```
> restart: #1243,1342
> aa:=proc(n,i,j) option remember: local s: if n=1 then return(1):
  fi: if n=2 then if i=1 and j=2 then return(1): fi: if i=2 and j=1
  then return(q): fi: return(0): fi: if n=3 then if i=1 and j=2 then
  return(1): fi: if (i=1 and j=3) or (i=2 and j=1) or (i=2 and j=3)
  or (i=3 and j=1) then return(q): fi: if i=3 and j=2 then
  return(q^2): fi: return(0): fi: if 0<i and i<j and j<n then
  s:=q*add(aa(n-1,i,k),k=i+1..j-1)+add(add(add(q^(c+1)*binomial(i-a-
  1,c)*aa(n-2-c,a,b),b=a+1..j-2-c),c=0..i-1-a),a=1..i-1): if j=i+1
  then
  s:=s+aa(n-1,i,i+1)+add(add(q^(c+1)*binomial(i-1-a,c)*aa(n-2-c,a,i-
  c),c=0..i-1-a),a=1..i-1): fi: return(simplify(s)): fi: if j=n and
  i>0 and i<n then
  return(simplify(add(aa(n-1,i,jj),jj=1..i-1)+q*add(aa(n-1,i,jj),jj=
  i+1..n-1))): fi: if j>0 and j<i and i<n+1 then
  return(simplify(add(q*aa(n-1,j,k),k=1..n-1))): fi: return(0): end:
> NN:=14:
> AA:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=1..n),i=1..n),n=2
  ..NN):
  AAN:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=1..i-1),i=1..n),
  n=2..NN):
  AAP:=(x,v,w)->add(add(add(aa(n,i,j)*x^n*v^i*w^j,j=i+1..n),i=1..n-1
  ),n=2..NN):
  CC:=(x,v)->add(add(aa(n,i,i+1)*x^n*v^i,i=1..n-1),n=3..NN):
> #EQ1
> simplify(taylor(-AAN(x,v,w)+q*v^2*w*x^2+v*x*q/(1-v)*AA(x,v*w,1)-v^
  2*x*q/(1-v)*AA(v*x,w,1),x,14));
```

$$O(x^{14})$$

```
> #EQ2
> simplify(taylor(-AAP(x,v,w)+v*w^2*x^2+w*x*AAN(w*x,v,1)+q*w*x*AAP(w
  *x,v,1)+q*w*x/(1-w)*(AAP(x,v,w)-AAP(w*x,v,1))+q*x^2/(q*v*w*x+v-1)*
  (v^2*w^2/(q*v*w*x+v*w-1)*AAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)+v*w^2/
  (1-w)*AAP(w*x,v,1)-v^2*w^2/(q*v*w*x+v*w-1)*AAP(x,1-q*v*w*x,v*w/(1-
  q*v*w*x))-v*w^2/(1-w)*AAP(x,v,w))
  +w*CC(x,v*w)-q*v^2*w^3*x^3-q*v*w^2*x^2*AA(v*w*x,1,1)-q*v*w^2*x^2/(
  q*v*w*x+v*w-1)*(AAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)-AAP(x,1-q*v*w*x
  ,v*w/(1-q*v*w*x))),x,14));
```

$$O(x^{14})$$

```
> #EQ3
> simplify(taylor(-w*CC(x,v*w)+q*v^2*w^3*x^3+q*v*w^2*x^2*AA(v*w*x,1,
  1)+q*v*w^2*x^2/(q*v*w*x+v*w-1)*(AAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)
  -AAP(x,1-q*v*w*x,v*w/(1-q*v*w*x)))    +w*x*CC(x,v*w)+v*w^2*x^3
```

```
+q*w*x^2*AAP(x,1-q*v*w*x,v*w/(1-q*v*w*x)),x,14));
```

$$O(x^{14})$$

```
> eq1:=-FAAN(x,v,w)+q*v^2*w*x^2+v*x*q/(1-v)*FAA(x,v*w,1)-v^2*x*q/(1-
v)*FAA(v*x,w,1);
simplify(taylor(subs(FAAN(x,v,w)=AAN(x,v,w),FAA(x,v*w,1)=AA(x,v*w,
1),FAA(v*x,w,1)=AA(v*x,w,1),eq1),x,14));
```

$$eq1 := -\mathrm{FAAN}(x, v, w) + q\, v^2\, w\, x^2 + \frac{v\, x\, q\, \mathrm{FAA}(x, v\, w, 1)}{1 - v} - \frac{v^2\, x\, q\, \mathrm{FAA}(v\, x, w, 1)}{1 - v}$$

$$O(x^{14})$$

```
> eq2:=-FAAP(x,v,w)+v*w^2*x^2+w*x*FAAN(w*x,v,1)+q*w*x*FAAP(w*x,v,1)+
q*w*x/(1-w)*(FAAP(x,v,w)-FAAP(w*x,v,1))+q*x^2/(q*v*w*x+v-1)*(v^2*w
^2/(q*v*w*x+v*w-1)*FAAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)+v*w^2/(1-w)
*FAAP(w*x,v,1)-v^2*w^2/(q*v*w*x+v*w-1)*FAAP(x,1-q*v*w*x,v*w/(1-q*v
*w*x))-v*w^2/(1-w)*FAAP(x,v,w))
+w*FCC(x,v*w)-q*v^2*w^3*x^3-q*v*w^2*x^2*FAA(v*w*x,1,1)-
q*v*w^2*x^2/(q*v*w*x+v*w-1)*(FAAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)-F
AAP(x,1-q*v*w*x,v*w/(1-q*v*w*x)));
simplify(taylor(subs(FAAP(x,v,w)=AAP(x,v,w),FAAN(w*x,v,1)=AAN(w*x,
v,1),FAAP(w*x,v,1)=AAP(w*x,v,1),FAAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1
)=AAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1),FCC(x,v*w)=CC(x,v*w),FAAP(x,1
-q*v*w*x,v*w/(1-q*v*w*x))=AAP(x,1-q*v*w*x,v*w/(1-q*v*w*x)),FAA(v*w
*x,1,1)=AA(v*w*x,1,1),eq2),x,14));
```

$$\begin{aligned}
eq2 := {} & -\mathrm{FAAP}(x, v, w) + v\, w^2\, x^2 + w\, x\, \mathrm{FAAN}(w\, x, v, 1) + q\, w\, x\, \mathrm{FAAP}(w\, x, v, 1) \\
& + \frac{q\, w\, x\, (\mathrm{FAAP}(x, v, w) - \mathrm{FAAP}(w\, x, v, 1))}{1 - w} + q\, x^2 \left(\frac{v^2\, w^2\, \mathrm{FAAP}\left(\frac{v\, w\, x}{-q\, v\, w\, x + 1}, -q\, v\, w\, x + 1, 1\right)}{q\, v\, w\, x + v\, w - 1} \right. \\
& \left. + \frac{v\, w^2\, \mathrm{FAAP}(w\, x, v, 1)}{1 - w} - \frac{v^2\, w^2\, \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right)}{q\, v\, w\, x + v\, w - 1} - \frac{v\, w^2\, \mathrm{FAAP}(x, v, w)}{1 - w} \right) / \\
& (q\, v\, w\, x + v - 1) + w\, \mathrm{FCC}(x, v\, w) - q\, v^2\, w^3\, x^3 - q\, v\, w^2\, x^2\, \mathrm{FAA}(v\, w\, x, 1, 1) \\
& - \frac{q\, v\, w^2\, x^2 \left(\mathrm{FAAP}\left(\frac{v\, w\, x}{-q\, v\, w\, x + 1}, -q\, v\, w\, x + 1, 1\right) - \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right) \right)}{q\, v\, w\, x + v\, w - 1}
\end{aligned}$$

$$O(x^{14})$$

```
> eq3:=-w*FCC(x,v*w)+q*v^2*w^3*x^3+q*v*w^2*x^2*FAA(v*w*x,1,1)+q*v*w^
2*x^2/(q*v*w*x+v*w-1)*(FAAP(v*w*x/(1-q*v*w*x),1-q*v*w*x,1)-FAAP(x,
1-q*v*w*x,v*w/(1-q*v*w*x)))     +w*x*FCC(x,v*w)+v*w^2*x^3
+q*w*x^2*FAAP(x,1-q*v*w*x,v*w/(1-q*v*w*x));
simplify(taylor(subs(FCC(x,v*w)=CC(x,v*w),FAA(v*w*x,1,1)=AA(v*w*x,
```

```
1,1),FAAP(v*w*x/(-q*v*w*x+1),-q*v*w*x+1,1)=AAP(v*w*x/(-q*v*w*x+1),
-q*v*w*x+1,1),FAAP(x,-q*v*w*x+1,v*w/(-q*v*w*x+1))=AAP(x,-q*v*w*x+1
,v*w/(-q*v*w*x+1)),eq3),x,14));
```

$$eq3 := -w\,\mathrm{FCC}(x, v\,w) + q\,v^2\,w^3\,x^3 + q\,v\,w^2\,x^2\,\mathrm{FAA}(v\,w\,x, 1, 1) + \frac{q\,v\,w^2\,x^2\left(\mathrm{FAAP}\left(\frac{v\,w\,x}{-q\,v\,w\,x+1}, -q\,v\,w\,x+1, 1\right) - \mathrm{FAAP}\left(x, -q\,v\,w\,x+1, \frac{v\,w}{-q\,v\,w\,x+1}\right)\right)}{q\,v\,w\,x + v\,w - 1} + w\,x\,\mathrm{FCC}(x, v\,w) + x^3\,v\,w^2 + q\,w\,x^2\,\mathrm{FAAP}\left(x, -q\,v\,w\,x+1, \frac{v\,w}{-q\,v\,w\,x+1}\right)$$

$$\mathrm{O}(x^{14})$$

```
> #formulas
> AAAP := (x, v, w) ->
1/2*x^2*v*w^2*(-1+v+w*x+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2
*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v-6*v^2*w+v^3*w^2*x^2-9*v^2*w^2*x^
2-v^3*w^3*x^3+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v
*w*x-2*v*w*x+1)^(1/2)*w*x+2*v^3*w^3*x^2-4*v^2*w^3*x^2-2*v^3*w^2*x-
4*v*w^2*x-5*v*w*x+4*v*w^2*x^2+2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2
+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v^2*w+3*w^3*x^3*v^2+6*v^2*
w*x+10*v^2*w^2*x-2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-
4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v*w^2*x^2-2*(4*q^2*v^2*w^2*x^2-4*q*v^
2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q^2*v^2*w^3*x^2+2
*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+
1)^(1/2)*q*v^2*w^3*x^2+2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^
2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v*w^2*x-(4*q^2*v^2*w^2*x^2-4*q*
v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)+(4*q^2*v^2*w^2*
x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v*w*x
+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+
1)^(1/2)*q^2*v^2*w^3*x^3-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^
2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v^2*w^3*x^3-q*v^2*w^3*x^3+q*v^3
*w^4*x^4+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-
2*v*w*x+1)^(1/2)*v*w^2*x^2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*
w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v^2*w*x+v^2*w^2*x^2*(4*q^2*v^2*
w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)+7*q
*v^2*w^2*x^2-2*q^2*v^2*w^2*x^2+3*q*v*w*x+6*q^2*v^3*w^4*x^3-2*q*v^3
*w^2*x^2+2*q*v^3*w^3*x^3-2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*
w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v^2*w^2*x-4*v^2*w^2*x*q-2*q*v^2
*w*x+6*q^2*v^2*w^3*x^2-2*q*v*w^2*x-3*w^3*x^3*q^2*v^2+2*q^3*v^3*w^4
*x^4-4*w^3*x^2*q*v^3-3*w^4*x^4*q^2*v^3-2*w^4*x^3*q*v^3+4*q*v^3*w^2
*x-4*q^3*v^3*w^4*x^3-2*(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*
x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v*w*x+4*v*w)/(q*v*w*x^2-2*q*v*w*x-v
*w*x^2+3*v*w*x-2*v*w-x+1)/(q^2*v*w^2*x^2-q*v*w^2*x^2-q*w*x+v*w*x-w
```

```
*x-v+1)/(q*v*w*x-v*w*x-1);
```

$$AAAP := (x, v, w) \rightarrow \frac{1}{2} x^2 v w^2 (-1 - 9 v^2 w^2 x^2 - q v^2 w^3 x^3 + q v^3 w^4 x^4$$
$$+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v w^2 x^2$$
$$- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v^2 w x$$
$$+ v^2 w^2 x^2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1} - 2 q^2 v^2 w^2 x^2$$
$$+ 7 q v^2 w^2 x^2 + 3 q v w x + 6 q^2 v^3 w^4 x^3 - 2 q v^3 w^2 x^2 + 2 q v^3 w^3 x^3$$
$$- 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v^2 w^2 x + v^3 w^2 x^2 - v^3 w^3 x^3$$
$$+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ w x - 5 v w x + 2 v^3 w^3 x^2$$
$$- 4 v^2 w^3 x^2 - 2 v^3 w^2 x - 4 v w^2 x + 4 v w^2 x^2$$
$$+ 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v^2 w + 3 w^3 x^3 v^2 + 6 v^2 w x$$
$$+ 10 v^2 w^2 x + v - 4 v^2 w^2 x q - 2 q v^2 w x + 6 q^2 v^2 w^3 x^2 - 2 q v w^2 x - 3 w^3 x^3 q^2 v^2$$
$$+ 2 q^3 v^3 w^4 x^4 - 4 w^3 x^2 q v^3 - 3 w^4 x^4 q^2 v^3 - 2 w^4 x^3 q v^3 + 4 q v^3 w^2 x - 4 q^3 v^3 w^4 x^3$$
$$- 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v w x + w x$$
$$+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ v - 6 v^2 w + 4 v w$$
$$+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q v w x$$
$$+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q^2 v^2 w^3 x^3$$
$$- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q v^2 w^3 x^3$$
$$- 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q v w^2 x^2$$
$$- 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q^2 v^2 w^3 x^2$$
$$+ 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q v^2 w^3 x^2$$
$$+ 2 \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ q v w^2 x$$
$$- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\ ) \Big/ ($$
$$(q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1)$$
$$(q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1)(q v w x - v w x - 1))$$

```
> AAAN:=(x,v,w)->1/2*x^2*w*v^2*q*(-2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*
x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v^2*w*x^2+(4*q^2*v^2*w^
2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v*x
^2+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*
x+1)^(1/2)*v^2*w*x-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-
4*q*v*w*x-2*v*w*x+1)^(1/2)*v*w*x^2-2*q^2*v*x^2+2*q*v*x-3*q*v*x^2+2
*v*w+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*
w*x+1)^(1/2)*q^2*v^2*w^2*x^2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^
```

```
2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q^2*v^2*w*x^3+(4*q^2*v^2*w^2*
x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*q*v^2*w
*x^3-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*
w*x+1)^(1/2)*q*v*w^2*x-2*q*x^2*v^2*w^2+2*q*x^3*v^2*w^2-q^2*x^3*v^3
*w^3-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*
w*x+1)^(1/2)*q*v^2*w^2*x+x+2*w+v^3*w^2*x^2-v^2*w^2*x^3-v^2*w^3*x+v
^2*w*x^3-v^2*w*x-2*q*w*x-v^3*w^2*x^3+(4*q^2*v^2*w^2*x^2-4*q*v^2*w^
2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v*w^2+(4*q^2*v^2*w^2*
x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*v*x^2+(
4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-4*q*v*w*x-2*v*w*x+1)
^(1/2)*w*x+4*q*v^2*w*x^2-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^
2*x^2-4*q*v*w*x-2*v*w*x+1)^(1/2)*x+v*x+q*v^2*w^2*x+4*q*v*w*x^2+2*q
*w^2*v^3*x^3-2*q^2*w*v^2*x^2+q*w^2*v*x-q^2*v^2*w^2*x^2-2*q^2*v^2*w
^2*x^3-2*q^2*v*w*x^2-2*q*x^3*v^2*w-2*q^2*x^3*v^3*w^2-3*q^2*x^4*v^3
*w^2-3*v*w^2+2*q*x-2*v*w*x+v*w^2*x^2-w*x-2*q^4*v^3*w^2*x^4+2*q^3*v
^3*w^3*x^3+4*q^3*v^3*w^2*x^4+q*v^3*w^2*x^4+4*q^3*v^2*w*x^3+2*q^2*v
^3*w^2*x^2-2*q^2*v^2*w^3*x^2-q^2*v^2*w*x^3-2*q*v^3*w^2*x^2+q*v^2*w
^3*x^2+2*q^2*v*w^2*x^2+2*q*v^2*w^3*x-2*q*v*w^2*x^2-2*q*v^2*w*x+2*v
^2*w^2*x+2*w^2*v*x-(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^2-
4*q*v*w*x-2*v*w*x+1)^(1/2)*v*x-x^2*v+x^2*w^3*v^3*q-2*q^2*v^3*w^3*x
^2)/(q^2*v^2*w*x^2-q*v^2*w*x^2-q*v*x+v*w*x-v*x-w+1)/(q*v*w*x-v*w*x
-1)/(q^2*v*w*x^2-q*v*w*x^2+v*w*x-q*x-v*w-x+1);
```

$$
\begin{aligned}
AAAN := (x, v, w) \to \frac{1}{2} x^2 w v^2 q\, (&-2 \\
&+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; v^2 w x + 4 q v w x^2 \\
&+ 2 q^2 v w^2 x^2 - 2 q v w^2 x^2 - q^2 v^2 w^2 x^2 - 2 q v^2 w^2 x^2 - 2 q v^3 w^2 x^2 + v^3 w^2 x^2 \\
&+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; w x - 2 q w x - 2 v w x \\
&+ 2 v w^2 x + v w^2 x^2 - v^2 w x + 2 v^2 w^2 x - 3 q v x^2 - 2 q^2 v x^2 + 2 q v x + x + 2 w + v^2 w^2 x q \\
&- 2 q v^2 w x - 2 q^2 v^2 w^3 x^2 + q v w^2 x + w^3 x^2 q v^3 - q^2 x^3 v^3 w^3 + q v^3 w^2 x^4 - q^2 v^2 w x^3 \\
&+ q v^2 w^3 x^2 + 2 q x^3 v^2 w^2 + 2 q w^2 v^3 x^3 - 2 q^2 w v^2 x^2 - 2 q^2 v^2 w^2 x^3 - 2 q^2 v w x^2 - 2 q x^3 v^2 w \\
&- 2 q^2 x^3 v^3 w^2 - 3 q^2 x^4 v^3 w^2 - 2 q^4 v^3 w^2 x^4 + 2 q^3 v^3 w^3 x^3 + 4 q^3 v^3 w^2 x^4 + 4 q^3 v^2 w x^3 \\
&+ 2 q^2 v^3 w^2 x^2 + 2 q v^2 w^3 x - 2 q^2 v^3 w^3 x^2 \\
&- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; v^2 w x^2 \\
&+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; q v x^2 \\
&- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; v w x^2 \\
&+ \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; q^2 v^2 w^2 x^2 \\
&- \sqrt{4 q^2 v^2 w^2 x^2 - 4 q v^2 w^2 x^2 + v^2 w^2 x^2 - 4 q v w x - 2 v w x + 1}\; q^2 v^2 w x^3
\end{aligned}
$$

$$+\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;q\,v^2\,w\,x^3$$
$$-\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;q\,v^2\,w^2\,x+4\,q\,v^2\,w\,x^2$$
$$-\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;x-3\,v\,w^2-w\,x+2\,v\,w+2\,q\,x$$
$$+v\,x-v\,x^2-v^2\,w^2\,x^3-v^2\,w^3\,x+v^2\,w\,x^3-v^3\,w^2\,x^3$$
$$+\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;v\,w^2$$
$$+\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;v\,x^2$$
$$-\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;v\,x$$
$$-\sqrt{4\,q^2\,v^2\,w^2\,x^2-4\,q\,v^2\,w^2\,x^2+v^2\,w^2\,x^2-4\,q\,v\,w\,x-2\,v\,w\,x+1}\;q\,v\,w^2\,x)\Big/($$
$$(q^2\,w\,v^2\,x^2-q\,v^2\,w\,x^2-q\,v\,x+v\,w\,x-v\,x-w+1)\,(q\,v\,w\,x-v\,w\,x-1)$$
$$(q^2\,v\,w\,x^2-q\,v\,w\,x^2+v\,w\,x-q\,x-v\,w-x+1))$$

```
> AAA:=(x,v,w)->AAAP(x,v,w)+AAAN(x,v,w);
```

$$AAA := (x, v, w) \to \mathrm{AAAP}(x, v, w) + \mathrm{AAAN}(x, v, w)$$

```
> solve(subs(w=1,eq3)=0,FCC(x,v));
FINCC:=(x,v)->-x^2*(q^2*v^3*x^2+AAA(v*x,1,1)*q^2*v^2*x+AAAP(x,-q*v
*x+1,v/(-q*v*x+1))*q^2*v*x+q*v^3*x+q*x^2*v^2+AAA(v*x,1,1)*q*v^2-q*
v^2*x-q*v*AAA(v*x,1,1)+q*v*AAAP(v*x/(-q*v*x+1),-q*v*x+1,1)+v^2*x-q
*AAAP(x,-q*v*x+1,v/(-q*v*x+1))-v*x)/(q*v*x+v-1)/(-1+x);
```

$$-x^2\left(q^2\,v^3\,x^2+\mathrm{FAA}(v\,x,1,1)\,q^2\,v^2\,x+\mathrm{FAAP}\left(x,-q\,v\,x+1,\frac{v}{-q\,v\,x+1}\right)q^2\,v\,x+q\,v^3\,x+q\,x^2\,v^2\right.$$
$$+\mathrm{FAA}(v\,x,1,1)\,q\,v^2-q\,v^2\,x-q\,v\,\mathrm{FAA}(v\,x,1,1)+q\,v\,\mathrm{FAAP}\left(\frac{v\,x}{-q\,v\,x+1},-q\,v\,x+1,1\right)+v^2\,x$$
$$\left.-q\,\mathrm{FAAP}\left(x,-q\,v\,x+1,\frac{v}{-q\,v\,x+1}\right)-v\,x\right)/((q\,v\,x+v-1)\,(-1+x))$$

$$FINCC := (x, v) \to -x^2\left(q^2\,v^3\,x^2+\mathrm{AAA}(v\,x,1,1)\,q^2\,v^2\,x\right.$$
$$+\mathrm{AAAP}\left(x,-q\,v\,x+1,\frac{v}{-q\,v\,x+1}\right)q^2\,v\,x+q\,v^3\,x+q\,x^2\,v^2+\mathrm{AAA}(v\,x,1,1)\,q\,v^2-q\,v^2\,x$$
$$-q\,v\,\mathrm{AAA}(v\,x,1,1)+q\,v\,\mathrm{AAAP}\left(\frac{v\,x}{-q\,v\,x+1},-q\,v\,x+1,1\right)+v^2\,x$$
$$\left.-q\,\mathrm{AAAP}\left(x,-q\,v\,x+1,\frac{v}{-q\,v\,x+1}\right)-v\,x\right)/((q\,v\,x+v-1)\,(x-1))$$

```
> ##checking equations;
> eq1;
simplify(-AAAN(x,v,w)+q*v^2*w*x^2+v*x*q/(1-v)*AAA(x,v*w,1)-v^2*x*q
/(1-v)*AAA(v*x,w,1));
```

$$-\mathrm{FAAN}(x, v, w) + q\, v^2\, w\, x^2 + \frac{v\, x\, q\, \mathrm{FAA}(x, v\, w, 1)}{1 - v} - \frac{v^2\, x\, q\, \mathrm{FAA}(v\, x, w, 1)}{1 - v}$$

$$0$$

```
> eq2;
simplify(-AAAP(x,v,w)+v*w^2*x^2+w*x*AAAN(w*x,v,1)+q*w*x*AAAP(w*x,v
,1)+q*w*x/(1-w)*(AAAP(x,v,w)-AAAP(w*x,v,1))+q*x^2/(q*v*w*x+v-1)*(v
^2*w^2/(q*v*w*x+v*w-1)*AAAP(v*w*x/(-q*v*w*x+1),-q*v*w*x+1,1)+v*w^2
/(1-w)*AAAP(w*x,v,1)-v^2*w^2/(q*v*w*x+v*w-1)*AAAP(x,-q*v*w*x+1,v*w
/(-q*v*w*x+1))-v*w^2/(1-w)*AAAP(x,v,w))+w*FINCC(x,v*w)-q*v^2*w^3*x
^3-q*v*w^2*x^2*AAA(v*w*x,1,1)-q*v*w^2*x^2/(q*v*w*x+v*w-1)*(AAAP(v*
w*x/(-q*v*w*x+1),-q*v*w*x+1,1)-AAAP(x,-q*v*w*x+1,v*w/(-q*v*w*x+1))
));
```

$$-\mathrm{FAAP}(x, v, w) + v\, w^2\, x^2 + w\, x\, \mathrm{FAAN}(w\, x, v, 1) + q\, w\, x\, \mathrm{FAAP}(w\, x, v, 1) + \frac{q\, w\, x\, (\mathrm{FAAP}(x, v, w) - \mathrm{FAAP}(w\, x, v, 1))}{1 - w} + q\, x^2 \left(\frac{v^2\, w^2\, \mathrm{FAAP}\left(\frac{v\, w\, x}{-q\, v\, w\, x + 1}, -q\, v\, w\, x + 1, 1\right)}{q\, v\, w\, x + v\, w - 1} + \frac{v\, w^2\, \mathrm{FAAP}(w\, x, v, 1)}{1 - w} - \frac{v^2\, w^2\, \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right)}{q\, v\, w\, x + v\, w - 1} - \frac{v\, w^2\, \mathrm{FAAP}(x, v, w)}{1 - w} \right) / (q\, v\, w\, x + v - 1) + w\, \mathrm{FCC}(x, v\, w) - q\, v^2\, w^3\, x^3 - q\, v\, w^2\, x^2\, \mathrm{FAA}(v\, w\, x, 1, 1) - \frac{q\, v\, w^2\, x^2 \left(\mathrm{FAAP}\left(\frac{v\, w\, x}{-q\, v\, w\, x + 1}, -q\, v\, w\, x + 1, 1\right) - \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right)\right)}{q\, v\, w\, x + v\, w - 1}$$

$$0$$

```
> eq3;
simplify(-w*FINCC(x,v*w)+q*v^2*w^3*x^3+q*v*w^2*x^2*AAA(v*w*x,1,1)+
q*v*w^2*x^2/(q*v*w*x+v*w-1)*(AAAP(v*w*x/(-q*v*w*x+1),-q*v*w*x+1,1)
-AAAP(x,-q*v*w*x+1,v*w/(-q*v*w*x+1)))+w*x*FINCC(x,v*w)+x^3*v*w^2+q
*w*x^2*AAAP(x,-q*v*w*x+1,v*w/(-q*v*w*x+1)));
```

$$-w\, \mathrm{FCC}(x, v\, w) + q\, v^2\, w^3\, x^3 + q\, v\, w^2\, x^2\, \mathrm{FAA}(v\, w\, x, 1, 1) + \frac{q\, v\, w^2\, x^2 \left(\mathrm{FAAP}\left(\frac{v\, w\, x}{-q\, v\, w\, x + 1}, -q\, v\, w\, x + 1, 1\right) - \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right)\right)}{q\, v\, w\, x + v\, w - 1} + w\, x\, \mathrm{FCC}(x, v\, w) + x^3\, v\, w^2 + q\, w\, x^2\, \mathrm{FAAP}\left(x, -q\, v\, w\, x + 1, \frac{v\, w}{-q\, v\, w\, x + 1}\right)$$

$$0$$

```
> #presentation of A^+=AAAP
> s:=coeff(AAAP(x,v,w),(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^
```

```
2-4*q*v*w*x-2*v*w*x+1)^(1/2),1);
factor(taylor(numer(s)/x^2/v/w^2,x,10));
```

$$s := x^2 v w^2 (q^2 v^2 w^3 x^3 - 2 q^2 v^2 w^3 x^2 - q v^2 w^3 x^3 + 2 q v^2 w^3 x^2 - 2 q v w^2 x^2 + v^2 w^2 x^2 + 2 q v w^2 x - 2 v^2 w^2 x + v w^2 x^2 + q v w x - v^2 w x + 2 v^2 w - 2 v w x + w x + v - 1) \Big/ (2 (q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1) (q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1) (q v w x - v w x - 1)) (2 v^2 w + v - 1) + w (2 q v w - 2 v^2 w + q v - v^2 - 2 v + 1) x - v w^2 (2 q^2 v w - 2 q v w + 2 q - v - 1) x^2 + q w^3 v^2 (q - 1) x^3$$

```
> s:=coeff(AAAP(x,v,w),(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^
2-4*q*v*w*x-2*v*w*x+1)^(1/2),0);
factor(taylor(numer(s)/v/w^2/x^2,x,10));
```

$$s := x^2 v w^2 (2 q^3 v^3 w^4 x^4 - 4 q^3 v^3 w^4 x^3 - 3 q^2 v^3 w^4 x^4 + 6 q^2 v^3 w^4 x^3 + q v^3 w^4 x^4 - 2 q v^3 w^4 x^3 - 3 q^2 v^2 w^3 x^3 + 2 q v^3 w^3 x^3 + 6 q^2 v^2 w^3 x^2 - 4 q v^3 w^3 x^2 - q v^2 w^3 x^3 - v^3 w^3 x^3 - 2 q^2 v^2 w^2 x^2 - 2 q v^3 w^2 x^2 + 2 v^3 w^3 x^2 + 3 v^2 w^3 x^3 + 4 q v^3 w^2 x + 7 q v^2 w^2 x^2 + v^3 w^2 x^2 - 4 v^2 w^3 x^2 - 4 q v^2 w^2 x - 2 v^3 w^2 x - 9 v^2 w^2 x^2 - 2 q v^2 w x - 2 q v w^2 x + 10 v^2 w^2 x + 4 v w^2 x^2 + 3 q v w x + 6 v^2 w x - 4 v w^2 x - 6 v^2 w - 5 v w x + 4 v w + w x + v - 1) \Big/ (2 (q v w x^2 - 2 q v w x - v w x^2 + 3 v w x - 2 v w - x + 1) (q^2 v w^2 x^2 - q v w^2 x^2 - q w x + v w x - w x - v + 1) (q v w x - v w x - 1)) (-6 v^2 w + 4 v w + v - 1) + w (4 q v^3 w - 4 q v^2 w - 2 v^3 w - 2 q v^2 - 2 q v w + 10 v^2 w + 3 q v + 6 v^2 - 4 v w - 5 v + 1) x + v w^2 (6 q^2 v w - 4 q v^2 w - 2 q^2 v - 2 q v^2 + 2 v^2 w + 7 q v + v^2 - 4 v w - 9 v + 4) x^2 - v^2 w^3 (4 q^3 v w - 6 q^2 v w + 2 q v w + 3 q^2 - 2 q v + q + v - 3) x^3 + q v^3 w^4 (2 q - 1) (q - 1) x^4$$

```
> #presentation of A^-=AAAN
> s:=coeff(AAAN(x,v,w),(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^
2-4*q*v*w*x-2*v*w*x+1)^(1/2),1);
factor(taylor(numer(s)/x^2/v^2/w/q,x,10));
```

$$s := x^2 w v^2 q (q^2 v^2 w^2 x^2 - q^2 v^2 w x^3 + q v^2 w x^3 - q v^2 w^2 x - q v w^2 x - v^2 w x^2 + q v x^2 + v^2 w x - v w x^2 + v w^2 + v x^2 - v x + w x - x) \Big/ (2 (q^2 v^2 w x^2 - q v^2 w x^2 - q v x + v w x - v x - w + 1) (q v w x - v w x - 1) (q^2 v w x^2 - q v w x^2 + v w x - q x - v w - x + 1)) v w^2 + (-q v^2 w^2 - q v w^2 + v^2 w - v + w - 1) x + v (q^2 v w^2 - v w + q - w + 1) x^2 - w q v^2 (q - 1) x^3$$

```
> s:=coeff(AAAN(x,v,w),(4*q^2*v^2*w^2*x^2-4*q*v^2*w^2*x^2+v^2*w^2*x^
2-4*q*v*w*x-2*v*w*x+1)^(1/2),0);
factor(taylor(numer(s)/x^2/v^2/w/q,x,10));
```

$$s := x^2 w v^2 q (-2 q^4 v^3 w^2 x^4 + 2 q^3 v^3 w^3 x^3 + 4 q^3 v^3 w^2 x^4 - q^2 v^3 w^3 x^3 - 3 q^2 v^3 w^2 x^4$$

$$
\begin{aligned}
&- 2 q^2 v^3 w^3 x^2 - 2 q^2 v^3 w^2 x^3 + q v^3 w^2 x^4 + 4 q^3 v^2 w x^3 + 2 q^2 v^3 w^2 x^2 - 2 q^2 v^2 w^3 x^2 \\
&- 2 q^2 v^2 w^2 x^3 + q v^3 w^3 x^2 + 2 q v^3 w^2 x^3 - q^2 v^2 w^2 x^2 - q^2 v^2 w x^3 - 2 q v^3 w^2 x^2 + q v^2 w^3 x^2 \\
&+ 2 q v^2 w^2 x^3 - v^3 w^2 x^3 - 2 q^2 v^2 w x^2 + 2 q^2 v w^2 x^2 + 2 q v^2 w^3 x - 2 q v^2 w^2 x^2 - 2 q v^2 w x^3 \\
&+ v^3 w^2 x^2 - v^2 w^2 x^3 - 2 q^2 v w x^2 + q v^2 w^2 x + 4 q v^2 w x^2 - 2 q v w^2 x^2 - v^2 w^3 x + v^2 w x^3 \\
&- 2 q^2 v x^2 - 2 q v^2 w x + q v w^2 x + 4 q v w x^2 + 2 v^2 w^2 x + v w^2 x^2 - 3 q v x^2 - v^2 w x + 2 v w^2 x \\
&+ 2 q v x - 2 q w x - 3 v w^2 - 2 v w x - v x^2 + 2 q x + 2 v w + v x - w x + 2 w + x - 2) \Big/ (2 \\
&(q^2 v^2 w x^2 - q v^2 w x^2 - q v x + v w x - v x - w + 1)(q v w x - v w x - 1) \\
&(q^2 v w x^2 - q v w x^2 + v w x - q x - v w - x + 1)) \\
(&-3 v w^2 + 2 v w + 2 w - 2) + (2 q v^2 w^3 + q v^2 w^2 - v^2 w^3 - 2 q v^2 w + q v w^2 + 2 v^2 w^2 - v^2 w \\
&+ 2 v w^2 + 2 q v - 2 q w - 2 v w + 2 q + v - w + 1) x - v (2 q^2 v^2 w^3 - 2 q^2 v^2 w^2 + 2 q^2 v w^3 \\
&- q v^2 w^3 + q^2 v w^2 + 2 q v^2 w^2 - q v w^3 + 2 q^2 v w - 2 q^2 w^2 + 2 q v w^2 - v^2 w^2 + 2 q^2 w - 4 q v w \\
&+ 2 q w^2 + 2 q^2 - 4 q w - w^2 + 3 q + 1) x^2 + \\
&v^2 w (2 q^3 v w^2 - q^2 v w^2 - 2 q^2 v w + 4 q^3 - 2 q^2 w + 2 q v w - q^2 + 2 q w - v w - 2 q - w + 1) x^3 \\
&- q v^3 w^2 (q - 1)(2 q^2 - 2 q + 1) x^4
\end{aligned}
$$

```
>
>
>
```