Abstract

This thesis is motivated by the treatment planning problem in intensity modulated radiation therapy (IMRT). The IMRT planning problem, like many others arising in real life applications, is a convex constrained multicriteria optimization problem. In this thesis we tackle the task of minimizing multiple objective functions in two different ways. We either optimize them in a lexicographic way, ranked according to their importance, or accumulate the objectives in a weighted sum. Both approaches yield one or several convex single criteria optimization problems. Our strategy to solve the resulting single criteria optimization problems in an approximate way is to translate them into a sequence of convex feasibility problems via the level set scheme and then solve each feasibility problem using projection methods.

Characteristics of some real life problems, in particular IMRT treatment planning, are challenging to this strategy. Our focus in this thesis are two of these challenges. Depending on the correlation of the objective functions a solution of one lexicographic optimization level picked by the solution algorithm might be far less favorable with respect to the objective function of the next level than a different solution. Additionally, projection methods often exhibit zigzagging behavior when the problem is ill-conditioned. Both phenomena lead to slow convergence of the overall procedure.

To mitigate these disadvantages, we exploit the fact that projection methods are bounded perturbation resilient. We accelerate lexicographic optimization via superiorization, which is a special kind of perturbation, with respect to the objective function of the next optimization level. The zigzagging behavior of the projection methods is avoided by three new perturbations we introduce. Two of these perturbations use gradient information from previous iterates in the spirit of k-step methods. The third perturbation uses the approach of surrogate constraint methods combined with relaxed, averaged projections.

We study both the theoretical and computational impact of the suggested perturbed iteration schemes. The most important new theoretical result presented in this context states that the third perturbation guarantees a significantly bigger minimum progress per iteration than the simultaneous subgradient projection method when subsequent simultaneous projection steps are strongly opposing.

We demonstrate our methods on linear examples and also apply them to nonlinear optimization problems arising from IMRT treatment planning on cases where the tumor is situated in the head-neck area. We present two different choices of superiorization parameter sets suited to yield fast convergence for each case individually or robust behavior for all four cases. Furthermore we illustrate that the anti-zigzag perturbations can significantly accelerate the convergence of the projection methods. For the IMRT optimization problems the perturbed projection methods find an approximate solution within up to 84.1% fewer iterations than the unperturbed methods while at the same time achieving objective function values which are up to 5.2% lower.