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[ > restart: #1243,1324
[ > aa:=proc(n,i,j) option remember: local s: if n=1 then return(1):
fi: if n=2 then if i=1 and j=2 then return(1): fi: if i=2 and j=1
then return(q): fi: return(0): fi: if n=3 then if i=1 and j=2
then return(1): fi: if (i=1 and j=3) or (i=2 and j=1) or (i=2 and
j=3) or (i=3 and j=1) then return(q): fi: if i=3 and j=2 then
return(q^2): fi: return(0): fi: if 0<i and i<n-1 and j=i+1 then
s:=aa(n-1,i,i+1)+add(add(add(q^(c+1)*binomial(i-1-a,c)*aa(n-2-c,a,
b),c=0..i-b),b=a+1..i),a=1..i-1): return(simplify(s)): fi: if
2<i+2 and i+1<j and j<n then
s:=aa(n-1,i,j)+add(add(q^(c+1)*binomial(i-1-a,c)*aa(n-2-c,a,j-1-c)
,c=0..i-1-a),a=1..i-1): return(simplify(s)): fi: if 0<i and i<n
and j=n then
s:=add(aa(n-1,i,kk),kk=1..i-1)+q*add(aa(n-1,i,kk),kk=i+1..n-1):
return(simplify(s)): fi: if 0<j and j<i and i<n+1 then
return(simplify(q*add(aa(n-1,j,kk),kk=1..n-1))): fi: return(0):
end:
[ > NN:=14:
[ > AA:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=1..n),i=1..n),n=2
..NN):
AAN:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=1..i-1),i=1..n),
n=2..NN):
AAP:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=i+1..n),i=1..n-1
),n=2..NN):
CC:=(x,v)->add(add(aa(n,i,i+1)*x^n*v^i,i=1..n-1),n=3..NN):
[ > AAAw1:=(x,v)->1/2*v*x*(-2*q^3*v^2*x^3+4*q^2*v^2*x^3+2*q^2*v^2*x^2-
2*q*v^2*x^3+2*q^2*v*x^2-3*q*v^2*x^2+(4*q^2*v^2*x^2-4*q*v^2*x^2+v^2
*x^2-4*q*v*x-2*v*x+1)^(1/2)*q*v*x+2*v^2*x^2-3*q*v*x-v^2*x-3*v*x^2-
(4*q^2*v^2*x^2-4*q*v^2*x^2+v^2*x^2-4*q*v*x-2*v*x+1)^(1/2)*v-(4*q^2
*v^2*x^2-4*q*v^2*x^2+v^2*x^2-4*q*v*x-2*v*x+1)^(1/2)*x+2*v*x+v-x)/
(q*v*x-v*x-1)/(q^2*v*x^2-q*v*x^2-q*x+v*x-v-x+1);
AAAw1:=(x,v)→ $\frac{1}{2}vx(-2q^3v^2x^3+4q^2v^2x^3+2q^2v^2x^2-2qv^2x^3+2q^2vx^2-3qv^2x^2$ 
 $+ \sqrt{4q^2v^2x^2-4qv^2x^2+v^2x^2-4qvx-2vx+1}qv x+2v^2x^2-3qv x-v^2x-3vx^2$ 
 $-\sqrt{4q^2v^2x^2-4qv^2x^2+v^2x^2-4qvx-2vx+1}v$ 
 $-\sqrt{4q^2v^2x^2-4qv^2x^2+v^2x^2-4qvx-2vx+1}x+2vx+v-x)/((qv x-vx-1)$ 
 $(q^2vx^2-qvx^2-qx+vx-v-x+1))$ 
[ >
[ > #EQ1
[ > simplify(taylor(-AAN(x,v,w)+q*v^2*w*x^2+v*x*q/(1-v)*AA(x,v*w,1)-v^
2*x*q/(1-v)*AA(v*x,w,1),x,14));

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$$O(x^{14})$$

> #EQ2

> simplify(taylor(-w*CC(x,v*w)+q*v^2*w^3*x^3+q*v*w^2*x^2*AA(v*w*x,1,1)+x*w*CC(x,v*w)+v*w^2*x^3+q*w*x^2/(q*v*w*x+v*w-1)*(v*w*AA(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)-(1-q*v*w*x)*AA(x,1-q*v*w*x,v*w/(1-q*v*w*x))),x,14));

$$O(x^{14})$$

> #EQ3

> simplify(taylor(-AA(x,v,w)+w*x*AAAN(w*x,v,1)+q*w*x*AAAP(w*x,v,1)+v*w^2*x^2-q*v^2*w^3*x^3-v*w^2*x^3+w*CC(x,v*w)-q*v*w^2*x^2*AA(v*w*x,1,1)+x*AA(x,v,w)-q*v*w^2*x^3*AA(v*w*x,1,1)-w*x*CC(x,v*w)+q*v*w^2*x^3*AA(v*w*x,1,1)+q*w*x^2/(q*v*w*x+v-1)*((1-q*v*w*x)*AA(x,1-q*v*w*x,v*w/(1-q*v*w*x))-v*AA(x,v,w)),x,14));

$$O(x^{14})$$

> eq1:=-FAAN(x,v,w)+q*v^2*w*x^2+v*x*q/(1-v)*FAA(x,v*w,1)-v^2*x*q/(1-v)*FAA(v*x,w,1);

$$eq1 := -FAAN(x, v, w) + q v^2 w x^2 + \frac{v x q FAA(x, v w, 1)}{1 - v} - \frac{v^2 x q FAA(v x, w, 1)}{1 - v}$$

> eq2:=-w*FCC(x,v*w)+q*v^2*w^3*x^3+q*v*w^2*x^2*FAA(v*w*x,1,1)+x*w*FCC(x,v*w)+v*w^2*x^3+q*w*x^2/(q*v*w*x+v*w-1)*(v*w*FAAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)-(1-q*v*w*x)*FAAP(x,1-q*v*w*x,v*w/(1-q*v*w*x)));

$$eq2 := -w FCC(x, v w) + q v^2 w^3 x^3 + q v w^2 x^2 FAA(v w x, 1, 1) + x w FCC(x, v w) + v w^2 x^3 + q$$

$$w x^2 \left(\right.$$

$$v w FAAP\left(\frac{v w x}{-q v w x + 1}, -q v w x + 1, 1\right) - (-q v w x + 1) FAAP\left(x, -q v w x + 1, \frac{v w}{-q v w x + 1}\right) \Bigg) \\ / (q v w x + v w - 1)$$

> eq3:=-FAAP(x,v,w)+w*x*FAAN(w*x,v,1)+q*w*x*FAAP(w*x,v,1)+v*w^2*x^2-q*v^2*w^3*x^3-v*w^2*x^3+w*FCC(x,v*w)-q*v*w^2*x^2*FAA(v*w*x,1,1)+x*FAAP(x,v,w)-q*v*w^2*x^3*FAA(v*w*x,1,1)-w*x*FCC(x,v*w)+q*v*w^2*x^3*FAA(v*w*x,1,1)+q*w*x^2/(q*v*w*x+v-1)*((1-q*v*w*x)*FAAP(x,1-q*v*w*x,v*w/(1-q*v*w*x))-v*FAAP(x,v,w));

$$eq3 := -FAAP(x, v, w) + w x FAAN(w x, v, 1) + q w x FAAP(w x, v, 1) + v w^2 x^2 - q v^2 w^3 x^3 \\ - v w^2 x^3 + w FCC(x, v w) - q v w^2 x^2 FAA(v w x, 1, 1) + x FAAP(x, v, w) - x w FCC(x, v w) \\ + \frac{q w x^2 \left((-q v w x + 1) FAAP\left(x, -q v w x + 1, \frac{v w}{-q v w x + 1}\right) - v FAAP(x, v, w) \right)}{q v w x + v - 1}$$

> AAAN:=(x,v,w)->1/2*q*v^2*w*x^2*(-2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q^2*v^2*w*x^3-(4*q^2*v^2

$$\begin{aligned}
& 2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*q \\
& *v*w^2*x+2*q*v*x-2*q^2*v*x^2+v*x-v*x^2+x+2*w+2*v^2*w^2*x-w*x-v^3*w \\
& ^2*x^3-v^2*w^2*x^3+q*v^2*w^2*x+v^2*w*x^3+q*v*w^2*x-3*q*v*x^2-v*w*x \\
& ^2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w* \\
& x+1)^{(1/2)}-2*q^2*v^3*w^2*x^3-q^2*v^3*w^3*x^3+v^3*w^2*x^2-2*q^2*v^2 \\
& *w^3*x^2-2*q^2*v^2*w*x^2+2*q^2*v*w^2*x^2+2*q*v^2*w^3*x-2*q*v*w^2*x \\
& ^2-2*q*v^2*w*x-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q* \\
& v*w*x-2*v*w*x+1)^{(1/2)}*x+2*q*x+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+ \\
& v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*v*w^2+(4*q^2*v^2*w^2*x^2-4* \\
& q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*v*x^2+(4*q^2* \\
& v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)} \\
& *w*x+q^2*v^2*w^2*x^2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2- \\
& 4*q*v*w*x-2*v*w*x+1)^{(1/2)}-q*v^2*w^2*x*(4*q^2*v^2*w^2*x^2-4*q*v^2* \\
& w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}+q*v^2*w*x^3*(4*q^2* \\
& v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)} \\
& +4*q*v^2*w*x^2-q^2*v^2*w^2*x^2-2*q^2*v^2*w^2*x^3-2*q^2*v*w*x^2+4 \\
& *q*v*w*x^2+q*v^2*w^3*x^2-v^2*w^3*x-v^2*w*x+2*v*w^2*x-2*q*w*x+x^2*v \\
& *w^2+2*v*w+2*q^3*v^3*w^3*x^3+4*q^3*v^3*w^2*x^4-3*q^2*v^3*w^2*x^4-2 \\
& *q^2*v^3*w^3*x^2+q*v^3*w^2*x^4+2*q^2*v^3*w^2*x^2+q*v^3*w^3*x^2+2*q \\
& *v^3*w^2*x^3-q^2*v^2*w*x^3-2*q*v^3*w^2*x^2-2*q*v^2*w*x^3-(4*q^2*v^2* \\
& w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*v \\
& *x+2*q*x^3*v^2*w^2-2*q^4*v^3*w^2*x^4+4*q^3*v^2*w*x^3-2*q*w^2*x^2*v \\
& ^2-2*v*w*x-3*v*w^2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2- \\
& 4*q*v*w*x-2*v*w*x+1)^{(1/2)}*v^2*w*x^2+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2* \\
& x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*q*v*x^2+(4*q^2*v^2*w^2* \\
& x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^{(1/2)}*v^2*w \\
& *x)/(q^2*v^2*w*x^2-q*v^2*w*x^2-q*v*x+v*w*x-v*x-w+1)/(q*v*w*x-v*w*x \\
& -1)/(q^2*v*w*x^2-q*v*w*x^2+v*w*x-q*x-v*w-x+1);
\end{aligned}$$

$$AAAN := (x, v, w) \rightarrow \frac{1}{2} q v^2 w x^2 (-2 + 2 q v x - 2 q^2 v x^2 - 3 q v x^2 + 2 q x + v x - v x^2 + x + 2 w$$

$$\begin{aligned}
& - v^3 w^2 x^3 - v^2 w^2 x^3 + v^2 w x^3 + v^3 w^2 x^2 \\
& + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v w^2 \\
& + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v x^2 \\
& + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} w x - v^2 w^3 x - v^2 w x + x^2 v w^2 \\
& - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v x - 2 v w x + 2 v^2 w^2 x \\
& + 2 v w^2 x - 2 q w x - w x - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} x \\
& + 2 v w - 3 v w^2 - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q^2 v^2 w x^3 \\
& - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q v w^2 x \\
& + q^2 v^2 w^2 x^2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}
\end{aligned}$$

$$(1/2) * q * v * w^2 * x^2 + (4 * q^2 * v^2 * w^2 * x^2 - 4 * q * v^2 * w^2 * x^2 + v^2 * w^2 * x^2 - 4 * q * v * w * x - 2 * v * w * x + 1) ^ (1/2) * q * v * w * x - 5 * v * w * x - (4 * q^2 * v^2 * w^2 * x^2 - 4 * q * v^2 * w^2 * x^2 + v^2 * w^2 * x^2 - 4 * q * v * w * x - 2 * v * w * x + 1) ^ (1/2) * v^2 * w * x / (q * v * w * x - v * w * x - 1) / (q * v * w * x^2 - 2 * q * v * w * x - v * w * x^2 + 3 * v * w * x - 2 * v * w - x + 1) / (q^2 * v * w^2 * x^2 - q * v * w^2 * x^2 - q * w * x + v * w * x - w * x - v + 1) ;$$

$$AAAP := (x, v, w) \rightarrow \frac{1}{2} x^2 v w^2 (-1 + v$$

$$\begin{aligned} & - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} - 9 v^2 w^2 x^2 + v^3 w^2 x^2 \\ & + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} w x + 6 v^2 w x + 4 x^2 v w^2 \\ & - 5 v w x + 10 v^2 w^2 x - 4 v w^2 x - v^3 w^3 x^3 + 2 v^3 w^3 x^2 - 4 v^2 w^3 x^2 + 3 v^2 w^3 x^3 - 2 v^3 w^2 x \\ & + 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v^2 w + w x + 4 v w \\ & + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v - 6 v^2 w \\ & + v w^2 x^2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} - q v^2 w^3 x^3 \\ & + q v^3 w^4 x^4 + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v^2 w^2 x^2 \\ & + 4 q v^3 w^2 x - 2 v w x \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} \\ & - 2 v^2 w^2 x \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} + 6 q^2 v^3 w^4 x^3 \\ & + 2 q v^3 w^3 x^3 - 3 q^2 v^2 w^3 x^3 + 2 q^3 v^3 w^4 x^4 - 3 q^2 v^3 w^4 x^4 - 4 q^3 v^3 w^4 x^3 - 2 q v^3 w^4 x^3 \\ & + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q^2 v^2 w^3 x^3 \\ & - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q v^2 w^3 x^3 \\ & + \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q v w x \\ & - 2 q^2 v^2 w^3 x^2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} \\ & + 2 q v^2 w^3 x^2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} \\ & - 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q v w^2 x^2 \\ & + 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} q v w^2 x - 4 q v^3 w^3 x^2 \\ & - \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} v^2 w x - 2 q^2 v^2 w^2 x^2 \\ & + 7 q v^2 w^2 x^2 + 3 q v w x + 6 q^2 v^2 w^3 x^2 - 2 q v^2 w x - 2 q v^3 w^2 x^2 - 4 q v^2 w^2 x - 2 q v w^2 x) / (\\ & (q v w x - v w x - 1) (q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1) \\ & (q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1)) \end{aligned}$$

> solve (subs (w=1, eq2)=0, FCC (x, v)) ;

$$\text{FINCC} := (x, v) \rightarrow -x^2 * (q^2 * v^3 * x^2 + \text{AAA}(v * x, 1, 1) * q^2 * v^2 * x + \text{AAAP}(x, -q * v * x + 1, v / (-q * v * x + 1)) * q^2 * v * x + q * v^3 * x + q * v^2 * x^2 + \text{AAA}(v * x, 1, 1) * q * v^2 - q * v^2 * x - q * v * \text{AAA}(v * x, 1, 1) + \text{AAAP}(v * x / (-q * v * x + 1), -q * v * x + 1, 1) * q * v + v^2 * x - \text{AAAP}(x, -q * v * x + 1, v / (-q * v * x + 1)) * q - v * x) / (q * v * x + v - 1) / (-1 + x) ;$$

$$\begin{aligned}
& -x^2 \left(q^2 v^3 x^2 + \text{FAA}(vx, 1, 1) q^2 v^2 x + \text{FAAP} \left(x, -q vx + 1, \frac{v}{-q vx + 1} \right) q^2 vx + q v^3 x + q v^2 x^2 \right. \\
& \quad + \text{FAA}(vx, 1, 1) q v^2 - q v^2 x - q v \text{FAA}(vx, 1, 1) + \text{FAAP} \left(\frac{vx}{-q vx + 1}, -q vx + 1, 1 \right) q v + v^2 x \\
& \quad \left. - \text{FAAP} \left(x, -q vx + 1, \frac{v}{-q vx + 1} \right) q - vx \right) / ((q vx + v - 1)(-1 + x))
\end{aligned}$$

$$\begin{aligned}
\text{FINCC} := (x, v) \rightarrow & -x^2 \left(q^2 v^3 x^2 + \text{AAA}(vx, 1, 1) q^2 v^2 x \right. \\
& + \text{AAAP} \left(x, -q vx + 1, \frac{v}{-q vx + 1} \right) q^2 vx + q v^3 x + q v^2 x^2 + \text{AAA}(vx, 1, 1) q v^2 - q v^2 x \\
& - q v \text{AAA}(vx, 1, 1) + \text{AAAP} \left(\frac{vx}{-q vx + 1}, -q vx + 1, 1 \right) q v + v^2 x \\
& \left. - \text{AAAP} \left(x, -q vx + 1, \frac{v}{-q vx + 1} \right) q - vx \right) / ((q vx + v - 1)(x - 1))
\end{aligned}$$

> **AAA := (x, v, w) -> AAAN(x, v, w) + AAAP(x, v, w) ;**

$$AAA := (x, v, w) \rightarrow \text{AAAN}(x, v, w) + \text{AAAP}(x, v, w)$$

> **###checking equations**

> **eq1;**

simplify(-AAAN(x, v, w) + q*v^2*w*x^2 + v*x*q/(1-v)*AAA(x, v*w, 1) - v^2*x*q/(1-v)*AAA(v*x, w, 1)) ;

$$\begin{aligned}
& -\text{FAAN}(x, v, w) + q v^2 w x^2 + \frac{v x q \text{FAA}(x, v w, 1)}{1 - v} - \frac{v^2 x q \text{FAA}(v x, w, 1)}{1 - v} \\
& 0
\end{aligned}$$

> **eq2;**

simplify(-w*FINCC(x, v*w) + q*v^2*w^3*x^3 + q*v*w^2*x^2*AAA(v*w*x, 1, 1) + x*w*FINCC(x, v*w) + v*w^2*x^3 + q*w*x^2/(q*v*w*x + v*w - 1) * (v*w*AAAP(v*w*x/(-q*v*w*x + 1), -q*v*w*x + 1, 1) - (-q*v*w*x + 1)*AAAP(x, -q*v*w*x + 1, v*w/(-q*v*w*x + 1)))) ;

$$\begin{aligned}
& -w \text{FCC}(x, v w) + q v^2 w^3 x^3 + q v w^2 x^2 \text{FAA}(v w x, 1, 1) + x w \text{FCC}(x, v w) + v w^2 x^3 + q w x^2 \left(\right. \\
& \quad v w \text{FAAP} \left(\frac{v w x}{-q v w x + 1}, -q v w x + 1, 1 \right) - (-q v w x + 1) \text{FAAP} \left(x, -q v w x + 1, \frac{v w}{-q v w x + 1} \right) \left. \right) \\
& \quad / (q v w x + v w - 1)
\end{aligned}$$

$$0$$

> **eq3;**

simplify(-AAAP(x, v, w) + w*x*AAAN(w*x, v, 1) + q*w*x*AAAP(w*x, v, 1) + v*w^2*x^2 - q*v^2*w^3*x^3 - v*w^2*x^3 + w*FINCC(x, v*w) - q*v*w^2*x^2*AAA(v*w*x, 1, 1) + x*AAAP(x, v, w) - x*w*FINCC(x, v*w) + q*w*x^2/(q*v*w*x + v - 1) * ((-q*v*w*

$x+1) * AAAP(x, -q*v*w*x+1, v*w/(-q*v*w*x+1)) - v*AAAP(x, v, w))) ;$

$$-FAAP(x, v, w) + w x FAAN(w x, v, 1) + q w x FAAP(w x, v, 1) + v w^2 x^2 - q v^2 w^3 x^3 - v w^2 x^3 \\ + w FCC(x, v w) - q v w^2 x^2 FAA(v w x, 1, 1) + x FAAP(x, v, w) - x w FCC(x, v w) \\ + \frac{q w x^2 \left((-q v w x + 1) FAAP\left(x, -q v w x + 1, \frac{v w}{-q v w x + 1}\right) - v FAAP(x, v, w) \right)}{q v w x + v - 1}$$

0

> ##presentation of the functions A^+=AAAP

> factor(coeff(AAAP(x,v,w), (4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2), 1)) ;

factor(taylor(q^2*v^2*w^3*x^3-2*q^2*v^2*w^3*x^2-q*v^2*w^3*x^3+2*q*v^2*w^3*x^2-2*q*v*w^2*x^2+v^2*w^2*x^2+2*q*v*w^2*x-2*v^2*w^2*x+v*w^2*x^2+q*v*w*x-v^2*w*x+2*v^2*w-2*v*w*x+w*x+v-1, x, 10)) ;

$$x^2 v w^2 (q^2 v^2 w^3 x^3 - 2 q^2 v^2 w^3 x^2 - q v^2 w^3 x^3 + 2 q v^2 w^3 x^2 - 2 q v w^2 x^2 + v^2 w^2 x^2 + 2 q v w^2 x \\ - 2 v^2 w^2 x + v w^2 x^2 + q v w x - v^2 w x + 2 v^2 w - 2 v w x + w x + v - 1) / (2 \\ (q v w x - v w x - 1) (q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1) \\ (q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1))$$

$$(2 v^2 w + v - 1) + w (2 q v w - 2 v^2 w + q v - v^2 - 2 v + 1) x -$$

$$v w^2 (2 q^2 v w - 2 q v w + 2 q - v - 1) x^2 + q v^2 w^3 (q - 1) x^3$$

> factor(coeff(AAAP(x,v,w), (4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2), 0)) ;

factor(taylor(2*q^3*v^3*w^4*x^4-4*q^3*v^3*w^4*x^3-3*q^2*v^3*w^4*x^4+6*q^2*v^3*w^4*x^3+q*v^3*w^4*x^4-2*q^2*v^3*w^4*x^3-3*q^2*v^2*w^3*x^4+3+2*q^2*v^3*w^3*x^3+6*q^2*v^2*w^3*x^2-4*q^2*v^3*w^3*x^2-q*v^2*w^3*x^3-v^3*w^3*x^3-2*q^2*v^2*w^2*x^2-2*q^2*v^3*w^2*x^2+2*v^3*w^3*x^2+3*v^2*w^3*x^3+4*q^2*v^3*w^2*x+7*q^2*v^2*w^2*x^2+v^3*w^2*x^2-4*v^2*w^3*x^2-4*q^2*v^2*w^2*x-2*v^3*w^2*x-9*v^2*w^2*x^2-2*q^2*v^2*w*x-2*q*v*w^2*x+10*v^2*w^2*x+4*v*w^2*x^2+3*q*v*w*x+6*v^2*w*x-4*v*w^2*x-6*v^2*w-5*v*w*x+4*v*w+w*x+v-1, x, 10)) ;

$$x^2 v w^2 (2 q^3 v^3 w^4 x^4 - 4 q^3 v^3 w^4 x^3 - 3 q^2 v^3 w^4 x^4 + 6 q^2 v^3 w^4 x^3 + q v^3 w^4 x^4 - 2 q v^3 w^4 x^3 \\ - 3 q^2 v^2 w^3 x^3 + 2 q v^3 w^3 x^3 + 6 q^2 v^2 w^3 x^2 - 4 q v^3 w^3 x^2 - q v^2 w^3 x^3 - v^3 w^3 x^3 - 2 q^2 v^2 w^2 x^2 \\ - 2 q v^3 w^2 x^2 + 2 v^3 w^3 x^2 + 3 v^2 w^3 x^3 + 4 q v^3 w^2 x + 7 q v^2 w^2 x^2 + v^3 w^2 x^2 - 4 v^2 w^3 x^2 \\ - 4 q v^2 w^2 x - 2 v^3 w^2 x - 9 v^2 w^2 x^2 - 2 q v^2 w x - 2 q v w^2 x + 10 v^2 w^2 x + 4 v w^2 x^2 + 3 q v w x \\ + 6 v^2 w x - 4 v w^2 x - 6 v^2 w - 5 v w x + 4 v w + w x + v - 1) / (2 (q v w x - v w x - 1)$$

$$(q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1)$$

$$(q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1))$$

$$(-6 v^2 w + 4 v w + v - 1) +$$

$$w(4q^3v^3w - 4q^3v^2w - 2v^3w - 2q^3v^2 - 2qv^2w + 10v^2w + 3qv + 6v^2 - 4vw - 5v + 1)x + \\ v w^2(6q^2vw - 4q^2v^2w - 2q^2v - 2q^2v^2 + 2v^2w + 7qv + v^2 - 4vw - 9v + 4)x^2 - \\ v^2w^3(4q^3vw - 6q^2vw + 2qv^2w + 3q^2 - 2qv + q + v - 3)x^3 + qv^3w^4(2q - 1)(q - 1)x^4$$

> ##presentation of the functions A^+=AAAN

> factor(coeff(AAAN(x,v,w), (4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2), 1));
factor(taylor(q^2*v^2*w^2*x^2-q^2*v^2*w*x^3+q*v^2*w*x^3-q*v^2*w^2*x-q*v*w^2*x-v^2*w*x^2+q*v*x^2+v^2*w*x-v*w*x^2+v*w^2+v*x^2-v*x+w*x-x,x,10));

$$qv^2wx^2(q^2v^2w^2x^2 - q^2v^2wx^3 + qv^2wx^3 - qv^2w^2x - qvw^2x - v^2wx^2 + qvx^2 + v^2wx \\ - vwx^2 + vw^2 + vx^2 - vx + wx - x) / (2(q^2v^2wx^2 - qv^2wx^2 - qvx + vwx - vx - w + 1) \\ (qvwx - vwx - 1)(q^2vwx^2 - qvwx^2 + vwx - qx - vw - x + 1)) \\ vw^2 + (-qv^2w^2 - qvw^2 + v^2w - v + w - 1)x + v(q^2vw^2 - vw + q - w + 1)x^2 - qv^2w(q - 1) \\ x^3$$

> factor(coeff(AAAN(x,v,w), (4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2), 0));
factor(taylor(-(2*q^4*v^3*w^2*x^4-2*q^3*v^3*w^3*x^3-4*q^3*v^3*w^2*x^4+q^2*v^3*w^3*x^3+3*q^2*v^3*w^2*x^4+2*q^2*v^3*w^3*x^2+2*q^2*v^3*w^2*x^3-q*v^3*w^2*x^4-4*q^3*v^2*w*x^3-2*q^2*v^3*w^2*x^2+2*q^2*v^2*w^3*x^2+2*q^2*v^2*w^2*x^3-q*v^3*w^3*x^2-2*q*v^3*w^2*x^3+q^2*v^2*w^2*x^2+q^2*v^2*w*x^3+2*q*v^3*w^2*x^2-q*v^2*w^3*x^2-2*q*v^2*w^2*x^3+v^3*w^2*x^3+2*q^2*v^2*w*x^2-2*q^2*v*w^2*x^2-2*q*v^2*w^3*x+2*q*v^2*w^2*x^2+2*q*v^2*w*x^3-v^3*w^2*x^2+v^2*w^2*x^3+2*q^2*v*w*x^2-qv^2w^2x-4qv^2wx^2+2qv^2w^3x+2qv^2w^2x^2+v^2w^3x-v^2wx^3+2q^2vx^2+2qv^2wx-qvw^2x-4qvwx^2-2v^2w^2x-vw^2x^2+3qv^2x+v^2wx-2vw^2x-2qv^2x+2qwx+3vw^2+2vwx+vx^2-2qx-2vw-vx+wx-2w-x+2),x,10));

$$-qv^2wx^2(2q^4v^3w^2x^4 - 2q^3v^3w^3x^3 - 4q^3v^3w^2x^4 + q^2v^3w^3x^3 + 3q^2v^3w^2x^4 + 2q^2v^3w^3x^2 \\ + 2q^2v^3w^2x^3 - qv^3w^2x^4 - 4q^3v^2wx^3 - 2q^2v^3w^2x^2 + 2q^2v^2w^3x^2 + 2q^2v^2w^2x^3 - qv^3w^3x^2 \\ - 2qv^3w^2x^3 + q^2v^2w^2x^2 + q^2v^2wx^3 + 2qv^3w^2x^2 - qv^2w^3x^2 - 2qv^2w^2x^3 + v^3w^2x^3 \\ + 2q^2v^2wx^2 - 2q^2vw^2x^2 - 2qv^2w^3x + 2qv^2w^2x^2 + 2qv^2wx^3 - v^3w^2x^2 + v^2w^2x^3 \\ + 2q^2vw^2x - qv^2w^2x - 4qv^2wx^2 + 2qv^2w^2x^2 + v^2w^3x - v^2wx^3 + 2q^2vx^2 + 2qv^2wx \\ - qvw^2x - 4qvwx^2 - 2v^2w^2x - vw^2x^2 + 3qv^2x + v^2wx - 2vw^2x - 2qv^2x + 2qwx \\ + 3vw^2 + 2vwx + vx^2 - 2qx - 2vw - vx + wx - 2w - x + 2) / (2 \\ (q^2v^2wx^2 - qv^2wx^2 - qvx + vwx - vx - w + 1)(qvwx - vwx - 1) \\ (q^2vwx^2 - qvwx^2 + vwx - qx - vw - x + 1)) \\ (-3vw^2 + 2vw + 2w - 2) + (2qv^2w^3 + qv^2w^2 - v^2w^3 - 2qv^2w + qvw^2 + 2v^2w^2 - v^2w$$

$$\begin{aligned}
& + 2 v w^2 + 2 q v - 2 q w - 2 v w + 2 q + v - w + 1) x - v (2 q^2 v^2 w^3 - 2 q^2 v^2 w^2 + 2 q^2 v w^3 \\
& - q v^2 w^3 + q^2 v w^2 + 2 q v^2 w^2 - q v w^3 + 2 q^2 v w - 2 q^2 w^2 + 2 q v w^2 - v^2 w^2 + 2 q^2 w - 4 q v w \\
& + 2 q w^2 + 2 q^2 - 4 q w - w^2 + 3 q + 1) x^2 + \\
& v^2 w (2 q^3 v w^2 - q^2 v w^2 - 2 q^2 v w + 4 q^3 - 2 q^2 w + 2 q v w - q^2 + 2 q w - v w - 2 q - w + 1) x^3 \\
& - q v^3 w^2 (q - 1) (2 q^2 - 2 q + 1) x^4
\end{aligned}$$

[>