M. Yamagishi and I. Yamada, Nonexpansiveness of a linearized augmented Lagrangian operator for hierarchical convex optimization, *Inverse Problems*, Vol. 33 (**2017**), 044003. https://doi.org/10.1088/1361-6420/33/4/044003.

Abstract

Hierarchical convex optimization concerns two-stage optimization problems: the *first stage problem* is a convex optimization; the *second stage problem* is the minimization of a convex function over the solution set of the first stage problem. For the hierarchical convex optimization, the *hybrid steepest descent method* (HSDM) can be applied, where the solution set of the first stage problem must be expressed as the fixed point set of a certain nonexpansive operator.

In this paper, we propose a nonexpansive operator that yields a computationally efficient update when it is plugged into the HSDM. The proposed operator is inspired by the update of the linearized augmented Lagrangian method. It is applicable to characterize the solution set of recent sophisticated convex optimization problems found in the context of inverse problems, where the sum of multiple proximable convex functions involving linear operators must be minimized to incorporate preferable properties into the minimizers. For such a problem formulation, there has not yet been reported any nonexpansive operator that yields an update free from the inversions of linear operators in cases where it is utilized in the HSDM. Unlike previously known nonexpansive operators, the proposed operator yields an inversion-free update in such cases. As an application of the proposed operator plugged into the HSDM, we also present, in the context of the so-called superiorization, an algorithmic solution to a convex optimization problem over the generalized convex feasible set where the intersection of the hard constraints is not necessarily simple.