A.J. Zaslavski, *Algorithms for Solving Common Fixed Point Problems*, Springer International Publishing AG, part of Springer Nature, (**2018**)

Preface

In this book, we study approximate solutions of common fixed point and convex feasibility problems in the presence of perturbations. A convex feasibility problem is to find a point which belongs to the intersection of a given finite family of subsets of a Hilbert space. This problem is a special case of a common fixed point problem which is to find a common fixed point of a finite family of self-mappings of a Hilbert space. The study of these problems has recently been a rapidly growing area of research. This is due not only to theoretical achievements in this area, but also because of numerous applications to engineering and, in particular, to computed tomography and radiation therapy planning. In the book, we consider a number of algorithms, which are known as important tools for solving convex feasibility and common fixed point problems. According to the results known in the literature, these algorithms should converge to a solution. But it is clear that in practice it is sufficient to find a good approximate solution instead of constructing a minimizing sequence. In our recent book Approximate Solutions of Common Fixed Point Problems, Springer, 2016, we analyzed these algorithms and showed that almost all exact iterates generated by them are approximate solutions. Moreover, we obtained an estimate of the number of iterates which are not approximate solutions. This estimate depends on the algorithm but does not depend on the starting point. In this book, our first goal is to generalize these results for perturbed algorithms in the case when perturbations are summable. These generalizations are important because such results find interesting applications and are important ingredients in superiorization and perturbation resilience of algorithms. The superiorization methodology works by taking an iterative algorithm, investigating its perturbation resilience, and then using proactively such perturbations in order to “force” the perturbed algorithm to do in addition to its original task something useful. Our second goal is to study approximate solutions of common fixed point problems in the presence of perturbations which are not necessarily summable. Note that in our recent book mentioned earlier it was shown that if perturbations are small enough, then we have an approximate solution during a certain number of iterates, and an estimate for this number of iterates was obtained. But these results do not show what happens with subsequent iterates, when an approximated solution is obtained. In this book, we show that if our algorithms are cyclic and a computational error is sufficiently small, then beginning from a certain instant of time iterates become approximate solutions. This instant of time depends on the algorithm but does not depend on its starting point. This book contains eight chapters. Chapter 1 is an introduction. In Chapter 2, we study iterative methods in metric spaces. The dynamic string-averaging methods for common fixed point problems in normed space are analyzed in Chapter 3. Dynamic string methods, for common fixed point problems in a metric space, are introduced and studied in Chapter 4. Chapter 5 is devoted to the study of the convergence of an abstract version of the algorithm which is called in the literature as component averaged row projections or CARP. In Chapter 6, we study a proximal algorithm for finding a common zero of a family of maximal monotone operators. In Chapter 7, we extend the results of Chapter 6 for a dynamic string-averaging version of the proximal algorithm. In Chapter 8, subgradient projection algorithms for convex feasibility problems are studied for infinite-dimensional Hilbert spaces. Theorems 2.1 and 3.1 were obtained in [125]. All other results are new.

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