Superiorization and Perturbation Resilience of Algorithms: A Continuously Updated Bibliography

Yair Censor Department of Mathematics University of Haifa Mt. Carmel, Haifa 3498838, Israel (yair@math.haifa.ac.il)

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Abstract

This document presents a, chronologically ordered, bibliography of scientific publications on the superiorization methodology and perturbation resilience of algorithms which is compiled and continuously updated by us at: http://math.haifa.ac.il/yair/bib-superiorizationcensor.html.

Since the topic is relatively new it is possible to trace the work that has been published about it since its inception. To the best of our knowledge this bibliography represents all available publications on this topic to date, and while the URL is continuously updated we will revise this document and bring it up to date on arXiv approximately once a year. Abstracts of the cited works, and some downloadable files of preprints are available on the above mentioned Internet page. If you know of a related scientific work in any form that should be included here kindly write to me on: yair@math.haifa.ac.il with full bibliographic details, a DOI if available, and a PDF copy of the work if possible. The Internet page was initiated on March 7, 2015, and has been last updated on March 6, 2017.

1 Trailer

The superiorization methodology works by taking an iterative algorithm, investigating its perturbation resilience, and then using proactively such perturbations in order to "force" the perturbed algorithm to do in addition to its original task something useful. The perturbed algorithm is called the "superiorized version" of the original unperturbed algorithm. If the original algorithm is computationally efficient and useful in terms of the application at hand, and if the perturbations are simple and not expensive to calculate, then the advantage of this method is that, for essentially the computational cost of the original algorithm, we are able to get something more by steering its iterates according to the perturbations.

This is a very general principle, which has been successfully used in some important practical applications such as image reconstruction from projections, intensity-modulated radiation therapy and nondestructive testing, and awaits to be implemented and tested in additional fields. An important case is when the original algorithm is a feasibility-seeking algorithm, or one that strives to find constraint-compatible points for a family of constraints, and the perturbations that are interlaced into the original algorithm aim at reducing (not necessarily minimizing) a given merit function.

To a novice on the superiorization methodology and perturbation resilience of algorithms we recommend to read first the recent reviews in [25] and [39] below.

For a recent detailed description of previous work that is related to superiorization but is not included in this bibliography we direct the reader to Section 3 of [24] below.

Naturally there is variability among the bibliography items below in their degree of relevance to the superiorization methodology and perturbation resilience of algorithms. In some, such as in, e.g., [23] below, superiorization appears only inside the work [23, Subsection 6.2.1: Optimization vs. Superiorization].

2 The Bibliography

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