important for the actual placement of the blocks on the chip, once their dimensions are set [22].

Theorem 2 *Let π be a Baxter permutation on $[n]$ and let f be its corresponding floorplan. Then f and the DRS of every block in f can be computed in $O(n)$ time.*

Proof: Algorithm `BP2FP` inserts n blocks one after the other. It is easy to update the DRS of the currently inserted block and of its neighbors while the block is being extended at their expense. For a certain block there could be many update operations, but every block can be chopped at most once from above and at most once from right. Hence, the total number of update operations is $O(n)$. $\square$

Some of the problems which are hard to solve for general or even mosaic floorplans are easier for slicing floorplans, due to their simple structure. Therefore, sometimes one wishes to determine whether a given floorplan is slicing [23]. When a floorplan f is represented as a permutation π, by Theorem 1 f is slicing if and only if π is separable. Thus, it can be determined in a linear time whether f is a slicing floorplan using the algorithm suggested in [4, pp. 282] to test if a permutation is separable.

4 Enumeration of Mosaic Floorplans According to Various Parameters

Baxter permutations are known to be enumerated according to various parameters [6, 11]. Algorithm `BP2FP` along with those results suggest enumerations of mosaic floorplans according to various parameters such as the number of vertical segments, and the number of blocks on the boundary of the floorplan.

Definition 4.1 (rise) *Given a permutation $\pi = (\sigma_1\sigma_2\ldots\sigma_n)$, a* rise *(resp.,* descent*) in π is sequence of two consecutive elements $\sigma_i\sigma_{i+1}$ such that $\sigma_i < \sigma_{i+1}$ (resp., $\sigma_i > \sigma_{i+1}$).*

According to Algorithm BP2FP, every rise in the input permutation is mapped to a vertical segment in the output floorplan. Mallows [11] considered the enumeration of Baxter permutations according to the number of rises. The next corollary follows from his result.

Corollary 4.2 *The number of mosaic floorplans with n blocks and r vertical segments is*

$$\frac{\binom{n+1}{r}\binom{n+1}{r+1}\binom{n+1}{r+2}}{\binom{n+1}{1}\binom{n+1}{2}}.$$

Definition 4.3 (left-to-right minimum/maximum) *Let $\pi = (\sigma_1\sigma_2\ldots\sigma_n)$ be a permutation on $[n]$. An element σ_k is a* left-to-right minimum *(resp.,* maximum*) if $\sigma_k < \sigma_i$ (resp., $\sigma_k > \sigma_i$) for every $1 \le i < k$.*

Algorithm BP2FP maps every left-to-right minimum to a block touching the left edge of the boundary of the output floorplan f. Similarly, every left-to-right maximum in π is mapped to a block touching the bottom edge of the boundary of f. Thus, according to a result of Dulucq and Guibert [6, Theorem 1] we have:

Corollary 4.4 *The number of mosaic floorplans with n blocks, r vertical segments, i blocks touching the left edge of the boundary of the floorplan, and s blocks touching the bottom edge of the boundary of the floorplan is*

$$\binom{n+1}{r+1}\frac{si}{n(n+1)}\left(\binom{n-s-1}{n-r-2}\binom{n-i-1}{r-1} - \binom{n-s-1}{n-r-1}\binom{n-i-1}{r}\right).$$

Dulucq and Guibert have considered the enumeration of Baxter permutations according to two other parameters.

Definition 4.5 [6, Definition 4] *Given a permutation $\pi = (\sigma_1\sigma_2\ldots\sigma_n)$,*

- *$rd(\pi)$ is the number of rises in $\sigma_i\sigma_{i+1}\ldots\sigma_n$, where $i = \max\{j|\exists k \ge 2 : \sigma_j < \sigma_{j+k} < \sigma_{j+1} < \ldots < \sigma_{j+k-1}\}$ with $\sigma_0 = -1$ and $\sigma_{n+1} = 0$.*
- *$dd(\pi)$ is the number of descents in $\sigma_i\sigma_{i+1}\ldots\sigma_n$, where $i = \max\{j|\exists k \ge 2 : \sigma_j > \sigma_{j+k} > \sigma_{j+1} > \ldots > \sigma_{j+k-1}\}$ with $\sigma_0 = n+2$ and $\sigma_{n+1} = n+1$.*

We define below the corresponding parameters for a mosaic floorplan f.

Definition 4.6 *Let f be a mosaic floorplan and let t be a '$\perp$'- (resp., '$\vdash$'-) junction in f. We say that t is the* last *'$\perp$'- (resp., '$\vdash$'-) junction in f if it is the last '$\perp$'- (resp., '$\vdash$'-) junction which is deleted when the blocks of f are removed from the bottom-left corner. Given a horizontal (resp., vertical) segment s in f, we say that s is* above *(resp.,* left of*) t if s is above the horizontal (resp., vertical) segment [2] of t.*

[2] A segment s_1 is *above* a segment s_2 in a floorplan f if: 1. s_1 is vertical and its lower endpoint is on s_2; or 2. s_2 is vertical and its upper endpoint is on s_1; or 3. s_1 and s_2 are both horizontal and contain opposite edges of a block (rectangle) in f. The relation *left of* is defined in a similar manner.

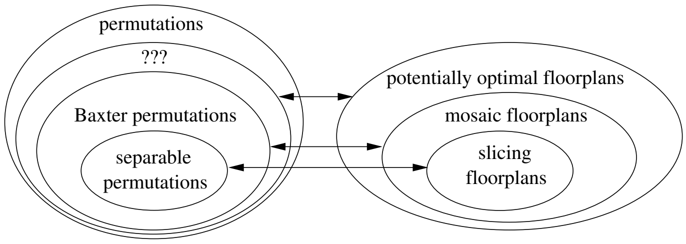


Figure 7: The hierarchy of bijections between permutations and floorplans.

A careful look at the definitions of $rd(\pi)$ and $dd(\pi)$, and the way Algorithm `BP2FP` works leads to the following observation.

Observation 4.7 *Let π be a Baxter permutation on $[n]$ and let f be the floorplan produced by the application of Algorithm `BP2FP` to π. Denote by t the last '$\perp$'- (resp., '$\vdash$'-) junction in f. Then, the number of horizontal (resp., vertical) segments in f above (resp., left of) t is $dd(\pi)$ (resp., $rd(\pi)$).*

It follows from this observation and from Theorem 5 in [6] that:

Corollary 4.8 *The number of mosaic floorplans with n blocks, r vertical segments, i blocks touching the left edge of the boundary of the floorplan, s blocks touching the bottom edge of the boundary of the floorplan, p vertical segments after the last '$\perp$'-junction, and q horizontal segments after the last '$\vdash$'-junction is*

$$\begin{vmatrix} \binom{n-1-i-p}{r-p} & \binom{n-1-p}{r-p} & \binom{n-1-s-p}{r-s-p} \\ \binom{n-1-i}{r} & \binom{n-1}{r} & \binom{n-1-s}{r-s} \\ \binom{n-1-i-q}{r} & \binom{n-1-q}{r} & \binom{n-1-s-q}{r-s} \end{vmatrix}.$$

5 Discussion

We have presented a bijection between Baxter permutations and mosaic floorplans. Moreover, from this bijection we have also deduced a similar correspondence between separable permutations and slicing floorplans (see Figure 7), and have suggested enumerations of mosaic floorplans according to various parameters, such as the number of vertical segments and the number of blocks on certain edges of the boundary of the floorplan. The algorithm we use to map Baxter permutations to mosaic floorplans has applications in integrated circuit (IC) design: it can be used for an easy and efficient construction of a floorplan from the permutation representing it.

Given a non-Baxter permutation, it is possible to convert it to a Baxter permutation by inserting *dummy* elements, as suggested by Murata et al. [10]. The new permutation can then be mapped to a floorplan containing *empty* rooms (which is a general, non-mosaic floorplan) using Algorithm `BP2FP`. Given a permutation $\pi = (\sigma_1\sigma_2 \ldots \sigma_n)$, the elimination of a forbidden subpattern of the form $\sigma_i \ldots \sigma_j\sigma_{j+1} \ldots \sigma_k$, such that $\sigma_{j+1} < \sigma_i + 1 = \sigma_k < \sigma_j$ (resp.,

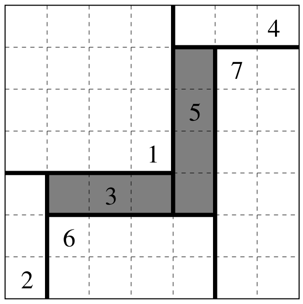


Figure 8: The floorplan with empty rooms corresponding to the non-Baxter permutation 24153.

$\sigma_j < \sigma_k + 1 = \sigma_i < \sigma_{j+1}$), is done by inserting the dummy element σ_k (resp., σ_i) between σ_j and σ_{j+1} and increasing by 1 each of the old elements greater than or equal to σ_k (resp., σ_i). For example, the permutation 2413 is converted to 25**3**14. In the mosaic floorplan that matches the new permutation we mark every block that corresponds to a dummy element as an empty room. Each of these empty rooms is the center of a 'pin-wheel' structure. Figure 8 shows an example with two pin-wheels and their corresponding empty rooms.

Finding a floorplan that minimizes criteria such as area or wire-length is a major problem in IC design. It is well-known that an optimal floorplan might contain empty rooms. However, Young et al. [18] showed that when searching for an optimal floorplan, it is enough to consider floorplans in which every empty room (if such exists) is at the center of pin-wheel structure and has no *room-room* neighbor (that is, a touching room) which is an empty room. Characterizing and enumerating permutations that are mapped to such floorplans is an interesting open problem (as indicated in Figure 7).

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