ABSTRACTS

ERAN ASSAF: THE EVEN CLIFFORD FUNCTOR AND MODULARITY

We consider the even Clifford functor on quaternary quadratic modules. By adding data from the odd Clifford bimodule, we relate the quadratic modules to quaternion orders over quadratic modules, with descent data. This is used in the context of positive definite quadratic lattices to establish an explicit bijection between certain spaces of modular forms, which in turn sheds light on questions related to representation numbers of quadratic forms in four variables.

Tomer Bauer: Ideal growth in amalgamated powers of nilpotent Rings

In a seminal paper Grunewald, Segal and Smith (1988) introduced the zeta functions of groups and rings enumerating different types of sub-objects, such as subgroups of finite index or two-sided ideals. These zeta functions are connected to various enumeration problems in algebra and combinatorics.

Our main focus will be on zeta functions enumerating ideals of finite (additive) index, in nilpotent rings of class 2. It is well known that in this case there is a decomposition into an Euler product of local factors indexed by primes. The local factors are rational functions over the rationals, but their explicit computation is usually a very hard problem.

We show that the complexity of computing the ideal zeta function of an amalgamated direct power of such rings does not increase by the amalgamation. More generally, we prove this for the zeta functions of quiver representations introduced by Lee and Voll (2021). The proof of rationality of the local factors relies on techniques from model theory, and their explicit computation uses tools from algebraic combinatorics. Such methods are likely to be applicable to other combinatorial enumeration problems in which one wants to prove polynomiality, rationality or uniformity.

This is a joint work with M. Schein.

Alexander Duncan: Separable algebras associated to arithmetic toric varieties

"Ordinary" toric varieties over the complex numbers exhibit many of the behaviors of more complicated varieties but are computationally manageable. Arithmetic toric varieties play a similar role for questions over non-closed fields. In particular, while toric varieties can be seen as generalizations of projective spaces, arithmetic toric varieties can be seen as generalizations of Severi-Brauer varieties. Thus, it is natural to try to generalize the connection to central simple simple algebras. I will discuss both the successes and limitations of this approach with connections to derived categories and rationality.

This is based on joint work with Matthew Ballard, Alicia Lamarche, and Patrick McFaddin.

ABSTRACTS

IDO EFRAT: STEINBERG RELATIONS FOR MASSEY PRODUCTS

Let F be a field of characteristic prime to m which contains the mth roots of unity, and let G_F be its absolute Galois group. As shown by Tate, for $a \neq 0, 1$ in F, the Kummer elements $(a)_F$, $(1-a)_F$ in $H^1(G_F, \mathbb{Z}/m)$ have trivial cup product. In fact, by the celebrated Voevodsky-Rost theorem, this relation completely determines the cohomology ring $H^{\bullet}(G_F, \mathbb{Z}/m)$ with the cup product. A natural generalization of the cup product is the *n*-fold Massey product, where $n \geq 2$. Extending results by Kirsten Wickelgren, we show how Tate's relation generalizes to the Massey product context.

Alexander Guterman: Noncommutative version of Frobenius theorem on matrix maps and beyond

In 1897 Frobenius described the structure of linear maps T preserving the determinant function, i.e., det $X = \det T(X)$ for all X. Later on there were several extension of this result which are due to Diedonné, Schur, Dynkin and others. Maps preserving different matrix properties, invariants, relations on operator or matrix algebras over various algebraic structures were actively investigated, see [1] and references therein.

We are going to discuss the analogs of Frobenius and Diedonné theorems for matrices over division rings and several related topics.

References

 S. Pierce and others.: A Survey of Linear Preserver Problems Linear and Multilinear Algebra. 33 (1992), 1-119.

DIEGO IZQUIERDO: MILNOR K-THEORY AND ZERO-CYCLES OVER P-ADIC FUNCTION FIELDS

In 1986, Kato and Kuzumaki introduced a set of conjectures in order to characterize the cohomological dimension of fields in diophantine terms. The conjectures are known to be wrong in full generality, but they provide interesting arithmetical problems over various usual fields in arithmetic geometry. The goal of this talk is to discuss the case of function fields of *p*-adic curves. This is joint work with G. Lucchini Arteche.

GIANCARLO LUCCHINI ARTECHE: HIGHER K-THEORETIC VERSIONS OF SERRE'S CONJECTURE II

Serre's Conjecture I states that, if K is a perfect field of cohomological dimension 1, then every homogeneous space under a connected linear K-group has a rational point. Replacing "rational point" by "zero-cycle of degree one", one can restate this conjecture in terms of the 0-th Milnor K-theory group $K_M^0(K)$. With this in mind, in previous work with Diego izquierdo, we proved "higher K-theoretic versions" of Serre's Conjecture I that characterized perfect fields of cohomological dimension q+1 by relating the group $K_M^q(K)$ with extensions of K over which homogeneous spaces have rational points.

In this talk, we will present a work in progress with Diego Izquierdo which attempts to prove "higher K-theoretic versions" of Serre's Conjecture II. This conjecture states that, if K is a perfect field of cohomological dimension 2, then every principal homogeneous space under a semisimple simply connected K-group has a rational point. Our results characterize perfect fields of cohomological dimension q+2 by relating the group $K_M^q(K)$ with extensions of K over which these particular homogeneous spaces have rational points. In particular, the statement for q = 0

ABSTRACTS

recovers once again Serre's Conjecture II replacing "rational point" by "zero-cycle of degree one".

We will try to explain how one can use known results on Serre's Conjecture II to prove these higher K-theoretic versions and present our current state on these questions.

DAVID SALTMAN: CYCLIC MATTERS

This work was motivated by the problem of describing cyclic Galois extensions and differential crossed product algebras in mixed characteristic, with the the goal of lifting from arbitrary characteristic p rings to suitable characteristic 0 rings. The first step was the construction of Artin-Schreier like polynomials and extensions in mixed characteristic, where the group acts by $\sigma(x) = \rho x + 1$ ($\rho^p = 1$). This leads to Azumaya algebras A defined by $xy - \rho yx = 1$. Of course, we want to generalize to higher degrees than p. This leads to an Albert like criterion for extending cyclic Galois extensions of rings and to the definition and study of almost-cyclic Azumaya algebras, generalizing A above.

SOLOMON VISHKAUTSAN: POLYNOMIAL DYNAMICS OVER DIVISION RINGS

We generalize aspects of classical complex dynamics of polynomials with complex coefficients to polynomials over division rings (which can be alternative, and not necessarily associative). In particular, we are interested in real Quaternion and real Octonion algebras. We discuss the fixed and periodic points and their properties, and under which conditions these points behave as expected (say, as in the complex case). This is a joint work with Adam Chapman.

Angelo Vistoli: The Lang-Nishimura Theorem and fields of moduli

I would report on joint work with Giulio Bresciani; we give a new criterion for a variety, possibly with extra structure (for example, a polarization, or marked points) to be defined on its field of definition; this criterion was known for smooth curves, but it is completely new in higher dimension.