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> #Equivalence of the descents statistic on some (4,4)-avoidance
  classes of permutations
> #April 18, 2021
>
> #function t(x)
> t:=(x)->sqrt((1-2*q)^2*x^2-2*x*(1+2*q)+1);
```

$$t := x \to \sqrt{(1-2\,q)^2\,x^2 - 2\,x\,(1+2\,q) + 1}$$

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> #Theorem 4
> U0:=(x,v)->(v*x^2*(2*(v-1)*v*q*x-(v-3)*v*x-3*v+1)+v*(v+1)*x^2*t(v*
  x))/(2*(2*v*q*x-v*x+v+x-1)*(v*q*x-v*x-1));
  UP:=(x,v,w)->-1/2*x^2*w^2*v*q*(q*v*w*x-v+x)/(2*q*v*w*x-v*w*x+v*w+x
  -1)/(q^2*v*w^2*x^2-q*v*w^2*x^2-q*w*x+v*w*x-w*x-v+1)*t(v*w*x)-1/2*x
  ^2*w^2*v*q*(v+(-2*q*v^2*w-q*v*w+v^2*w-2*v*w-2*v+1)*x+v*w*(2*q^2*v*
  w-q*v*w+1)*x^2)/(q^2*v*w^2*x^2-q*v*w^2*x^2-q*w*x+v*w*x-w*x-v+1)/(2
  *q*v*w*x-v*w*x+v*w+x-1);
  UN:=(x,v,w)->1/2*x^2*w*v^2*q*(q*v*w*x-v*w*x+v*w+w-1)/(2*q*v*w*x-v*
  w*x+v*w+x-1)/(q^2*v^2*w*x^2-q*v^2*w*x^2-q*v*x+v*w*x-v*x-w+1)*t(v*w
  *x)+((-2*q*v*x^2+q*v*x-2*v*x^2+2*v*x+2*x-1)+(2*q^2*v^2*x^3-3*q^2*v
  ^2*x^2-2*q*v^2*x^3-q*v^2*x+2*v^2*x^2+2*q*v*x-2*v^2*x+2*v*x^2-4*v*x
  +v-2*x+1)*w+v*x*(2*q-1)*(q*v*x-1)*(q*v*x-v*x+v+1)*w^2)*q*v^2*w*x^2
  /2/(q^2*v^2*w*x^2-q*v^2*w*x^2-q*v*x+v*w*x-v*x-w+1)/(q*v*x-1)/(2*q*
  v*w*x-v*w*x+v*w+x-1);   U:=(x,v,w)->UP(x,v,w)+UN(x,v,w)+U0(x,v*w);
```

$$U0 := (x, v) \to \frac{1}{2}\,\frac{v\,x^2\,(2\,(v-1)\,v\,q\,x - (v-3)\,v\,x - 3\,v + 1) + v\,(v+1)\,x^2\,\mathrm{t}(v\,x)}{(2\,v\,q\,x - v\,x + v + x - 1)\,(v\,q\,x - v\,x - 1)}$$

$$UP := (x, v, w) \to$$

$$-\frac{1}{2}\,\frac{x^2\,w^2\,v\,q\,(q\,v\,w\,x - v + x)\,\mathrm{t}(v\,w\,x)}{(2\,q\,v\,w\,x - v\,w\,x + v\,w + x - 1)\,(q^2\,v\,w^2\,x^2 - q\,v\,w^2\,x^2 - q\,w\,x + v\,w\,x - w\,x - v + 1)}$$

$$-\frac{1}{2}\,\frac{x^2\,w^2\,v\,q\,(v + (-2\,q\,v^2\,w - q\,v\,w + v^2\,w - 2\,v\,w - 2\,v + 1)\,x + v\,w\,(2\,q^2\,v\,w - q\,v\,w + 1)\,x^2)}{(q^2\,v\,w^2\,x^2 - q\,v\,w^2\,x^2 - q\,w\,x + v\,w\,x - w\,x - v + 1)\,(2\,q\,v\,w\,x - v\,w\,x + v\,w + x - 1)}$$

$$UN := (x, v, w) \to$$

$$\frac{1}{2}\,\frac{x^2\,w\,v^2\,q\,(q\,v\,w\,x - v\,w\,x + v\,w + w - 1)\,\mathrm{t}(v\,w\,x)}{(2\,q\,v\,w\,x - v\,w\,x + v\,w + x - 1)\,(v^2\,w\,q^2\,x^2 - v^2\,w\,q\,x^2 - v\,q\,x + v\,w\,x - v\,x - w + 1)} + \frac{1}{2}\,($$

$$-2\,q\,v\,x^2 + v\,q\,x - 2\,v\,x^2 + 2\,v\,x + 2\,x - 1 +$$

$$(2\,q^2\,v^2\,x^3 - 3\,q^2\,v^2\,x^2 - 2\,q\,v^2\,x^3 - q\,v^2\,x + 2\,v^2\,x^2 + 2\,v\,q\,x - 2\,v^2\,x + 2\,v\,x^2 - 4\,v\,x + v - 2\,x + 1)$$

$$w + v\,x\,(2\,q - 1)\,(v\,q\,x - 1)\,(v\,q\,x - v\,x + v + 1)\,w^2)\,q\,v^2\,w\,x^2 \Big/ ($$

$$(v^2\,w\,q^2\,x^2 - v^2\,w\,q\,x^2 - v\,q\,x + v\,w\,x - v\,x - w + 1)\,(v\,q\,x - 1)\,(2\,q\,v\,w\,x - v\,w\,x + v\,w + x - 1))$$

$$U := (x, v, w) \to \mathrm{UP}(x, v, w) + \mathrm{UN}(x, v, w) + \mathrm{U0}(x, v\,w)$$

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> #checking eq U^-,U^+,U^0
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> simplify(-UP(x,v,w)+v*w^2*q*x^3/((1-w*q*x)*(1-v*w*q*x))+x*UN(x,w,v
)+v*w*q*x^2/(1-v)*(UN(x,w,v)-UN(x,v*w,1))+v*w*q*x/(1-v)*(UN(x,w,v)
-UN(x,v*w,1)-w*q*x*U(w*x,1,v)+v*w*q*x*U(v*w*x,1,1)));
```

$$0$$

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> simplify(-UN(x,v,w)+v*q*x/(1-v)*(UN(x,v,w)-v*UN(v*x,1,w)+U0(x,v*w)
-v*U0(v*x,w))+v*q*x/(1-v)*(UP(x,1,v*w)-v*UP(v*x,1,w)));
```

$$0$$

```
> simplify(-U0(x,v)+v*x^2/(1-v*q*x)+x*UP(x,1,v)+x*U0(x,v)+x*UN(x,v,1
));
```

$$0$$

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> U0(x,v);
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$$\frac{v x^2 (2 (v-1) v q x-(v-3) v x-3 v+1)+v (v+1) x^2 \sqrt{(1-2 q)^2 v^2 x^2-2 v x (1+2 q)+1}}{2 (2 q v x-v x+v+x-1) (q v x-v x-1)}$$

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> UP(x,v,w);
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$$-\frac{x^2 w^2 v q (q v w x-v+x) \sqrt{(1-2 q)^2 v^2 w^2 x^2-2 v w x (1+2 q)+1}}{2 (2 q v w x-v w x+v w+x-1) (q^2 v w^2 x^2-q v w^2 x^2-q w x+v w x-w x-v+1)} - \frac{x^2 w^2 v q (v+(-2 q v^2 w-q v w+v^2 w-2 v w-2 v+1) x+v w (2 q^2 v w-q v w+1) x^2)}{2 (q^2 v w^2 x^2-q v w^2 x^2-q w x+v w x-w x-v+1) (2 q v w x-v w x+v w+x-1)}$$

```
> UN(x,v,w);
```

$$\frac{x^2 w v^2 q (q v w x-v w x+v w+w-1) \sqrt{(1-2 q)^2 v^2 w^2 x^2-2 v w x (1+2 q)+1}}{2 (2 q v w x-v w x+v w+x-1) (q^2 v^2 w x^2-q v^2 w x^2-q v x+v w x-v x-w+1)} + (-2 q v x^2+q v x-2 v x^2+2 v x+2 x-1+ (2 q^2 v^2 x^3-3 q^2 v^2 x^2-2 q v^2 x^3-q v^2 x+2 v^2 x^2+2 q v x-2 v^2 x+2 v x^2-4 v x+v-2 x+1) w+v x (2 q-1) (q v x-1) (q v x-v x+v+1) w^2) q v^2 w x^2 \Big/ (2 (q^2 v^2 w x^2-q v^2 w x^2-q v x+v w x-v x-w+1) (q v x-1) (2 q v w x-v w x+v w+x-1))$$

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>
>
> s:=1/2*x*v*(q*v*x-v-x)/(q^2*v*x^2-q*v*x^2-q*x+v*x-v-x+1)/(q*v*x-v*
x-1)*(4*q^2*v^2*x^2-4*q*v^2*x^2+v^2*x^2-4*q*v*x-2*v*x+1)^(1/2)+1/2
*(x+(q*x^2+q*x+3*x^2-2*x-1)*v+(-x*(2*q^2*x^2+3*q^2*x-q*x^2-q*x-3*q
+2*x-1))*v^2+q*x^2*(2*q-1)*(q*x-1)*v^3)*v*x/(q*v*x-1)/(q*v*x-v*x-1
)/(q^2*v*x^2-q*v*x^2-q*x+v*x-v-x+1); simplify(s-U(x,v,1));
```

$$s := \frac{x v (q v x-v-x) \sqrt{4 q^2 v^2 x^2-4 q v^2 x^2+v^2 x^2-4 q v x-2 v x+1}}{2 (q^2 v x^2-q v x^2-q x+v x-v-x+1) (q v x-v x-1)} + (x + (q x^2+q x+3 x^2-2 x-1) v-x (2 q^2 x^2+3 q^2 x-q x^2-q x-3 q+2 x-1) v^2 + q x^2 (2 q-1) (q x-1) v^3) v x \Big/ (2 (q v x-1) (q v x-v x-1)$$

$$(q^2 v x^2 - q v x^2 - q x + v x - v - x + 1))$$

$$0$$

```
> s:=1/2/q/(q*x-x-1)*(4*q^2*x^2-4*q*x^2-4*q*x+x^2-2*x+1)^(1/2)+1/2*(
2*q^2*x^2-q*x^2-3*q*x-x+1)/q/(q*x-x-1)/(q*x-1);
simplify(s-U(x,1,1));
```

$$s := \frac{\sqrt{4 q^2 x^2 - 4 q x^2 - 4 q x + x^2 - 2 x + 1}}{2 q (q x - x - 1)} + \frac{2 q^2 x^2 - q x^2 - 3 q x - x + 1}{2 q (q x - x - 1)(q x - 1)}$$

$$0$$

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