```
> ##Avoiding 021 and another 5 letter pattern
> #Done019=12340,12330,12304,12303,12300,12230,12220,12203,12202,122
  00,12034,12033,12030,12023,12022,12020,12003,12002,12000
> #Done019=11230,11220,11203,11202,11200,11120,11110,11102,11101,111
  00,11010,11011,11001,10122,10120,10110,10102,10100,10010
> #Done019=11023,11022,11000,11012,10234,10233,10230,10223,10222,102
  20,10203,10202,10200,10123,10112,10111,10101,10011,10001
> #Done019=10000,10023,10022,10020,10012,10002,11002,11020,01234,012
  33,01230,01223,01222,01202,01220,01203,01200,01123,01122
> #Done019=01120,01112,01012,01111,01101,01011,01001,01110,01010,011
  02,01100,01023,01022,01020,01002,01000,00123,00122,00120
> #Done010=00112,00111,00110,00102,00101,00100,00012,00011,00010,000
  01,
>
> #12340: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: 0,0,0,0,0,1,2,3,4,  Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*DD:
  eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x/(1-v)*E:
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*E:
> Exv:=SolveKE(E,E1,eqE,2): Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),1):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
  =x,Bxv))+1)-1)); taylor(FinA1,x,12);
```

$$FinA1 := -x\,(16\,x^6\sqrt{1-4\,x}+256\,x^7-32\sqrt{1-4\,x}\,x^5-896\,x^6+38\,x^4\sqrt{1-4\,x}+1432\,x^5 -26\sqrt{1-4\,x}\,x^3-1274\,x^4+8\,x^2\sqrt{1-4\,x}+658\,x^3-x\sqrt{1-4\,x}-196\,x^2+31\,x-2)\Big/(2\,(-1+4\,x)^2\,(-1+2\,x)^3\,(-1+x)^3)$$

$$x+2\,x^2+6\,x^3+22\,x^4+90\,x^5+393\,x^6+1777\,x^7+8130\,x^8+37082\,x^9+167376\,x^{10}+745780\,x^{11}+O(x^{12})$$

```
> #12330: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: 0,0,0,0,0,1,2,3,3,:
  f=0,1,2,3,3,5  Reg5
> #12300: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: 0,0,0,0,0,1,2,3,0,:
  f=0,1,2,3,0,5  Reg5
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*DD:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x*E+x/(1-v)*DD:
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*E+x/(1-v)*F:
  eqF:=-F+x+x*E1+x*F:
> FF:=solve(eqF=0,F): Exv:=SolveKE(E,E1,subs(F=FF,eqE),2):
  Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),2):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
  =x,Bxv))+1)-1)); taylor(FinA1,x,12);
```

$$FinA1 := -\left(26\sqrt{1-4x}\,x^4 + 72x^5 - 54\sqrt{1-4x}\,x^3 - 190x^4 + 41\sqrt{1-4x}\,x^2 + 186x^3 - 12\sqrt{1-4x}\,x - 83x^2 + \sqrt{1-4x} + 16x - 1\right) \Big/ \left(2(-1+2x)(-1+4x)^2(x-1)^2\right)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 393x^6 + 1782x^7 + 8234x^8 + 38294x^9 + 177915x^{10} + 822358x^{11} + O(x^{12})$$

```
> #12304: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: 0,0,0,0,0,1,2,3,4,:
  f=01230: g=012340: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*DD:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*E+x/(1-v)*F:
  eqE:=-E+x/(1-v)*(1+G)+x/v*(E-E1)+x/(1-v)*E: F:=x/(1-2*x):
  G:=x/(1-x):
> Exv:=SolveKE(E,E1,eqE,2): Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),1):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
  =x,Bxv))+1)-1)); taylor(FinA1,x,12);
```

$$FinA1 := x\,(-256x^{11} + 32\sqrt{1-4x}\,x^9 + 1760x^{10} - 192x^8\sqrt{1-4x} - 5976x^9 + 512x^7\sqrt{1-4x} + 12292x^8 - 800\sqrt{1-4x}\,x^6 - 16836x^7 + 810\sqrt{1-4x}\,x^5 + 16104x^6 - 552\sqrt{1-4x}\,x^4 - 10942x^5 + 253\sqrt{1-4x}\,x^3 + 5268x^4 - 75\sqrt{1-4x}\,x^2 - 1761x^3 + 13\sqrt{1-4x}\,x + 389x^2$$

$$-\sqrt{1-4x}-51x+3)\Big/(2(-1+4x)(-1+2x)^5(-1+x)^6)$$

$$x+2x^2+6x^3+22x^4+90x^5+392x^6+1754x^7+7845x^8+34533x^9+148840x^{10}+628842x^{11}+O(x^{12})$$

```
> #12303: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: 0,0,0,0,0,1,2,3,0,:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*DD:
  eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*DD:
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/v*(DD-DD1)+x/(1-v)*DD:
> eqD1:=subs(E=solve(eqD=0,E),eqE):
  KK:=factor(coeff(eqD1,DD)*x*v^2*(1-v)); taylor(KK,v,10);
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$KK := v^3 - v^2 x + v x^2 - v^2 + 2 v x - x^2$$

$$-x^2 + (x^2 + 2x) v + (-x - 1) v^2 + v^3$$

```
> map(factor,solve({subs(v=u1,subs(DD=0,eqD1)),subs(v=u2,subs(DD=0,e
  qD1))},{DD1,E1})); Dx0:= u1*u2/x/(-1+u2)/(-1+u1): Ex0:=
  -(u2*x+u2-x)*(u1*x+u1-x)/(-1+u1)/(-1+u2)/x^2:
  Dxv:=factor(solve(subs(DD1=Dx0,E1=Ex0,eqD1)=0,DD)):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
  =x,Bxv))+1)-1)): uu:=solve(u1+x^2/u1/u3=1+x-u3,u1):
  FinA1:=factor(simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2]
  ,FinA1))+1)-1)); simplify(series(subs(u3=vv,FinA1),x,12)) assuming
  x>0 and x<0.1;
```

$$\{DD1 = \frac{u1\,u2}{x(-1+u2)(-1+u1)}, E1 = -\frac{(u2\,x+u2-x)(u1\,x+u1-x)}{(-1+u1)(-1+u2)x^2}\}$$

$$FinA1 := -x(72\,u3^2 x^4 - 72\,u3\,x^5 + 72\,x^6 - 102\,u3^2 x^3 + 102\,u3\,x^4 - 78\,x^5 + 64\,u3^2 x^2 - 68\,u3\,x^3 + 50\,x^4 - 15\,u3^2 x + 13\,u3\,x^2 - 9\,x^3 + u3^2)\Big/((-1+2x)(6x-1)^2(-1+x)^2(u3^2 - u3\,x + x^2))$$

$$x+2x^2+6x^3+22x^4+90x^5+393x^6+1784x^7+8287x^8+39049x^9+185789x^{10}+890311x^{11}+O(x^{12})$$

```
> #12230: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,2,: 0,0,0,0,0,1,2,2,3,:
  f:=01224: Reg5
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*C+x*DD:
  eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x/(1-v)*(E+F):
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+F): F:=(x+x*E1)/(1-x):
> Exv:=SolveKE(E,E1,eqE,2): Ex0:=subs(v=0,Exv):
  Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=Ex0,eqD),1):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),2):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
  =x,Bxv))+1)-1)); taylor(FinA1,x,12);
```

$$FinA1 := -\frac{7\sqrt{1-4x}\,x^2+18x^3-6\sqrt{1-4x}\,x-24x^2+\sqrt{1-4x}+9x-1}{(-1+x)(-1+4x)^2}$$

$$x+2x^2+6x^3+22x^4+90x^5+392x^6+1764x^7+8046x^8+36790x^9+167660x^{10}+759448x^{11}+O(x^{12})$$

```
> #12220: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,2,: 0,0,0,0,0,1,2,2,2,:
  f=012224, g=012225: Reg5
> #12020: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,0,: 0,0,0,0,0,1,2,0,2,:
  f=012024, g=012025: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*C+x*DD:
  eqD:=-DD+x/(1-v)+x/v*(DD+C-DD1-C1)+x/(1-v)*C+x*E:
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+F+G):
  eqF:=-FF+x+x*E1+x*FF+x*GG: eqG:=-GG+x+x*FF+x*GG:
> solve({eqF,eqG},{FF,GG}); F:=x*(E1*x-E1-1)/(2*x-1):
  G:=-x*(E1*x+1)/(2*x-1): Exv:=SolveKE(E,E1,eqE,2):
```

$$\{FF=\frac{x(E1\,x-E1-1)}{2x-1},\ GG=-\frac{x(E1\,x+1)}{2x-1}\}$$

```
> eqC1:=subs(DD=solve(eqC=0,DD),subs(E=Exv,eqD)):
  KK:=factor(coeff(eqC1,C)*x*v*v*(1-v)): taylor(KK,v,10);
```

```
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2x)\,v+(-x-1)\,v^2+v^3$$

```
> ss:=map(factor,map(rationalize,solve({subs(v=u1,subs(C=0,eqC1)),su
bs(v=u2,subs(C=0,eqC1))},{C1,DD1}))); Cx0:=op(op(ss)[1])[2]:
Dx0:=op(op(ss)[2])[2]:
Cxv:=factor(simplify(simplify(rationalize(subs(u1=uu1,u2=uu2,subs(
C1=Cx0,DD1=Dx0,solve(eqC1=0,C))))+1)-1)):
Cxv:=subs(vv=u3,simplify(subs(u3=vv,Cxv))):
```

$$ss := \{C1 = -(2\,u1\,u2+\sqrt{1-4x}\,u1+\sqrt{1-4x}\,u2-u1-u2-2x-\sqrt{1-4x}+1)$$
$$(\sqrt{1-4x}+2x-3)\,u1\,u2\,(2\,u1\,u2\,x^2+\sqrt{1-4x}\,u1\,x+\sqrt{1-4x}\,u2\,x-4\,u1\,u2\,x-2\,u1\,x^2$$
$$-2\,u2\,x^2-\sqrt{1-4x}\,u1-\sqrt{1-4x}\,u2-2x\sqrt{1-4x}+4\,u1\,u2+5\,u1\,x+5\,u2\,x+4x^2$$
$$+\sqrt{1-4x}-3\,u1-3\,u2-8x+3)\Big/(8x\,(x^2-2x+2)\,(-1+u2)\,(u2^2-u2+x)\,(-1+u1)$$
$$(u1^2-u1+x)),\ DD1 = -(2\,u1\,u2+\sqrt{1-4x}\,u1+\sqrt{1-4x}\,u2-u1-u2-2x-\sqrt{1-4x}+1)$$
$$(\sqrt{1-4x}-2)\,(-4x\,u1^2+x^2\,u1^2+16\,u1\,u2\,x^3-4\,u1^2\,u2\,x^3-4\,u1\,u2^2\,x^3-3x^2$$
$$-\sqrt{1-4x}\,u1\,u2+\sqrt{1-4x}\,u1^2\,u2+\sqrt{1-4x}\,u1\,u2^2-\sqrt{1-4x}\,u2^2\,x^2-2\sqrt{1-4x}\,u2\,x^3$$
$$-2\sqrt{1-4x}\,u2^2\,x+2\sqrt{1-4x}\,u2\,x^2-2\sqrt{1-4x}\,u1^2\,x+2\sqrt{1-4x}\,u1\,x^2-\sqrt{1-4x}\,u1^2\,x^2$$
$$-2\sqrt{1-4x}\,u1\,x^3+2\sqrt{1-4x}\,u1\,x+2\sqrt{1-4x}\,u2\,x+2\sqrt{1-4x}\,x^3-3\sqrt{1-4x}\,x^2$$
$$+2\sqrt{1-4x}\,u1\,u2\,x^3-\sqrt{1-4x}\,u1\,u2\,x^2-4\sqrt{1-4x}\,u1\,u2\,x+3\sqrt{1-4x}\,u1^2\,u2\,x$$
$$+\sqrt{1-4x}\,u1\,u2^2\,x^2+3\sqrt{1-4x}\,u1\,u2^2\,x+\sqrt{1-4x}\,u1^2\,u2\,x^2-5\,u1\,u2+5\,u1^2\,u2$$
$$+7\,u1^2\,u2\,x-6\,u1\,u2\,x-5\,u1\,u2^2\,x^2+7\,u1\,u2^2\,x+15\,u1\,u2\,x^2-8\,u1^2\,u2^2\,x-5\,u1^2\,u2\,x^2$$
$$+5\,u1\,u2^2-6\,u1^2\,u2^2-4x\,u2^2+x^2\,u2^2+4\,u2^2\,x^3-12\,u2\,x^3-2\,u2\,x^2+4\,u1^2\,x^3-12\,u1\,x^3$$
$$-2\,u1\,x^2+4\,u2\,x+4\,u1\,x+8x^3)\Big/(4x^2\,(3+4x)\,(-1+u2)\,(u2^2-u2+x)\,(-1+u1)$$
$$(u1^2-u1+x))\}$$

```
> Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
=x,Bxv))+1)-1)); simplify(series(subs(u3=vv,FinA1),x,14)) assuming
x>0 and x<0.1;
```

$$FinA1 := (16\sqrt{1-4x}\,u3^2\,x^2-8\sqrt{1-4x}\,u3\,x^3-12\sqrt{1-4x}\,x^4-48x^5-8\sqrt{1-4x}\,u3^2\,x$$
$$-14\sqrt{1-4x}\,u3\,x^2+46\sqrt{1-4x}\,x^3-16\,u3^2\,x^2+8x^3\,u3+68x^4+\sqrt{1-4x}\,u3^2$$
$$+8x\sqrt{1-4x}\,u3-19\sqrt{1-4x}\,x^2+8x\,u3^2+14x^2\,u3-66x^3-\sqrt{1-4x}\,u3+2x\sqrt{1-4x}$$
$$-u3^2-8\,u3\,x+21x^2+u3-2x)\Big/(2x^2\,(x-1)\,(6x-1)\,(-1+4x))$$

$$x+2x^2+6x^3+22x^4+90x^5+393x^6+1786x^7+8318x^8+39332x^9+187778x^{10}+902185x^{11}$$
$$+O(x^{12})$$

```
> #12203: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,2,: 0,0,0,0,0,1,2,2,3,:
```

```
f=01220: g=01224: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*C+x*DD:
eqD:=-DD+x/(1-v)*(1+F+G)+x/v*(DD-DD1)+x/(1-v)*E:
eqE:=-E+x/(1-v)*(1+H+G)+x/v*(E-E1)+x/(1-v)*E:
eqF:=-FF+x+x*FF+x*FF: eqG:=-GG+x+x*HH+x*E1+x*GG: eqH:=-HH+x+x*HH:
> solve({eqF,eqG,eqH},{FF,GG,HH}); F:=-x/(2*x-1):
G:=-x*(E1*x-E1-1)/(x^2-2*x+1): H:=-x/(x-1):
```

$$\{FF=-\frac{x}{2x-1}, GG=-\frac{x(E1\,x-E1-1)}{x^2-2x+1}, HH=-\frac{x}{x-1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2): Ex0:=subs(v=0,Exv):
Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=Ex0,eqD),1):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),2):
Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
FinA1:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v
=x,Bxv))+1)-1)); taylor(FinA1,x,12);
```

$$FinA1 := -(16\sqrt{1-4x}\,x^8+128x^9-172\sqrt{1-4x}\,x^7-608x^8+532\sqrt{1-4x}\,x^6+1552x^7 -803\sqrt{1-4x}\,x^5-2340x^6+689\sqrt{1-4x}\,x^4+2245x^5-350\sqrt{1-4x}\,x^3-1397x^4 +103\sqrt{1-4x}\,x^2+556x^3-16\sqrt{1-4x}\,x-135x^2+\sqrt{1-4x}+18x-1)\Big/(2x(-1+4x)^2(2x-1)^2(x-1)^4)$$

$$x+2x^2+6x^3+22x^4+90x^5+391x^6+1744x^7+7821x^8+34903x^9+154433x^{10}+O(x^{11})$$

```
> #12202: a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m122, e_m=0^m1220,
regular=5
> #12022: a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m1202,
regular=5
> #12002: a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m1200,
regular=5
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
eqC:=-C(x,v)+x/(1-v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*C(x,v)+x*DD(x,v);
eqD:=-DD(x,v)+x/(1-v)+x*E(x,v)+x/v*(DD(x,v)-DD(x,0))+x/(1-v)*C(x,v
)+x/v*(C(x,v)-C(x,0));
eqE:=-E(x,v)+x/(1-v)+x/v*(E(x,v)-E(x,0))+x/v^2*(C(x,v)-C(x,0)-v*CC
1)+x/v*(C(x,v)-C(x,0))+x/(1-v)*C(x,v);
```

$$eqA := -\mathrm{A}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{A}(x,v) - \mathrm{A}(x,0))}{v} + \frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{B}(x,v) - \mathrm{B}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v} + x\,\mathrm{DD}(x,v)$$

$$eqD := -\mathrm{DD}(x,v) + \frac{x}{1-v} + x\,\mathrm{E}(x,v) + \frac{x\,(\mathrm{DD}(x,v) - \mathrm{DD}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v} + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v}$$

$$eqE := -\mathrm{E}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{E}(x,v) - \mathrm{E}(x,0))}{v} + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0) - v\,CC1)}{v^2} + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

```
>
> eqC1:=simplify(subs(E(x,v)=solve(subs(DD(x,v)=solve(eqC,DD(x,v)),eqD),E(x,v)),eqE));
```

$$eqC1 := -\left(v^3 x^2 + v^3 x + \mathrm{C}(x,v)\,v^4 - \mathrm{C}(x,0)\,x^3 + \mathrm{C}(x,v)\,x^3 - \mathrm{C}(x,v)\,v^3 - 2\,x^2 v^2 + x^3 v^3 + x^3 v - x^3 v^2 - 3\,\mathrm{C}(x,0)\,v^2 x^2 - \mathrm{DD}(x,0)\,v^2 x^2 + 3\,\mathrm{C}(x,v)\,v^2 x - 3\,\mathrm{C}(x,v)\,v\,x^2 - \mathrm{C}(x,0)\,v^2 x + 2\,\mathrm{C}(x,0)\,v\,x^2 + CC1\,v^3 x^3 - CC1\,v^2 x^3 + \mathrm{C}(x,0)\,v^3 x^3 + \mathrm{E}(x,0)\,v^3 x^3 + \mathrm{C}(x,0)\,v^3 x^2 - \mathrm{C}(x,0)\,v^2 x^3 + \mathrm{DD}(x,0)\,v^3 x^2 - \mathrm{DD}(x,0)\,v^2 x^3 - \mathrm{E}(x,0)\,v^2 x^3 + \mathrm{C}(x,0)\,v^3 x + \mathrm{C}(x,0)\,v\,x^3 + \mathrm{DD}(x,0)\,v\,x^3 - 2\,\mathrm{C}(x,v)\,v^3 x + 2\,\mathrm{C}(x,v)\,v^2 x^2\right) \Big/ \left(v^3 (-1+v)\,x^2\right)$$

```
> KK:=factor(coeff(eqC1,C(x,v))); alias(vv=RootOf(KK=0,v)):
VVV:=allvalues(vv): factor(taylor(-numer(KK),v,10));
```

$$KK := -\frac{v^4 - 2\,v^3 x + 2\,v^2 x^2 - v^3 + 3\,v^2 x - 3\,v\,x^2 + x^3}{v^3 (-1+v)\,x^2}$$

$$x^3 - 3\,x^2 v + x\,(3 + 2\,x)\,v^2 + (-2\,x - 1)\,v^3 + v^4$$

```
> eqC11:=subs(C(x,v)=0,eqC1):
factor(coeff(eqC11,DD(x,0)));factor(coeff(eqC11,C(x,0)));factor(coeff(eqC11,E(x,0)));factor(coeff(eqC11,CC1));
eqC12:=simplify(subs(CC1=CC2-E(x,0),eqC11));
```

$$-\frac{v-x}{v^2}$$

$$-\frac{v^2 x^2 + v^2 x + v^2 - 2\,v\,x + x^2}{x\,v^3}$$

$$-\frac{x}{v}$$

$$-\frac{x}{v}$$

$$eqC12 := -(v^3 x^2 CC2 + \mathrm{C}(x,0) v^3 x^2 - x^2 v^2 CC2 + \mathrm{C}(x,0) v^3 x - \mathrm{C}(x,0) v^2 x^2 + \mathrm{DD}(x,0) v^3 x - \mathrm{DD}(x,0) v^2 x^2 + v^3 x^2 + \mathrm{C}(x,0) v^3 - 3\,\mathrm{C}(x,0) v^2 x + \mathrm{C}(x,0) v x^2 - \mathrm{DD}(x,0) v^2 x + \mathrm{DD}(x,0) v x^2 + v^3 x - x^2 v^2 - \mathrm{C}(x,0) v^2 + 2\,\mathrm{C}(x,0) v x - \mathrm{C}(x,0) x^2 + v^3 - 2 v^2 x + x^2 v) \Big/ (x v^3 (-1+v))$$

```
> map(factor,solve({subs(v=u1,eqC12),subs(v=u2,eqC12),subs(v=u3,eqC12)},{DD(x,0),C(x,0),CC2}));
```

$$\{CC2 = -(u1\,u2\,u3\,x^3 - u1\,u2\,x^4 - u1\,u3\,x^4 - u2\,u3\,x^4 + u1\,u2\,u3\,x^2 - u1\,u2\,x^3 - u1\,u3\,x^3 + u1\,x^4 - u2\,u3\,x^3 + u2\,x^4 + u3\,x^4 - u1\,u2\,u3\,x + u1\,u2\,u3 - u1\,u2\,x - u1\,u3\,x + u1\,x^2 - u2\,u3\,x + u2\,x^2 + u3\,x^2 - x^3) \Big/ (x^4 (-1+u1)(-1+u3)(-1+u2)),$$

$$\mathrm{C}(x,0) = -\frac{(x^2 - x + 1)\,u1\,u2\,u3}{(-1+u3)(-1+u2)(-1+u1)\,x^2},\ \mathrm{DD}(x,0) = (u1\,u2\,u3\,x^2 - u1\,u2\,u3\,x + u1\,u2\,x^2 + u1\,u3\,x^2 - u1\,x^3 + u2\,u3\,x^2 - u2\,x^3 - u3\,x^3 + 2\,u1\,u2\,u3 - u1\,u2\,x - u1\,u3\,x - u2\,u3\,x + x^3) \Big/ (x^3 (-1+u2)(-1+u1)(-1+u3))\}$$

```
> Cx2:=-1/x^4*(u1*u2*u3*x^3-u1*u2*x^4-u1*u3*x^4-u2*u3*x^4+u1*u2*u3*x^2-u1*u2*x^3-u1*u3*x^3+u1*x^4-u2*u3*x^3+u2*x^4+u3*x^4-u1*u2*u3*x+u1*u2*u3-u1*u2*x-u1*u3*x+u1*x^2-u2*u3*x+u2*x^2+u3*x^2-x^3)/(-1+u1)/(-1+u3)/(-1+u2):
  Cx0:=-(x^2-x+1)*u1*u2*u3/(-1+u3)/(-1+u2)/(-1+u1)/x^2:
  Dx0:=(u1*u2*u3*x^2-u1*u2*u3*x+u1*u2*x^2+u1*u3*x^2-u1*x^3+u2*u3*x^2-u2*x^3-u3*x^3+2*u1*u2*u3-u1*u2*x-u1*u3*x-u2*u3*x+x^3)/x^3/(-1+u2)/(-1+u1)/(-1+u3):
> map(factor,solve({factor(subs(CC2=Cx2,C(x,0)=Cx0,subs(CC1=CC2-E(x,0),eqE))),subs(DD(x,0)=Dx0,C(x,0)=Cx0,eqD),subs(C(x,0)=Cx0,eqC)},{E(x,v),DD(x,v),C(x,v)})):
> Cxv:=-(x^2-x+1)*(u3-v)*(u2-v)*(u1-v)*x/(-1+u3)/(-1+u2)/(-1+u1)/(v^4-2*v^3*x+2*v^2*x^2-v^3+3*v^2*x-3*v*x^2+x^3);
```

$$Cxv := -\frac{(x^2 - x + 1)(u3 - v)(u2 - v)(u1 - v)\,x}{(-1+u3)(-1+u2)(-1+u1)(v^4 - 2 v^3 x + 2 v^2 x^2 - v^3 + 3 v^2 x - 3 v x^2 + x^3)}$$

```
> Bx0:=solve(subs(v=2*x,subs(C(x,v)=Cxv,eqB)),B(x,0)):
  Bxv:=factor(solve(subs(B(x,0)=Bx0,C(x,v)=Cxv,eqB)=0,B(x,v))):
> subs(v=x,eqA);
```

$$\frac{x}{-x+1} - \mathrm{A}(x,0) + \frac{x\,\mathrm{B}(x,x)}{-x+1}$$

```
> Ax0:=factor(x/(1-x)+x/(1-x)*subs(v=x,Bxv));
```

$$Ax0 := -(8\,u1\,u2\,u3\,x^3 - 4\,u1\,u2\,u3\,x^2 - 2\,u1\,u2\,x^3 - 2\,u1\,u3\,x^3 - 4\,u1\,x^4 - 2\,u2\,u3\,x^3 - 4\,u2\,x^4$$

$$-4\,u3\,x^4+6\,u1\,u2\,u3\,x-3\,u1\,u2\,x^2-3\,u1\,u3\,x^2+7\,u1\,x^3-3\,u2\,u3\,x^2+7\,u2\,x^3+7\,u3\,x^3+3\,x^4-u1\,u2\,u3+u1\,u2\,x+u1\,u3\,x-2\,u1\,x^2+u2\,u3\,x-2\,u2\,x^2-2\,u3\,x^2-6\,x^3+x^2)/(x\,(x-1)\,(-1+u1)\,(-1+u2)\,(-1+u3)\,(8\,x-1))$$

```
> FinalA:=factor(-1/x*(8*x^3/u4*x^3-4*x^3/u4*x^2+(x*(3+2*x)-(1+2*x-u4)*u4)*(-2*x^3-3*x^2+x)-4*(1+2*x-u4)*x^4+6*x^3/u4*x+7*(1+2*x-u4)*x^3+3*x^4-x^3/u4-2*(1+2*x-u4)*x^2-6*x^3+x^2)/(x-1)/(-1+(1+2*x-u4)-(x*(3+2*x)-(1+2*x-u4)*u4)+x^3/u4)/(8*x-1));
```

$$FinalA := -(2\,u4^3\,x^2-8\,u4^2\,x^3+12\,u4\,x^4-8\,x^5+3\,u4^3\,x-u4^2\,x^2-u4\,x^3+4\,x^4-u4^3-3\,u4^2\,x+10\,u4\,x^2-6\,x^3+u4^2-2\,u4\,x+x^2)\Big/((x-1)\,(u4^3-2\,u4^2\,x+2\,u4\,x^2-x^3+u4\,x)\,(8\,x-1))$$

```
> simplify(series(subs(u4=VVV[1],FinalA),x,15)) assuming x>0 and x<0.1;
```

$$x+2\,x^2+6\,x^3+22\,x^4+90\,x^5+393\,x^6+1789\,x^7+8381\,x^8+40105\,x^9+195101\,x^{10}+961773\,x^{11}+4793181\,x^{12}+24107561\,x^{13}+\mathrm{O}(x^{14})$$

```
> #12200 a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m122, e_m=0^m1220, f=01224, g=012204, h=012205 regular=5
> #12000 a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m1200, f=01204, g=012004, h=012005, regular=5
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
eqC:=-C(x,v)+x/(1-v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*C(x,v)+x*DD(x,v);
eqD:=-DD(x,v)+x/(1-v)+x*E(x,v)+x/v*(DD(x,v)-DD(x,0))+x/(1-v)*DD(x,v)+x/(1-v)*FF;
eqE:=-E(x,v)+x/(1-v)+x/v*(E(x,v)-E(x,0))+x/(1-v)*E(x,v)+x/(1-v)*(GG+HH); eqF:=-FF+x+x*GG+x*DD(x,0)+x*FF;
eqG:=-GG+x+x*E(x,0)+x*GG+x*HH; eqH:=-HH+x+x*GG+x*HH;
```

$$eqA := -\mathrm{A}(x,v)+\frac{x}{1-v}+\frac{x\,(\mathrm{A}(x,v)-\mathrm{A}(x,0))}{v}+\frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v)+\frac{x}{1-v}+\frac{2\,x\,(\mathrm{B}(x,v)-\mathrm{B}(x,0))}{v}+\frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v)+\frac{x}{1-v}+\frac{x\,(\mathrm{C}(x,v)-\mathrm{C}(x,0))}{v}+\frac{x\,\mathrm{C}(x,v)}{1-v}+x\,\mathrm{DD}(x,v)$$

$$eqD := -\mathrm{DD}(x,v)+\frac{x}{1-v}+x\,\mathrm{E}(x,v)+\frac{x\,(\mathrm{DD}(x,v)-\mathrm{DD}(x,0))}{v}+\frac{x\,\mathrm{DD}(x,v)}{1-v}+\frac{x\,FF}{1-v}$$

$$eqE := -\mathrm{E}(x,v)+\frac{x}{1-v}+\frac{x\,(\mathrm{E}(x,v)-\mathrm{E}(x,0))}{v}+\frac{x\,\mathrm{E}(x,v)}{1-v}+\frac{x\,(GG+HH)}{1-v}$$

$$eqF := -FF+x+x\,GG+x\,\mathrm{DD}(x,0)+x\,FF$$

$$eqG := -GG+x+x\,\mathrm{E}(x,0)+x\,GG+x\,HH$$

$$eqH := GG\,x+HH\,x-HH+x$$

```
> solve({eqF,eqG,eqH},{FF,GG,HH});
FF:=-x*(E(x,0)*x^2+2*x*DD(x,0)-x*E(x,0)-DD(x,0)+x-1)/(x-1)/(2*x-1)
: GG:=x*(x*E(x,0)-E(x,0)-1)/(2*x-1):
HH:=-1/(2*x-1)*x*(x*E(x,0)+1):
```

$$\{FF = -\frac{x\,(\mathrm{E}(x,0)\,x^2 + 2\,x\,\mathrm{DD}(x,0) - x\,\mathrm{E}(x,0) - \mathrm{DD}(x,0) + x - 1)}{(x-1)\,(2\,x-1)},$$

$$GG = \frac{x\,(x\,\mathrm{E}(x,0) - \mathrm{E}(x,0) - 1)}{2\,x-1},\ HH = -\frac{x\,(x\,\mathrm{E}(x,0)+1)}{2\,x-1}\}$$

```
> v0:=(1-sqrt(1-4*x))/2: Ex0:=solve(subs(v=v0,eqE)=0,E(x,0)):
Exv:=solve(subs(E(x,0)=Ex0,eqE)=0,E(x,v)):
> Dx0:=solve(simplify(limit(subs(E(x,v)=Exv,E(x,0)=Ex0,eqD),v=v0))=0
,DD(x,0)):
Dxv:=solve(subs(E(x,v)=Exv,E(x,0)=Ex0,DD(x,0)=Dx0,eqD),DD(x,v)):
> Cx0:=solve(simplify(limit(subs(DD(x,v)=Dxv,C(x,v)=0,eqC),v=v0))=0,
C(x,0)): Cxv:=solve(subs(C(x,0)=Cx0,DD(x,v)=Dxv,eqC)=0,C(x,v)):
Bx0:=solve(subs(v=2*x,subs(C(x,v)=Cxv,eqB))=0,B(x,0)):Bxv:=solve(s
ubs(B(x,0)=Bx0,C(x,v)=Cxv,eqB)=0,B(x,v)):
Ax0:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x
,Bxv))+1)-1)):
Ax0:=factor(coeff(Ax0,sqrt(1-4*x),1))*sqrt(1-4*x)+factor(coeff(Ax0
,sqrt(1-4*x),0)); taylor(Ax0,x,20);
```

$$Ax0 := \frac{(4\,x^5 - 154\,x^4 + 174\,x^3 - 74\,x^2 + 14\,x - 1)\sqrt{1-4\,x}}{2\,x^2\,(-1+4\,x)^3} - \frac{52\,x^4 - 92\,x^3 + 52\,x^2 - 12\,x + 1}{2\,x^2\,(-1+4\,x)^2}$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 393\,x^6 + 1784\,x^7 + 8272\,x^8 + 38722\,x^9 + 181636\,x^{10} + 849956\,x^{11} + 3957732\,x^{12} + 18314500\,x^{13} + 84180991\,x^{14} + 384298236\,x^{15} + 1742741332\,x^{16} + 7853095730\,x^{17} + \mathrm{O}(x^{18})$$

```
> #12034: a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m123,
f_m=0^m1230, g=01203, regular=6
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
eqC:=-C(x,v)+x/(1-v)+x*DD(x,v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*E(x,v);
eqD:=-DD(x,v)+x/(1-v)+2*x/v*(DD(x,v)-DD(x,0))+x*sum((m+1)*v^(m-1)*
GG,m=1..infinity);
eqE:=-E(x,v)+x/(1-v)+x*F(x,v)+x/v*(E(x,v)-E(x,0))+x/(1-v)*E(x,v);
eqF:=-F(x,v)+x/(1-v)+x/v*(F(x,v)-F(x,0))+x*sum((m+2)*v^(m-1)*GG,m=
1..infinity); eqG:=-GG+x+x*GG+x*GG;
```

$$eqA := -\mathrm{A}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{A}(x,v) - \mathrm{A}(x,0))}{v} + \frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{B}(x,v) - \mathrm{B}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v) + \frac{x}{1-v} + x\,\mathrm{DD}(x,v) + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{E}(x,v)}{1-v}$$

$$eqD := -\mathrm{DD}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{DD}(x,v) - \mathrm{DD}(x,0))}{v} - \frac{x\,GG\,(v-2)}{(v-1)^2}$$

$$eqE := -\mathrm{E}(x,v) + \frac{x}{1-v} + x\,\mathrm{F}(x,v) + \frac{x\,(\mathrm{E}(x,v) - \mathrm{E}(x,0))}{v} + \frac{x\,\mathrm{E}(x,v)}{1-v}$$

$$eqF := -\mathrm{F}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{F}(x,v) - \mathrm{F}(x,0))}{v} + \frac{x\,(-2\,v+3)\,GG}{(v-1)^2}$$

$$eqG := 2\,GG\,x - GG + x$$

```
> GG:=x/(1-2*x): Fx0:=solve(subs(v=x,eqF)=0,F(x,0)):
  Fxv:=solve(subs(F(x,0)=Fx0,eqF)=0,F(x,v)):
> v0:=(1-sqrt(1-4*x))/2:
  Ex0:=simplify(subs(v=v0,solve(subs(F(x,v)=Fxv,E(x,v)=0,eqE)=0,E(x,
  0)))):
  Exv:=factor(solve(subs(E(x,0)=Ex0,F(x,v)=Fxv,eqE)=0,E(x,v))):
> Dx0:=solve(subs(v=2*x,eqD)=0,DD(x,0)):
  Dxv:=factor(solve(subs(DD(x,0)=Dx0,eqD)=0,DD(x,v))):
> Cx0:=solve(subs(v=x,subs(E(x,v)=Exv,DD(x,v)=Dxv,eqC))=0,C(x,0)):Cx
  v:=solve(subs(C(x,0)=Cx0,subs(E(x,v)=Exv,DD(x,v)=Dxv,eqC))=0,C(x,v
  )):
  Bx0:=solve(subs(v=2*x,subs(C(x,v)=Cxv,eqB))=0,B(x,0)):Bxv:=solve(s
  ubs(B(x,0)=Bx0,C(x,v)=Cxv,eqB)=0,B(x,v)):
  Ax0:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x
  ,Bxv))+1)-1)):
  Ax0:=factor(coeff(Ax0,sqrt(1-4*x),1))*sqrt(1-4*x)+factor(coeff(Ax0
  ,sqrt(1-4*x),0)); taylor(Ax0,x,20);
```

$$Ax0 := \frac{(2\,x^2 - 2\,x + 1)\sqrt{1-4\,x}}{2\,(x-1)^3\,(-1+4\,x)} - (64\,x^{11} - 424\,x^{10} + 1444\,x^9 - 2940\,x^8 + 4008\,x^7 - 3848\,x^6 + 2640\,x^5 - 1296\,x^4 + 449\,x^3 - 105\,x^2 + 15\,x - 1) \Big/ (2\,(x-1)^6\,(-1+2\,x)^5)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1732\,x^7 + 7584\,x^8 + 32309\,x^9 + 133472\,x^{10} + 536722\,x^{11} + 2114755\,x^{12} + 8220487\,x^{13} + 31711653\,x^{14} + 121944196\,x^{15} + 468847971\,x^{16} + 1805526264\,x^{17} + 6970255412\,x^{18} + 26982021532\,x^{19} + \mathrm{O}(x^{20})$$

```
> #-----------------------------------------------
> #12033 a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m123,
  f_m=0^m1230, g=01203, h=01204 regular=6
> #12030 a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m123,
  f_m=0^m1230, g=01203, h=01204 regular=6
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
  eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
```

```
eqC:=-C(x,v)+x/(1-v)+x*DD(x,v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*E(x,v);
eqD:=-DD(x,v)+x/(1-v)+2*x/v*(DD(x,v)-DD(x,0))+x*v/(1-v)*F(x,v)+x/(
1-v)*(GG+HH);
eqE:=-E(x,v)+x/(1-v)+x*F(x,v)+x/v*(E(x,v)-E(x,0))+x/(1-v)*E(x,v);
eqF:=-F(x,v)+x/(1-v)+x/v*(F(x,v)-F(x,0))+x/(1-v)*F(x,v)+x/(1-v)*(H
H+GG); eqG:=-GG+x+x*F(x,0)+x*(GG+HH); eqH:=-HH+x+x*(GG+HH);
```

$$eqA := -\mathrm{A}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{A}(x,v) - \mathrm{A}(x,0))}{v} + \frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{B}(x,v) - \mathrm{B}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v) + \frac{x}{1-v} + x\,\mathrm{DD}(x,v) + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{E}(x,v)}{1-v}$$

$$eqD := -\mathrm{DD}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{DD}(x,v) - \mathrm{DD}(x,0))}{v} + \frac{x\,v\,\mathrm{F}(x,v)}{1-v} + \frac{x\,(GG+HH)}{1-v}$$

$$eqE := -\mathrm{E}(x,v) + \frac{x}{1-v} + x\,\mathrm{F}(x,v) + \frac{x\,(\mathrm{E}(x,v) - \mathrm{E}(x,0))}{v} + \frac{x\,\mathrm{E}(x,v)}{1-v}$$

$$eqF := -\mathrm{F}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{F}(x,v) - \mathrm{F}(x,0))}{v} + \frac{x\,\mathrm{F}(x,v)}{1-v} + \frac{x\,(GG+HH)}{1-v}$$

$$eqG := -GG + x + x\,\mathrm{F}(x,0) + x\,(GG+HH)$$

$$eqH := -HH + x + x\,(GG+HH)$$

```
> solve({eqG,eqH},{HH,GG});GG:= x*(x*F(x,0)-F(x,0)-1)/(2*x-1): HH:=
-1/(2*x-1)*x*(x*F(x,0)+1):
```

$$\left\{GG = \frac{x\,(x\,\mathrm{F}(x,0) - \mathrm{F}(x,0) - 1)}{2\,x-1},\ HH = -\frac{x\,(x\,\mathrm{F}(x,0)+1)}{2\,x-1}\right\}$$

```
> v0:=(1-sqrt(1-4*x))/2: Fx0:=solve(subs(v=v0,eqF)=0,F(x,0)):
Fxv:=solve(subs(F(x,0)=Fx0,eqF)=0,F(x,v)):
Ex0:=solve(limit(subs(F(x,v)=Fxv,E(x,v)=0,eqE),v=v0)=0,E(x,0)):
Exv:=simplify(factor(solve(subs(E(x,0)=Ex0,F(x,v)=Fxv,eqE)=0,E(x,v
)))): Dx0:=solve(limit(subs(F(x,v)=Fxv,eqD),v=2*x)=0,DD(x,0)):
Dxv:=simplify(factor(solve(subs(DD(x,0)=Dx0,F(x,v)=Fxv,F(x,0)=Fx0,
eqD)=0,DD(x,v)))):
Cx0:=solve(subs(v=x,subs(DD(x,v)=Dxv,E(x,v)=Exv,eqC))=0,C(x,0)):
Cxv:=solve(subs(C(x,0)=Cx0,E(x,v)=Exv,DD(x,v)=Dxv,eqC)=0,C(x,v)):
Bx0:=solve(subs(v=2*x,subs(C(x,v)=Cxv,eqB))=0,B(x,0));
Bxv:=solve(subs(B(x,0)=Bx0,C(x,v)=Cxv,eqB)=0,B(x,v)):
Ax0:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x
,Bxv))+1)-1)):
Ax0:=factor(coeff(Ax0,sqrt(1-4*x),1))*sqrt(1-4*x)+factor(coeff(Ax0
,sqrt(1-4*x),0)); taylor(Ax0,x,20);
```

$$Bx0 := -2\,(28\sqrt{1-4x}\,x^4 - 16x^5 - 53\sqrt{1-4x}\,x^3 + 90x^4 + 36\sqrt{1-4x}\,x^2 - 109x^3 - 10\sqrt{1-4x}\,x + 54x^2 + \sqrt{1-4x} - 12x + 1)\,x \Big/ \left(\left(2\sqrt{1-4x}\,x^2 - 4\sqrt{1-4x}\,x + 8x^2 + \sqrt{1-4x} - 6x + 1\right)(2x-1)\left(\sqrt{1-4x}\,x - \sqrt{1-4x} + 3x - 1\right)(-1+4x)\right)$$

$$Ax0 := -\frac{(118x^5 - 284x^4 + 264x^3 - 118x^2 + 25x - 2)\sqrt{1-4x}}{2x^2(2x-1)(-1+4x)^2(x-1)^2} - \frac{16x^6 - 2x^5 - 92x^4 + 136x^3 - 80x^2 + 21x - 2}{2(2x-1)(-1+4x)(x-1)^2x^2}$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 392x^6 + 1763x^7 + 8024x^8 + 36523x^9 + 165258x^{10} + 741449x^{11} + 3296436x^{12} + 14526983x^{13} + 63499268x^{14} + 275533680x^{15} + 1187797230x^{16} + 5090902102x^{17} + O(x^{18})$$

```
> #12023 a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e=1202, regular=4
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
  eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
  eqC:=-C(x,v)+x/(1-v)+x*DD(x,v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*C(x,v);
  eqD:=-DD(x,v)+x/(1-v)+x/v*(DD(x,v)+C(x,v)-DD(x,0)-C(x,0))+x/(1-v)*
  C(x,v)+x/(1-v)*EE; EE:=x/(1-2*x);
```

$$eqA := -\mathrm{A}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{A}(x,v) - \mathrm{A}(x,0))}{v} + \frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v) + \frac{x}{1-v} + \frac{2x\,(\mathrm{B}(x,v) - \mathrm{B}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v) + \frac{x}{1-v} + x\,\mathrm{DD}(x,v) + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqD := -\mathrm{DD}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{DD}(x,v) + \mathrm{C}(x,v) - \mathrm{DD}(x,0) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v} + \frac{x\,EE}{1-v}$$

$$EE := \frac{x}{1-2x}$$

```
> eqC1:=subs(DD(x,v)=solve(eqC,DD(x,v)),eqD):
  KK:=factor(coeff(eqC1,C(x,v))*x*v^2*(1-v));
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$KK := v^3 - v^2x + vx^2 - v^2 + 2vx - x^2$$

```
> map(factor,solve({subs(v=u1,subs(C(x,v)=0,eqC1)),subs(v=u2,subs(C(
  x,v)=0,eqC1))},{C(x,0),DD(x,0)}));
```

$$\{\mathrm{C}(x,0) = -\frac{(x-1)^2\,u1\,u2}{(-1+u2)(-1+2x)(-1+u1)\,x},$$

$$\mathrm{DD}(x,0) = -\frac{2\,u1\,u2\,x^2 - u1\,x^3 - u2\,x^3 - u1\,x^2 - u2\,x^2 + 2\,x^3 - u1\,u2 + u1\,x + u2\,x - x^2}{(-1+u1)\,(-1+2\,x)\,x^2\,(-1+u2)}\}$$

```
> Cx0:= -(x-1)^2*u1*u2/(-1+u2)/(-1+2*x)/(-1+u1)/x: Dx0:=
-(2*u1*u2*x^2-u1*x^3-u2*x^3-u1*x^2-u2*x^2+2*x^3-u1*u2+u1*x+u2*x-x^
2)/(-1+u1)/(-1+2*x)/x^2/(-1+u2):
```

```
> Cxv:=solve(subs(C(x,0)=Cx0,DD(x,0)=Dx0,eqC1)=0,C(x,v)):
Bx0:=solve(subs(v=2*x,subs(C(x,v)=Cxv,eqB))=0,B(x,0)):
Bxv:=solve(subs(B(x,0)=Bx0,C(x,v)=Cxv,eqB)=0,B(x,v)):
Ax0:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x
,Bxv))+1)-1));
```

$$Ax0 := -(24\,u1\,u2\,x^5 - 30\,u1\,u2\,x^4 - 32\,u1\,x^5 - 32\,u2\,x^5 + 4\,x^6 + 4\,u1\,u2\,x^3 + 60\,u1\,x^4 + 60\,u2\,x^4 + 16\,x^5 + 13\,u1\,u2\,x^2 - 47\,u1\,x^3 - 47\,u2\,x^3 - 35\,x^4 - 7\,u1\,u2\,x + 16\,u1\,x^2 + 16\,u2\,x^2 + 28\,x^3 + u1\,u2 - 2\,u1\,x - 2\,u2\,x - 9\,x^2 + x) \Big/ ((x-1)^2\,(6\,x-1)\,(-1+u1)\,(-1+2\,x)^2\,(-1+u2))$$

```
> ss:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=ss[1]: uu2:=ss[2]:
Ax0:=subs(vv=u3,factor(simplify(subs(u3=vv,factor(rationalize(subs
(u1=uu1,u2=uu2,Ax0))))))));
```

$$Ax0 := (8\,u3^2\,x^5 - 4\,u3\,x^6 - 30\,u3^2\,x^4 + 6\,u3\,x^5 - 20\,x^6 + 43\,u3^2\,x^3 + 12\,u3\,x^4 + 16\,x^5 - 29\,u3^2\,x^2 - 33\,u3\,x^3 + 14\,x^4 + 9\,u3^2\,x + 27\,u3\,x^2 - 23\,x^3 - u3^2 - 9\,u3\,x + 9\,x^2 + u3 - x) \Big/ (x\,(6\,x-1)\,(-1+2\,x)^2\,(x-1)^2)$$

```
> simplify(series(subs(u3=vv,Ax0),x,14)) assuming x>0 and x<0.1;
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1751\,x^7 + 7967\,x^8 + 36628\,x^9 + 169850\,x^{10} + 793875\,x^{11} + 3738000\,x^{12} + \mathrm{O}(x^{13})$$

```
> #-------------------------------------------
> #12003: a_m=0^m, b_m=0^m1, c_m=0^m12, d_m=0^m120, e_m=0^m123,
f_m=0^m1203, g=01200, h=01204, j=012030, regular=6
> restart: eqA:=-A(x,v)+x/(1-v)+x/v*(A(x,v)-A(x,0))+x/(1-v)*B(x,v);
eqB:=-B(x,v)+x/(1-v)+2*x/v*(B(x,v)-B(x,0))+x/(1-v)*C(x,v);
eqC:=-C(x,v)+x/(1-v)+x*DD(x,v)+x/v*(C(x,v)-C(x,0))+x/(1-v)*E(x,v);
eqD:=-DD(x,v)+x/(1-v)*(1+GG+HH)+x/v*(DD(x,v)-DD(x,0))+x/(1-v)*F(x,
v);
eqE:=-E(x,v)+x/(1-v)+x/v*(E(x,v)-E(x,0))+x*F(x,v)+x/(1-v)*E(x,v);
eqF:=-F(x,v)+x/(1-v)*(1+HH+JJ)+x/v*(F(x,v)-F(x,0))+x/(1-v)*F(x,v);
GG:=x/(1-2*x):   JJ:=x/(1-x): HH:=(x+x*JJ+x*F(x,0))/(1-x);
```

$$eqA := -\mathrm{A}(x,v) + \frac{x}{1-v} + \frac{x\,(\mathrm{A}(x,v) - \mathrm{A}(x,0))}{v} + \frac{x\,\mathrm{B}(x,v)}{1-v}$$

$$eqB := -\mathrm{B}(x,v) + \frac{x}{1-v} + \frac{2\,x\,(\mathrm{B}(x,v) - \mathrm{B}(x,0))}{v} + \frac{x\,\mathrm{C}(x,v)}{1-v}$$

$$eqC := -\mathrm{C}(x,v) + \frac{x}{1-v} + x\,\mathrm{DD}(x,v) + \frac{x\,(\mathrm{C}(x,v) - \mathrm{C}(x,0))}{v} + \frac{x\,\mathrm{E}(x,v)}{1-v}$$

$$eqD := -\mathrm{DD}(x, v) + \frac{x\,(1 + GG + HH)}{1 - v} + \frac{x\,(\mathrm{DD}(x, v) - \mathrm{DD}(x, 0))}{v} + \frac{x\,\mathrm{F}(x, v)}{1 - v}$$

$$eqE := -\mathrm{E}(x, v) + \frac{x}{1 - v} + \frac{x\,(\mathrm{E}(x, v) - \mathrm{E}(x, 0))}{v} + x\,\mathrm{F}(x, v) + \frac{x\,\mathrm{E}(x, v)}{1 - v}$$

$$eqF := -\mathrm{F}(x, v) + \frac{x\,(1 + HH + JJ)}{1 - v} + \frac{x\,(\mathrm{F}(x, v) - \mathrm{F}(x, 0))}{v} + \frac{x\,\mathrm{F}(x, v)}{1 - v}$$

$$HH := \frac{x + \frac{x^2}{1 - x} + x\,\mathrm{F}(x, 0)}{1 - x}$$

```
> v0:=(1-sqrt(1-4*x))/2:   Fx0:=solve(subs(v=v0,eqF)=0,F(x,0)):
  Fxv:=factor(solve(subs(F(x,0)=Fx0,eqF)=0,F(x,v))):
  Ex0:=solve(limit(subs(F(x,v)=Fxv,E(x,v)=0,eqE),v=v0)=0,E(x,0)):
  Exv:=simplify(factor(solve(subs(E(x,0)=Ex0,F(x,v)=Fxv,eqE)=0,E(x,v
  )))):
  Dx0:=solve(limit(subs(F(x,v)=Fxv,F(x,0)=Fx0,eqD),v=x)=0,DD(x,0)):
  Dxv:=factor(solve(subs(F(x,v)=Fxv,F(x,0)=Fx0,DD(x,0)=Dx0,eqD)=0,DD
  (x,v))):
  Cx0:=factor(solve(limit(subs(E(x,v)=Exv,DD(x,v)=Dxv,eqC),v=x)=0,C(
  x,0))):
  Cxv:=simplify(simplify(factor(rationalize(solve(subs(E(x,v)=Exv,DD
  (x,v)=Dxv,C(x,0)=Cx0,eqC)=0,C(x,v))))+1)-1):
  Bx0:=factor(solve(limit(subs(C(x,v)=Cxv,eqB),v=2*x)=0,B(x,0))):
  Bxv:=simplify(simplify(factor(rationalize(solve(subs(B(x,0)=Bx0,C(
  x,v)=Cxv,eqB)=0,B(x,v))))+1)-1):
  Ax0:=factor(simplify(simplify(rationalize(x/(1-x)+x/(1-x)*limit(Bx
  v,v=x))+1)-1));
  Ax0:=factor(coeff(Ax0,sqrt(1-4*x),1))*sqrt(1-4*x)+factor(coeff(Ax0
  ,sqrt(1-4*x),0)); taylor(Ax0,x,20);
```

$$Ax0 := -\left(1 - \sqrt{1-4x} + 20\sqrt{1-4x}\,x + 212x^2 - 22x + 256x^{12} - 2048x^{11} + 7168x^{10} + 2288\sqrt{1-4x}\,x^9 + 64\sqrt{1-4x}\,x^{11} - 576\sqrt{1-4x}\,x^{10} - 15716x^9 + 23616x^8 - 5288\sqrt{1-4x}\,x^8 + 7888\sqrt{1-4x}\,x^7 - 7958\sqrt{1-4x}\,x^6 + 5528\sqrt{1-4x}\,x^5 - 25242x^7 + 19510x^6 - 10898x^5 - 2637\sqrt{1-4x}\,x^4 + 4333x^4 + 844\sqrt{1-4x}\,x^3 - 172\sqrt{1-4x}\,x^2 - 1188x^3\right) \Big/ \left(2x(-1+2x)^3(-1+4x)^2(-1+x)^6\right)$$

$$Ax0 := -\frac{(2x^2 - 4x + 1)(4x^2 - 6x + 1)\sqrt{1-4x}}{2(-1+x)^2(-1+4x)^2 x} - \left(64x^{11} - 496x^{10} + 1668x^9 - 3512x^8 + 5026x^7 - 5054x^6 + 3614x^5 - 1821x^4 + 628x^3 - 140x^2 + 18x - 1\right) \Big/ \left(2(-1+x)^6(-1+4x)(-1+2x)^3 x\right)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 391x^6 + 1743x^7 + 7803x^8 + 34731x^9 + 153242x^{10} + 670337x^{11}$$

$$+ 2910038\, x^{12} + 12549935\, x^{13} + 53813093\, x^{14} + 229576826\, x^{15} + 974986767\, x^{16} + 4123843376\, x^{17} + 17378650044\, x^{18} + O(x^{19})$$

```
> #11230: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,3,:
  f=0113: g=01124: Reg5:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(DD+F):
  eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x/(1-v)*(E+G):
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+G):
  eqF:=-FF+x+x*DD1+x*DD1+x*GG: eqG:=-GG+x+x*E1+x*GG:
> solve({eqF,eqG},{FF,GG}); F:= x*(2*DD1*x-E1*x-2*DD1-1)/(x-1): G:=
  -x*(E1+1)/(x-1):
```

$$\left\{FF = \frac{x\,(2\,DD1\,x - E1\,x - 2\,DD1 - 1)}{x-1},\ GG = -\frac{x\,(E1+1)}{x-1}\right\}$$

```
> Exv:=SolveKE(E,E1,eqE,2): Ex0:=subs(v=0,Exv):
  Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=Ex0,eqD),1):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,E1=Ex0,DD1=subs(v=0,Dxv),eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
  FinA1:=factor(x/(1-x)*(1+limit(Bxv,v=x)));
  simplify(series(subs(u3=vv,FinA1),x,15)) assuming x>0 and x<0.1;
```

$$FinA1 := -x\,(30\,x^4\sqrt{1-4\,x} - 36\,x^5 - 54\sqrt{1-4\,x}\,x^3 + 108\,x^4 + 36\sqrt{1-4\,x}\,x^2 - 112\,x^3 - 10\sqrt{1-4\,x}\,x + 54\,x^2 + \sqrt{1-4\,x} - 12\,x + 1) \Big/ ((x-1)\,(2\,x-1)\,(-1+4\,x)\,(5\sqrt{1-4\,x}\,x^2 - 4\,x^3 - 5\sqrt{1-4\,x}\,x + 13\,x^2 + \sqrt{1-4\,x} - 7\,x + 1))$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1746\,x^7 + 7857\,x^8 + 35267\,x^9 + 157196\,x^{10} + 694788\,x^{11} + 3045072\,x^{12} + 13241392\,x^{13} + 57174546\,x^{14} + O(x^{15})$$

```
> #11220: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,2, f=0113,
  g=01133, h=011225, Reg5
> #11200: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,0, f=0113,
  g=01130, h=011205, Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
```

```
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(DD+F):
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*(DD+F)+x*E:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+G+H):
eqF:=-FF+x+x*DD1+x*FF+x*GG: eqG:=-GG+x+x*E1+x*GG+x*HH:
eqH:=-HH+x+x*GG+x*HH:
```

```
> solve({eqF,eqG,eqH},{FF,GG,HH}); F:=
-x*(E1*x^2+2*DD1*x-E1*x-DD1+x-1)/(2*x^2-3*x+1): G:=
x*(E1*x-E1-1)/(2*x-1): H:= -x*(E1*x+1)/(2*x-1):
```

$$\{FF = -\frac{x\,(E1\,x^2 + 2\,DD1\,x - E1\,x - DD1 + x - 1)}{2\,x^2 - 3\,x + 1},\ GG = \frac{x\,(E1\,x - E1 - 1)}{2\,x - 1},$$
$$HH = -\frac{x\,(E1\,x + 1)}{2\,x - 1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2):
Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=subs(v=0,Exv),eqD),2):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=subs(v=0,Dxv),E1=subs(v=0,Exv),e
qC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA1:=factor(simplify(simplify(rationalize(x/(1-x)*(1+limit(Bxv,v
=x)))+1)-1)); simplify(series(FinA1,x,15)) assuming x>0 and x<0.1;
```

$$FinA1 := (\sqrt{1-4\,x}\,x^4 - 40\sqrt{1-4\,x}\,x^3 - 53\,x^4 + 37\,x^2\sqrt{1-4\,x} + 98\,x^3 - 11\,x\sqrt{1-4\,x} - 57\,x^2 + \sqrt{1-4\,x} + 13\,x - 1) \Big/ (2\,x^2\,(-1+4\,x)^2)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 392\,x^6 + 1767\,x^7 + 8097\,x^8 + 37308\,x^9 + 171759\,x^{10} + 787376\,x^{11} + 3587992\,x^{12} + O(x^{13})$$

```
> #11203: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,3,:
f=0113: g=01120: h=01124: j=011230: Reg5:
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(DD+F):
eqD:=-DD+x/(1-v)*(1+G)+x/v*(DD-DD1)+x/(1-v)*(E+H):
eqE:=-E+x/(1-v)*(1+J)+x/v*(E-E1)+x/(1-v)*(E+H):
eqF:=-FF+x+x*DD1+x*HH+x*GG: eqG:=-GG+x+x*GG+x*GG:
eqH:=-HH+x+x*JJ+x*E1+x*HH: eqJ:=-JJ+x+x*JJ:
```

```
> map(factor,solve({eqF,eqG,eqH,eqJ},{FF,GG,HH,JJ})); F:=
x*(2*DD1*x^3-2*E1*x^3-5*DD1*x^2+3*E1*x^2+x^3+4*DD1*x-E1*x-x^2-DD1+
```

```
2*x-1)/(2*x-1)/(x-1)^2: G:= -x/(2*x-1): H:=
-x*(E1*x-E1-1)/(x-1)^2: J:= -x/(x-1):
```

$$\{FF = \frac{x\,(2\,DD1\,x^3 - 2\,E1\,x^3 - 5\,DD1\,x^2 + 3\,E1\,x^2 + x^3 + 4\,DD1\,x - E1\,x - x^2 - DD1 + 2\,x - 1)}{(2\,x-1)\,(x-1)^2},$$

$$GG = -\frac{x}{2\,x-1},\ HH = -\frac{x\,(E1\,x - E1 - 1)}{(x-1)^2},\ JJ = -\frac{x}{x-1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2):
Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=subs(v=0,Exv),eqD),1):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=subs(v=0,Dxv),E=Exv,E1=subs(v=0,
Exv),eqC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA1:=factor(simplify(simplify(rationalize(x/(1-x)*(1+limit(Bxv,v
=x)))+1)-1)); simplify(series(FinA1,x,15)) assuming x>0 and x<0.1;
```

$$FinA1 := -(8\,x^8\sqrt{1-4\,x} + 64\,x^9 - 106\,x^7\sqrt{1-4\,x} - 328\,x^8 + 382\sqrt{1-4\,x}\,x^6 + 946\,x^7 - 662\sqrt{1-4\,x}\,x^5 - 1660\,x^6 + 650\sqrt{1-4\,x}\,x^4 + 1858\,x^5 - 381\sqrt{1-4\,x}\,x^3 - 1348\,x^4 + 132\sqrt{1-4\,x}\,x^2 + 627\,x^3 - 25\sqrt{1-4\,x}\,x - 180\,x^2 + 2\sqrt{1-4\,x} + 29\,x - 2) \Big/ (2\,x\,(2\,x-1)^3\,(-1+4\,x)\,(x-1)^4)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 389\,x^6 + 1710\,x^7 + 7485\,x^8 + 32364\,x^9 + 138108\,x^{10} + 582744\,x^{11} + 2437269\,x^{12} + 10125882\,x^{13} + O(x^{14})$$

```
> #11202: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,0, f=0113:
g=01130: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(DD+F):
eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*(DD+F):
eqE:=-E+x/(1-v)+x/v*(E+DD-E1-DD1)+x/(1-v)*(DD+F):
eqF:=-FF+x+x*GG+x*DD1+x*FF: eqG:=-GG+x+x*E1+x*DD1+x*FF:
> solve({eqF,eqG},{FF,GG}); F:= -x*(DD1*x+E1*x+DD1+x+1)/(x^2+x-1):
G:= -x*(-E1*x+DD1+E1+1)/(x^2+x-1):
```

$$\{FF = -\frac{x\,(DD1\,x + E1\,x + DD1 + x + 1)}{x^2 + x - 1},\ GG = -\frac{x\,(-E1\,x + DD1 + E1 + 1)}{x^2 + x - 1}\}$$

```
> eqD1:=subs(E=solve(eqD,E),eqE):
KK:=factor(v^2*(1-v)*x*coeff(eqD1,DD)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

```
ss:=map(factor,solve({subs(v=u1,subs(DD=0,eqD1)),subs(v=u2,subs(DD
=0,eqD1))},{DD1,E1})); Dx0:=op(op(ss)[1])[2]:
Ex0:=op(op(ss)[2])[2]:
Dxv:=factor(solve(subs(E1=Ex0,DD1=Dx0,eqD1)=0,DD));
```

$$-x^2+(x^2+2x)v+(-x-1)v^2+v^3$$

$$ss := \{DD1 = \frac{u1\,u2}{x\,(u1\,u2-u1-u2-x+1)}, E1 = -\frac{(u2\,x+u2-x)\,(u1\,x+u1-x)}{x^2\,(u1\,u2-u1-u2-x+1)}\}$$

$$Dxv := -\frac{(u2-v)\,(u1-v)\,x}{(u1\,u2-u1-u2-x+1)\,(v^3-v^2\,x+v\,x^2-v^2+2\,v\,x-x^2)}$$

```
> Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=Dx0,E1=Ex0,eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA:=factor(simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],
x/(1-x)+x/(1-x)*subs(v=x,Bxv)))+1)-1));
simplify(series(subs(u3=vv,FinA),x,13)) assuming x>0 and x<0.1;
```

$$FinA := -\frac{(u3^2-5\,u3\,x+3\,x^2)\,(\sqrt{1-4\,x}+4\,x-1)}{2\,(6\,x-1)\,(u3-x)^2}$$

$$x+2\,x^2+6\,x^3+22\,x^4+90\,x^5+392\,x^6+1772\,x^7+8197\,x^8+38495\,x^9+182684\,x^{10}+873637\,x^{11}+4202782\,x^{12}+O(x^{13})$$

```
> #11120: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,1,: 0,0,0,0,0,1,1,1,2,:
f=01113, g=01114: Reg5
> #10122: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: 0,0,0,0,0,1,0,1,2,:
f=01013, g=01014: Reg5
> #10120: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: 0,0,0,0,0,1,0,1,2,:
f=01013, g=01014: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x/(1-v)*(E+F+G):
eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+F+G):
eqF:=-FF+x+x*E1+x*FF+x*GG: eqG:=-GG+x+x*GG+x*FF:
> solve({eqF,eqG},{FF,GG});F:= x*(E1*x-E1-1)/(2*x-1): G :=
-x*(E1*x+1)/(2*x-1):
```

$$\{FF = \frac{x\,(E1\,x - E1 - 1)}{2\,x - 1},\ GG = -\frac{x\,(E1\,x + 1)}{2\,x - 1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2):
  Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=subs(v=0,Exv),eqD),1):
  eqB1:=subs(DD=Dxv,C=solve(eqB,C),eqC):
  KK:=factor(v^2*x*(1-v)*coeff(eqB1,B)):  taylor(KK,v,10);
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
  uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
  ss:=map(factor,solve({subs(v=u1,subs(B=0,eqB1)),subs(v=u2,subs(B=0
  ,eqB1))},{C1,B1})): Bx0:=op(op(ss)[1])[2]: Cx0:=op(op(ss)[2])[2]:
  Bxv:=(solve(subs(C1=Cx0,B1=Bx0,eqB1)=0,B)):
```

$$-x^2 + (x^2 + 2\,x)\,v + (-x - 1)\,v^2 + v^3$$

```
> FinA:=factor(simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],
  x/(1-x)+x/(1-x)*subs(v=x,Bxv)))+1)-1)):
  FinA:=subs(vv=u3,simplify(subs(u3=vv,FinA)));
  simplify(series(subs(u3=vv,FinA),x,13)) assuming x>0 and x<0.1;
```

$$FinA := (\sqrt{1-4\,x}\,u3^2 - 2\,x^2\sqrt{1-4\,x} - 4\,x\,u3^2 - 8\,x^3 - \sqrt{1-4\,x}\,u3 + 2\,x\sqrt{1-4\,x} + u3^2 + 4\,u3\,x + 2\,x^2 - u3) \Big/ (2\,x^2\,(-1+4\,x))$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1756\,x^7 + 8037\,x^8 + 37208\,x^9 + 173558\,x^{10} + O(x^{11})$$

```
> #11110: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,1,: 0,0,0,0,0,1,1,1,1,:
  f=011113, g=011114, h=011115: Reg5:
> #11010: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,1,1,0,1,:
  f=011013, g=011014, h=011015: Reg5:
> #10110: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: 0,0,0,0,0,1,0,1,1,:
  f=010113, g=010114, h=011015: Reg5:
> #10010: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,0,: 0,0,0,0,0,1,0,0,1,:
  f=010013, g=010014, h=010015: Reg5:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x*E+x/v^2*(B-B1-B2*v)+x/v*(B-B1)+x/(
  1-v)*B: eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+F+G+H):
  eqF:=-FF+x+x*E1+x*FF+x*GG+x*HH: eqG:=-GG+x+x*FF+x*GG+x*HH:
```

```
eqH:=-HH+x+x*GG+x*HH:
> solve({eqF,eqG,eqH},{FF,GG,HH}); F:=-x*(2*E1*x-E1-1)/(x^2-3*x+1):
G:=-x*(E1*x^2-E1*x-1)/(x^2-3*x+1): H:=x*(E1*x^2-x+1)/(x^2-3*x+1):
```

$$\{FF=-\frac{x\,(2\,E1\,x-E1-1)}{x^2-3\,x+1},\ GG=-\frac{x\,(E1\,x^2-E1\,x-1)}{x^2-3\,x+1},\ HH=\frac{x\,(E1\,x^2-x+1)}{x^2-3\,x+1}\}$$

```
>
> Exv:=SolveKE(E,E1,eqE,2);
eqB1:=(subs(E=Exv,DD=solve(subs(C=solve(eqB,C),eqC)=0,DD),eqD));
KK:=factor(v^3*x^2*(1-v)*coeff(eqB1,B)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$Exv:=\frac{(\sqrt{1-4\,x}+2\,v-1)\,x}{(2\,x\sqrt{1-4\,x}-2\,x^2-1-\sqrt{1-4\,x}+4\,x)\,(v^2-v+x)}$$

$$eqB1:=-(B1\,v^2\,x^2+C1\,v^2\,x^2+B\,v^3-B\,v^2\,x+B\,v\,x^2+B1\,v^2\,x-2\,B1\,v\,x^2-C1\,v\,x^2+v^2\,x^2-B\,v^2+2\,B\,v\,x-B\,x^2-B1\,v\,x+B1\,x^2+v^2\,x-v\,x^2)\Big/(v^2\,(-1+v)\,x^2)+\frac{x}{1-v}+x\,((B1\,v^2\,x^2+C1\,v^2\,x^2+B\,v^3-B\,v^2\,x+B\,v\,x^2+B1\,v^2\,x-2\,B1\,v\,x^2-C1\,v\,x^2+v^2\,x^2-B\,v^2+2\,B\,v\,x-B\,x^2-B1\,v\,x+B1\,x^2+v^2\,x-v\,x^2)\Big/(v^2\,(-1+v)\,x^2)-DD1)/v+\frac{x^2\,(\sqrt{1-4\,x}+2\,v-1)}{(2\,x\sqrt{1-4\,x}-2\,x^2-1-\sqrt{1-4\,x}+4\,x)\,(v^2-v+x)}+\frac{x\,(-B2\,v+B-B1)}{v^2}+\frac{x\,(B-B1)}{v}+\frac{x\,B}{1-v}$$

$$x^3-3\,x^2\,v+(2\,x^2+3\,x)\,v^2+(-2\,x-1)\,v^3+v^4$$

```
> #simplify(taylor((v-u1)*(v-u2)*(v-u3)*(v-u4),v,10));
> factor(coeff(subs(B=0,eqB1),B1)),factor(coeff(subs(B=0,eqB1),B2)),
factor(coeff(subs(B=0,eqB1),C1)),factor(coeff(subs(B=0,eqB1),DD1))
; eqB1:=subs(B2=LB-DD1,eqB1):
```

$$-\frac{v^2\,x^2+v^2\,x+v^2-2\,v\,x+x^2}{x\,v^3},\ -\frac{x}{v},\ -\frac{v-x}{v^2},\ -\frac{x}{v}$$

```
> ss:=map(factor,solve({subs(v=u1,B=0,eqB1),subs(v=u2,B=0,eqB1),subs
(v=u3,B=0,eqB1)},{B1,C1,LB})):
> op(op(ss)[1])[1],op(op(ss)[2])[1],op(op(ss)[3])[1];
Bx0:=simplify(op(op(ss)[1])[2]):Cx0:=simplify(op(op(ss)[2])[2]):
LB0:=simplify(op(op(ss)[3])[2]):
```

$$B1,\ C1,\ LB$$

```
> ss1:=solve({f=LB0,u1*u2*u3=x^3/u4,u1*u2+u1*u3+u2*u3=2*x^2+3*x-(1+2
*x-u4)*u4,u1+u2+u3=1+2*x-u4},{u1,u2,u3,f}): op(op(ss1)[1])[1];
LB0:=subs(vv=u4,simplify(subs(u4=vv,op(op(ss1)[1])[2])));
```

$$f$$

$$LB0 := -(4\,u4^3\,x^3 - 6\,u4^3\,x^2 + 4\,u4^3\,x - 2\,u4^3 + u4^2\,x + 11\,u4^2\,x^3 - \sqrt{1-4\,x}\,x^2 - 2\sqrt{1-4\,x}\,x^4 + 2\sqrt{1-4\,x}\,x^3 + x^5\sqrt{1-4\,x} - 2\,x^6 + 4\,x^4 + x^5 - 2\sqrt{1-4\,x}\,x^4\,u4 - 3\sqrt{1-4\,x}\,x^2\,u4 - \sqrt{1-4\,x}\,u4^2\,x^2 + 3\sqrt{1-4\,x}\,u4\,x^3 - 4\,x^3 + 3\,x^2 - 5\,u4\,x + 2\,u4^2 + \sqrt{1-4\,x}\,u4^2\,x^3 + \sqrt{1-4\,x}\,u4\,x + \sqrt{1-4\,x}\,u4^2\,x + 3\,x^2\,u4 + 8\,x^5\,u4 - 8\,x^4\,u4 - 8\,u4^2\,x^4 - 5\,u4^2\,x^2 - u4\,x^3) \Big/ (3\,u4^3\,x^3 - 7\,u4^3\,x^2 + 5\,u4^3\,x - u4^3 + 3\,u4^2\,x + 17\,u4^2\,x^3 - 2\sqrt{1-4\,x}\,x^4 + \sqrt{1-4\,x}\,x^3 + 3\sqrt{1-4\,x}\,u4^3\,x + \sqrt{1-4\,x}\,u4^3\,x^3 - 3\sqrt{1-4\,x}\,u4^3\,x^2 + 2\,x^5\sqrt{1-4\,x} - 2\,x^6 - 4\,x^4 + 6\,x^5 - 8\sqrt{1-4\,x}\,x^4\,u4 - 3\sqrt{1-4\,x}\,x^2\,u4 + 2\,x^5\sqrt{1-4\,x}\,u4 - 2\sqrt{1-4\,x}\,u4^2\,x^4 - 7\sqrt{1-4\,x}\,u4^2\,x^2 + 7\sqrt{1-4\,x}\,u4\,x^3 + x^3 + 7\sqrt{1-4\,x}\,u4^2\,x^3 + 3\sqrt{1-4\,x}\,u4^2\,x - \sqrt{1-4\,x}\,u4^3 - 3\,x^2\,u4 + 10\,x^5\,u4 - 20\,x^4\,u4 - 8\,u4^2\,x^4 - 13\,u4^2\,x^2 + 13\,u4\,x^3)$$

```
> ss1:=solve({f=Cx0,u1*u2*u3=x^3/u4,u1*u2+u1*u3+u2*u3=2*x^2+3*x-(1+2
*x-u4)*u4,u1+u2+u3=1+2*x-u4},{u1,u2,u3,f}): op(op(ss1)[1])[1];
Cx0:=subs(vv=u4,simplify(subs(u4=vv,op(op(ss1)[1])[2])));
```

$$f$$

$$Cx0 := x\,(\sqrt{1-4\,x}\,u4^3\,x - 2\sqrt{1-4\,x}\,u4^2\,x^2 + 2\sqrt{1-4\,x}\,u4\,x^3 - \sqrt{1-4\,x}\,x^4 - 2\,u4^3\,x^2 + 4\,u4^2\,x^3 - 4\,x^4\,u4 - \sqrt{1-4\,x}\,u4^3 + 2\sqrt{1-4\,x}\,u4^2\,x - \sqrt{1-4\,x}\,x^2\,u4 + \sqrt{1-4\,x}\,x^3 + 3\,u4^3\,x - 6\,u4^2\,x^2 + 4\,u4\,x^3 + x^4 - \sqrt{1-4\,x}\,u4\,x - u4^3 + 2\,u4^2\,x + x^2\,u4 - 3\,x^3 - u4\,x + 2\,x^2) \Big/ (3\,u4^3\,x^3 - 7\,u4^3\,x^2 + 5\,u4^3\,x - u4^3 + 3\,u4^2\,x + 17\,u4^2\,x^3 - 2\sqrt{1-4\,x}\,x^4 + \sqrt{1-4\,x}\,x^3 + 3\sqrt{1-4\,x}\,u4^3\,x + \sqrt{1-4\,x}\,u4^3\,x^3 - 3\sqrt{1-4\,x}\,u4^3\,x^2 + 2\,x^5\sqrt{1-4\,x} - 2\,x^6 - 4\,x^4 + 6\,x^5 - 8\sqrt{1-4\,x}\,x^4\,u4 - 3\sqrt{1-4\,x}\,x^2\,u4 + 2\,x^5\sqrt{1-4\,x}\,u4 - 2\sqrt{1-4\,x}\,u4^2\,x^4 - 7\sqrt{1-4\,x}\,u4^2\,x^2 + 7\sqrt{1-4\,x}\,u4\,x^3 + x^3 + 7\sqrt{1-4\,x}\,u4^2\,x^3 + 3\sqrt{1-4\,x}\,u4^2\,x - \sqrt{1-4\,x}\,u4^3 - 3\,x^2\,u4 + 10\,x^5\,u4 - 20\,x^4\,u4 - 8\,u4^2\,x^4 - 13\,u4^2\,x^2 + 13\,u4\,x^3)$$

```
> ss1:=solve({f=Bx0,u1*u2*u3=x^3/u4,u1*u2+u1*u3+u2*u3=2*x^2+3*x-(1+2
*x-u4)*u4,u1+u2+u3=1+2*x-u4},{u1,u2,u3,f}): op(op(ss1)[1])[1];
Bx0:=subs(vv=u4,simplify(subs(u4=vv,op(op(ss1)[1])[2])));
```

$$f$$

$$Bx0 := -(\sqrt{1-4\,x}\,u4^3\,x^2 - 2\sqrt{1-4\,x}\,u4^2\,x^3 + 2\sqrt{1-4\,x}\,x^4\,u4 - 2\sqrt{1-4\,x}\,u4^3\,x + 4\sqrt{1-4\,x}\,u4^2\,x^2 - 4\sqrt{1-4\,x}\,u4\,x^3 + \sqrt{1-4\,x}\,x^4 - u4^3\,x^2 + 2\,x^4\,u4 + \sqrt{1-4\,x}\,u4^3 - 2\sqrt{1-4\,x}\,u4^2\,x + \sqrt{1-4\,x}\,x^2\,u4 + 2\,u4^2\,x^2 - 6\,u4\,x^3 + x^4 + u4^3 - 4\,u4^2\,x + 7\,x^2\,u4 - 2\,x^3)\,x \Big/ (3\,u4^3\,x^3 - 7\,u4^3\,x^2 + 5\,u4^3\,x - u4^3 + 3\,u4^2\,x + 17\,u4^2\,x^3 - 2\sqrt{1-4\,x}\,x^4 + \sqrt{1-4\,x}\,x^3 + 3\sqrt{1-4\,x}\,u4^3\,x + \sqrt{1-4\,x}\,u4^3\,x^3 - 3\sqrt{1-4\,x}\,u4^3\,x^2 + 2\,x^5\sqrt{1-4\,x} - 2\,x^6 - 4\,x^4 + 6\,x^5 - 8\sqrt{1-4\,x}\,x^4\,u4 - 3\sqrt{1-4\,x}\,x^2\,u4 + 2\,x^5\sqrt{1-4\,x}\,u4 - 2\sqrt{1-4\,x}\,u4^2\,x^4 - 7\sqrt{1-4\,x}\,u4^2\,x^2 + 7\sqrt{1-4\,x}\,u4\,x^3 + x^3 + 7\sqrt{1-4\,x}\,u4^2\,x^3 + 3\sqrt{1-4\,x}\,u4^2\,x - \sqrt{1-4\,x}\,u4^3 - 3\,x^2\,u4 + 10\,x^5\,u4 - 20\,x^4\,u4 - 8\,u4^2\,x^4 - 13\,u4^2\,x^2 + 13\,u4\,x^3)$$

```
>
```

```
> Bxv:=simplify(solve(subs(B1=Bx0,C1=Cx0,LB=LB0,eqB1)=0,B)):
> FinA:=factor(simplify(simplify(rationalize(simplify(x/(1-x)+x/(1-x
  )*subs(v=x,Bxv)))+1)-1)):
> FinA:=factor(simplify(subs(vv=u4,simplify(subs(u4=vv,FinA)+1)-1)))
  ;
```

$$FinA := -\left(\sqrt{1-4x}\,u4^3\,x - \sqrt{1-4x}\,u4^2\,x^2 + \sqrt{1-4x}\,u4\,x^3 - \sqrt{1-4x}\,u4^3 + \sqrt{1-4x}\,x^2\,u4 + \sqrt{1-4x}\,x^3 - u4^3\,x + u4^2\,x^2 - u4\,x^3 + 4x^4 + \sqrt{1-4x}\,u4^2 - 2\sqrt{1-4x}\,u4\,x + u4^3 - x^2\,u4 - 3x^3 - u4^2 + 2\,u4\,x\right) \Big/ \left(2x^4\right)$$

```
> simplify(series(subs(u4=vv,FinA),x,20));
```

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 393x^6 + 1788x^7 + 8361x^8 + 39874x^9 + 193044x^{10} + 946002x^{11} + 4683199x^{12} + 23387929x^{13} + 117697849x^{14} + 596342447x^{15} + O(x^{16})$$

```
> #11102: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,1,: 0,0,0,0,0,1,1,1,2:
  f=01113, g=01114, h=011120: j=01110: Reg5
> #10102: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: 0,0,0,0,0,1,0,1,2:
  f=01013, g=01014, h=010120: j=01010: Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*(E+F+G+H):
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+G+H+J):
  eqF:=-FF+x+x*FF+x*FF: eqG:=-GG+x+x*E1+x*JJ+x*GG+x*HH:
  eqH:=-HH+x+x*JJ+x*GG+x*HH: eqJ:=-JJ+x+x*JJ:
> solve({eqF,eqG,eqH,eqJ},{FF,GG,HH,JJ});F:=-x/(2*x-1):G:=(E1*x^2-2*
  E1*x+E1+1)*x/(x-1)/(2*x-1):H:=-x*(E1*x^2-E1*x-1)/(2*x^2-3*x+1):J:=
  -x/(x-1):
```

$$\left\{FF = -\frac{x}{2x-1},\ GG = \frac{(E1\,x^2 - 2\,E1\,x + E1 + 1)\,x}{(x-1)(2x-1)},\ HH = -\frac{x\,(E1\,x^2 - E1\,x - 1)}{2x^2 - 3x + 1},\ JJ = -\frac{x}{x-1}\right\}$$

```
> Exv:=SolveKE(E,E1,eqE,2):
  eqB1:=subs(E=Exv,E1=subs(v=0,Exv),DD=solve(subs(C=solve(eqB,C),eqC
  ),DD),eqD):
```

$$-\frac{v\,x + v - x}{v^2},\ -\frac{x}{v},\ -\frac{v^3 - v^2\,x + v\,x^2 - v^2 + 2\,v\,x - x^2}{v^2\,(-1+v)\,x}$$

$$-x^2 + (x^2 + 2x)\,v + (-x-1)\,v^2 + v^3$$

```
> ss:=map(factor,solve({subs(v=u1,B=0,eqB1),subs(v=u2,B=0,eqB1)},{B1
  ,C1})): op(op(ss)[1])[1],op(op(ss)[2])[1];
  Bx0:=op(op(ss)[1])[2]:Cx0:=op(op(ss)[2])[2]:
  Bxv:=solve(subs(B1=Bx0,C1=Cx0,DD1=Dx0,eqB1)=0,B):
```

$$B1, C1$$

```
> FinA:=factor(simplify(simplify(rationalize(simplify(x/(1-x)+x/(1-x
  )*subs(v=x,Bxv)))+1)-1)):
> ss1:=solve({f=FinA,u1+u2=1+x-u3,u1*u2=x^2/u3},{u1,u2,f}):
  FinA:=factor(subs(vv=u3,simplify(subs(u3=vv,op(ss1[1])[2]))));
  taylor(subs(u3=vv,FinA),x,20);
```

$$FinA := -(-11x^2 + 2x + 22x^3 + 2x^6 u3 - 6x^5 u3 + 10x^4 u3 - 6x^3 u3 - 3x^2 u3 - u3 - 2u3^2 x^6 + 6u3^2 x^5 - 10u3^2 x^4 + 6u3^2 x^3 + 3u3^2 x^2 - 4u3^2 x + 4x^7 - 8x^6 + 14x^5 - 4\sqrt{1-4x}\, u3\, x - 21x^4 + 9x^2\sqrt{1-4x} - 2x\sqrt{1-4x} + \sqrt{1-4x}\, u3 - \sqrt{1-4x}\, u3^2 + 2\sqrt{1-4x}\, u3^2 x^3 - 5\sqrt{1-4x}\, u3^2 x^2 - 2\sqrt{1-4x}\, x^3 u3 + 4\sqrt{1-4x}\, u3^2 x + 5x^2\sqrt{1-4x}\, u3 + 9\sqrt{1-4x}\, x^4 - 14\sqrt{1-4x}\, x^3 - 2x^5\sqrt{1-4x} + 4x\, u3 + u3^2) \Big/ (2x^3(2x-1)(x-1)^3)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 390x^6 + 1738x^7 + 7859x^8 + 35874x^9 + 164987x^{10} + 763840x^{11} + 3558013x^{12} + 16668042x^{13} + 78501085x^{14} + 371566461x^{15} + 1766983725x^{16} + O(x^{17})$$

```
>
> #11101: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,1,: Reg4
> #11011: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: Reg4
> #11001: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: Reg4
> #10001,10011: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,0,: Reg4
> #10101,10111: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*B+x/v*(B-B1):
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/v^3*(A-A1-A2*v-A3*v^2)+x/v^2*(B-B1
  -v*B2)+x/v*(B-B1)+x/(1-v)*B:
> BB:=solve(eqA,B): eqBB:=subs(B=BB,eqB): CC:=solve(eqBB,C):
  eqCC:=subs(C=CC,B=BB,eqC): DDD:=solve(eqCC,DD):
  eqA1:=simplify(subs(DD=DDD,subs(C=CC,B=BB,eqD)));
```

$$eqA1 := (-v^3 x^4 - A x^4 + A1 x^4 - A v^4 + A v^5 + v^4 x + 3 v^2 x^3 - x^3 v^3 - x^4 v - 3 v^3 x^2 - B1 v^3 x^3 - C1 v^3 x^3 - 3 A v^4 x + 4 A v^3 x^2 - 2 A v^2 x^3 + A1 v^4 x - 2 A1 v^3 x^2 + 2 A1 v^2 x^3 - B1 v^3 x^2 + 2 B1 v^2 x^3 + 4 A v^3 x - 6 A v^2 x^2 + 4 A v x^3 - A1 v^3 x + 3 A1 v^2 x^2 - 3 A1 v x^3 - B1 v^3 x^4 - DD1 v^3 x^4 + C1 v^2 x^4 + A v x^4 - A1 v x^4 - B1 v x^4 - A3 v^3 x^4 - A2 v^2 x^4 - B2 v^3 x^4) \Big/ (v^4 x^3)$$

```
> KK:=factor(coeff(eqA1,A)*v^4*x^3): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$-x^4 + (x^4 + 4 x^3) v + (-2 x^3 - 6 x^2) v^2 + (4 x^2 + 4 x) v^3 + (-3 x - 1) v^4 + v^5$$

```
> factor(coeff(eqA1,A1)),factor(coeff(eqA1,A2)),factor(coeff(eqA1,A3
)),factor(coeff(eqA1,B1)),factor(coeff(eqA1,B2)),factor(coeff(eqA1
,C1)),factor(coeff(eqA1,DD1));
```

$$\frac{(v-x)(v^3 - v^2 x + v x^2 - v^2 + 2 v x - x^2)}{v^4 x^2}, -\frac{x}{v^2}, -\frac{x}{v}, -\frac{v^2 x^2 + v^2 x + v^2 - 2 v x + x^2}{x v^3}, -\frac{x}{v}, -\frac{v-x}{v^2}, -\frac{x}{v}$$

```
> eqA2:=simplify(subs(A=0,A3=LA-B2-DD1,eqA1)):
> solve({subs(v=u1,eqA2),subs(v=u2,eqA2),subs(v=u3,eqA2),subs(v=u4,e
qA2)},{A1,A2,B1,LA});
```

$$\{A1 = -\frac{u1\, u2\, u3\, u4}{u1\, u2\, u3\, u4 - x^3}, A2 = (C1\, u1\, u2\, u3\, u4\, x^2 - C1\, x^5 + u1\, u2\, u3\, u4\, x + 3\, u1\, u2\, u3\, u4 - 2\, u1\, u2\, u3\, x - 2\, u1\, u2\, u4\, x + u1\, u2\, x^2 - 2\, u1\, u3\, u4\, x + u1\, u3\, x^2 + u1\, u4\, x^2 - 2\, u2\, u3\, u4\, x + u2\, u3\, x^2 + u2\, u4\, x^2 + u3\, u4\, x^2 - x^4) \Big/ (x^2 (u1\, u2\, u3\, u4 - x^3)),$$

$$B1 = \frac{3\, u1\, u2\, u3\, u4 - u1\, u2\, u3\, x - u1\, u2\, u4\, x - u1\, u3\, u4\, x - u2\, u3\, u4\, x + x^4}{x (u1\, u2\, u3\, u4 - x^3)}, LA = -(C1\, u1\, u2\, u3\, u4\, x^2 + u1\, u2\, u3\, u4\, x^3 - C1\, x^5 + 4\, u1\, u2\, u3\, u4\, x^2 - u1\, u2\, u3\, x^3 - u1\, u2\, u4\, x^3 - u1\, u3\, u4\, x^3 - u2\, u3\, u4\, x^3 + 4\, u1\, u2\, u3\, u4\, x - u1\, u2\, u3\, x^2 - u1\, u2\, u4\, x^2 - u1\, u3\, u4\, x^2 - u2\, u3\, u4\, x^2 + 2\, u1\, u2\, u3\, u4 - u1\, u2\, u3\, x - u1\, u2\, u4\, x - u1\, u3\, u4\, x + u1\, x^3 - u2\, u3\, u4\, x + u2\, x^3 + u3\, x^3 + u4\, x^3 - 2\, x^4) \Big/ (x^3 (u1\, u2\, u3\, u4 - x^3))\}$$

```
> FinA:=-u1*u2*u3*u4/(u1*u2*u3*u4-x^3):
FinA:=factor(-x^4/u5/(x^4/u5-x^3)); series(subs(u5=vv,FinA),x,11);
```

$$FinA := \frac{x}{u5 - x}$$

$$x + 2 x^2 + 6 x^3 + 22 x^4 + 90 x^5 + 393 x^6 + 1792 x^7 + 8430 x^8 + 40608 x^9 + 199298 x^{10} + O(x^{11})$$

```
> #11100:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
0,0,0,0,0,0,1,1,1,: 0,0,0,0,0,1,1,1,0,: f=01113: g=01114:
h=011103: j=011104: k=011105: Reg5:
> #10100:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
```

```
0,0,0,0,0,0,1,0,1,: 0,0,0,0,0,1,0,1,0,: f=01013: g=01014:
h=010103: j=010104: k=010105: Reg5:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*B+x/v*(B-B1):
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x*E+x/(1-v)*(DD+F+G):
eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+H+J+K):
eqF:=-FF+x*(1+HH+DD1+FF+GG): eqG:=-GG+x*(1+JJ+FF+GG):
eqH:=-HH+x*(1+E1+HH+JJ+KK): eqJ:=-JJ+x*(1+HH+JJ+KK):
eqK:=-KK+x*(1+JJ+KK):
> solve({eqF,eqG,eqH,eqJ,eqK},{FF,GG,HH,JJ,KK});
F:=(E1*x^3+DD1*x^2-2*E1*x^2-3*DD1*x+E1*x+DD1-x+1)*(x-1)*x/(x^2-3*x
+1)/(2*x-1):
G:=-x*(E1*x^4+DD1*x^3-4*E1*x^3-3*DD1*x^2+2*E1*x^2+DD1*x+x^2-2*x+1)
/(2*x^3-7*x^2+5*x-1): H:=-x*(2*E1*x-E1-1)/(x^2-3*x+1):
J:=-x*(E1*x^2-E1*x-1)/(x^2-3*x+1): K:= x*(E1*x^2-x+1)/(x^2-3*x+1):
```

$$\{FF=\frac{(E1\,x^3+DD1\,x^2-2\,E1\,x^2-3\,DD1\,x+E1\,x+DD1-x+1)\,(x-1)\,x}{(x^2-3\,x+1)\,(2\,x-1)},$$

$$GG=-\frac{x\,(E1\,x^4+DD1\,x^3-4\,E1\,x^3-3\,DD1\,x^2+2\,E1\,x^2+DD1\,x+x^2-2\,x+1)}{2\,x^3-7\,x^2+5\,x-1},$$

$$HH=-\frac{x\,(2\,E1\,x-E1-1)}{x^2-3\,x+1},\ JJ=-\frac{x\,(E1\,x^2-E1\,x-1)}{x^2-3\,x+1},\ KK=\frac{x\,(E1\,x^2-x+1)}{x^2-3\,x+1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2):
Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=subs(v=0,Exv),eqD),2):
> eqB1:=subs(DD=Dxv,C=solve(eqB,C),eqC):
KK:=factor(x*v^2*(1-v)*coeff(eqB1,B)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2\,x)\,v+(-x-1)\,v^2+v^3$$

```
> ss:=map(factor,solve({subs(v=u1,B=0,eqB1),subs(v=u2,B=0,eqB1)},{B1
,C1})):
> op(op(ss)[1])[1],op(op(ss)[2])[1];
Bx0:=op(op(ss)[1])[2]:Cx0:=op(op(ss)[2])[2]:
Bxv:=solve(subs(B1=Bx0,C1=Cx0,eqB1)=0,B):
```

$$B1,\ C1$$

```
> Bxv:=simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],Bxv))+1)
```

```
-1):
> FinA:=subs(vv=u3,simplify(subs(u3=vv,x/(1-x)*(1+subs(v=x,Bxv)))));
  taylor(subs(u3=vv,FinA),x,14);
```

$$FinA := (5\,x\sqrt{1-4\,x}\,u3^2 + 4\,u3^2\,x^2 - 13\,x^3\sqrt{1-4\,x} - 12\,x^4 - \sqrt{1-4\,x}\,u3^2 - 5\,u3^2\,x - 5\,x\sqrt{1-4\,x}\,u3 - 4\,u3\,x^2 + 25\,x^2\sqrt{1-4\,x} + 47\,x^3 + u3^2 + u3\sqrt{1-4\,x} + 5\,u3\,x - 9\,x\sqrt{1-4\,x} - 39\,x^2 - u3 + \sqrt{1-4\,x} + 11\,x - 1) \Big/ (2\,x^3\,(-1+4\,x))$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 393\,x^6 + 1785\,x^7 + 8300\,x^8 + 39132\,x^9 + 186011\,x^{10} + O(x^{11})$$

```
> #11023: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,0,1,1,2,:
  0,0,0,0,0,1,1,2,0,: g=0113, h=01102, j=01130: Reg6
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*(E+G):
  eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x*sum((m+2)*v^(m-1)*H,m=1..infinit
  y): eqE:=-E+x/(1-v)+x*F+x/v*(E-E1)+x/(1-v)*(E+G):
  eqF:=-F+x/(1-v)+x/v*(F-F1)+x*sum((m+3)*v^(m-1)*H,m=1..infinity):
  eqG:=-GG+x*(1+JJ+E1+GG): eqH:=-HH+x*(1+HH+HH):
  eqJ:=-JJ+x*(1+F1+3*HH):
> solve({eqG,eqH,eqJ},{GG,HH,JJ});G:=-x*(2*F1*x^2+2*E1*x-F1*x-x^2-E1
  +x-1)/(2*x^2-3*x+1):H:=-x/(2*x-1): J:=(2*F1*x-F1-x-1)*x/(2*x-1):
```

$$\left\{GG = -\frac{x\,(2\,F1\,x^2 + 2\,E1\,x - F1\,x - x^2 - E1 + x - 1)}{2\,x^2 - 3\,x + 1},\ HH = -\frac{x}{2\,x-1},\ JJ = \frac{(2\,F1\,x - F1 - x - 1)\,x}{2\,x-1}\right\}$$

```
> Fxv:=SolveKE(F,F1,eqF,1):   G:=subs(F1=subs(v=0,Fxv),G):
  Exv:=SolveKE(E,E1,subs(F=Fxv,eqE),2): Dxv:=SolveKE(DD,DD1,eqD,1):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,E=Exv,E1=subs(v=0,Exv),eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
> FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
  )))+1)-1)); taylor(FinA,x,14);
```

$$FinA := -(8\sqrt{1-4\,x}\,x^9 + 64\,x^{10} - 146\sqrt{1-4\,x}\,x^8 - 344\,x^9 + 530\sqrt{1-4\,x}\,x^7 + 1122\,x^8 - 946\sqrt{1-4\,x}\,x^6 - 2116\,x^7 + 1012\sqrt{1-4\,x}\,x^5 + 2584\,x^6 - 691\,x^4\sqrt{1-4\,x} - 2134\,x^5 + 306\,x^3\sqrt{1-4\,x} + 1201\,x^4 - 86\,x^2\sqrt{1-4\,x} - 456\,x^3 + 14\,x\sqrt{1-4\,x} + 112\,x^2 - \sqrt{1-4\,x} - 16\,x + 1) \Big/ (2\,x^2\,(-1+4\,x)\,(2\,x-1)^3\,(x-1)^4)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 388\,x^6 + 1688\,x^7 + 7240\,x^8 + 30406\,x^9 + 125233\,x^{10} + 508056\,x^{11}$$

$$+ O(x^{12})$$

```
> #11022:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
  0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,0,1,1,2,: 0,0,0,0,0,1,1,2,0,:
  g=0113, h=01102, j=01103, k=01104, Reg6
> #11020: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,0,1,1,2,:
  0,0,0,0,0,1,1,2,0,:  g=0113, h=01102, j=01103, k=01104, Reg6
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*(E+G):
  eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x*v/(1-v)*F+x/(1-v)*(H+J+K):
  eqE:=-E+x/(1-v)+x*F+x/v*(E-E1)+x/(1-v)*(E+G):
  eqF:=-F+x/(1-v)+x/v*(F-F1)+x/(1-v)*(F+H+J+K):
  eqG:=-GG+x*(1+HH+E1+GG): eqH:=-HH+x*(1+F1+HH+JJ+KK):
  eqJ:=-JJ+x*(1+HH+JJ+KK): eqK:=-KK+x*(1+JJ+KK):
> solve({eqG,eqH,eqJ,eqK},{GG,HH,JJ,KK});G:=-x*(E1*x^2-2*F1*x^2-3*E1
  *x+F1*x+x^2+E1-2*x+1)/(x^3-4*x^2+4*x-1):
  H:=-x*(2*F1*x-F1-1)/(x^2-3*x+1): J:=
  -x*(F1*x^2-F1*x-1)/(x^2-3*x+1):K:= x*(F1*x^2-x+1)/(x^2-3*x+1):
```

$$\{GG = -\frac{x\,(E1\,x^2 - 2\,F1\,x^2 - 3\,E1\,x + F1\,x + x^2 + E1 - 2\,x + 1)}{x^3 - 4\,x^2 + 4\,x - 1},\ HH = -\frac{x\,(2\,F1\,x - F1 - 1)}{x^2 - 3\,x + 1},$$

$$JJ = -\frac{x\,(F1\,x^2 - F1\,x - 1)}{x^2 - 3\,x + 1},\ KK = \frac{x\,(F1\,x^2 - x + 1)}{x^2 - 3\,x + 1}\}$$

```
> Fxv:=SolveKE(F,F1,eqF,2): Fx0:=subs(v=0,Fxv):
  Exv:=SolveKE(E,E1,subs(F=Fxv,F1=Fx0,eqE),2):
  Dxv:=SolveKE(DD,DD1,subs(F=Fxv,F1=Fx0,eqD),1):
  Cxv:=SolveKE(C,C1,subs(F=Fxv,F1=Fx0,DD=Dxv,E1=subs(v=0,Exv),E=Exv,
  eqC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
> FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
  )))+1)-1)); taylor(FinA,x,15);
```

$$FinA := -(7\sqrt{1-4\,x}\,x^5 + 36\,x^6 + 98\sqrt{1-4\,x}\,x^4 + 63\,x^5 - 165\sqrt{1-4\,x}\,x^3 - 310\,x^4 + 95\,x^2\sqrt{1-4\,x} + 317\,x^3 - 23\,x\sqrt{1-4\,x} - 137\,x^2 + 2\sqrt{1-4\,x} + 27\,x - 2) \Big/ (2\,x^4\,(-1+4\,x)^2)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1749\,x^7 + 7906\,x^8 + 35743\,x^9 + 160788\,x^{10} + O(x^{11})$$

```
> #11012:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: e=01101: Reg4:
```

```
> #10112:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: e=01011: Reg4:
> #10012:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,0,: e=01001: Reg4:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/v^2*(B-B1-B2*v)+x/v*(B-B1)+x/(1-v)
  *(B+E): E:=x/(1-2*x):
> eqB1:=subs(DD=solve(subs(C=solve(eqB,C),eqC),DD),eqD):
  KK:=factor(x^2*v^3*(1-v)*coeff(eqB1,B)): taylor(KK,v,10);
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$x^3 - 3\,x^2\,v + (2\,x^2 + 3\,x)\,v^2 + (-2\,x - 1)\,v^3 + v^4$$

```
> factor(coeff(eqB1,B1)),factor(coeff(eqB1,B2)),factor(coeff(eqB1,C1
  )),factor(coeff(eqB1,DD1));
```

$$-\frac{v^2\,x^2 + v^2\,x + v^2 - 2\,v\,x + x^2}{x\,v^3}, -\frac{x}{v}, -\frac{v - x}{v^2}, -\frac{x}{v}$$

```
> eqB2:=subs(B=0,B2=LB-DD1,eqB1):
  ss:=map(factor,solve({subs(v=u1,eqB2),subs(v=u2,eqB2),subs(v=u3,eq
  B2)},{B1,LB,C1})):
> op(op(ss)[1])[1],op(op(ss)[2])[1],op(op(ss)[3])[1];
  Bx0:=op(op(ss)[1])[2]: Cx0:=op(op(ss)[2])[2]:
  LB0:=op(op(ss)[3])[2]:
  Bxv:=solve(subs(C1=Cx0,B1=Bx0,LB=LB0,subs(B2=LB-DD1,eqB1))=0,B):
```

$$B1, C1, LB$$

```
> FinA:=simplify(x/(1-x)+x/(1-x)*subs(v=x,Bxv));
```

$$FinA := -(2\,u1\,u2\,u3\,x^4 - 2\,u1\,u2\,u3\,x^3 - u1\,u2\,x^4 - u1\,u3\,x^4 - u1\,x^5 - u2\,u3\,x^4 - u2\,x^5 - u3\,x^5 + x^6 + 3\,u1\,u2\,u3\,x^2 - 2\,u1\,u2\,x^3 - 2\,u1\,u3\,x^3 + 5\,u1\,x^4 - 2\,u2\,u3\,x^3 + 5\,u2\,x^4 + 5\,u3\,x^4 - 3\,x^5 - 3\,u1\,u2\,u3\,x + 3\,u1\,u2\,x^2 + 3\,u1\,u3\,x^2 - 4\,u1\,x^3 + 3\,u2\,u3\,x^2 - 4\,u2\,x^3 - 4\,u3\,x^3 + x^4 + u1\,u2\,u3 - u1\,u2\,x - u1\,u3\,x + u1\,x^2 - u2\,u3\,x + u2\,x^2 + u3\,x^2) \Big/ (x^2\,(x-1)\,(-1+u2)\,(-1+2\,x)\,(-1+u1)\,(-1+u3))$$

```
> m3:=x^3/u4: m2:=3*x+2*x^2-(1+2*x-u4)*u4: m1:=1+2*x-u4:
> FinA:=subs(vv=u4,simplify(subs(u4=vv,factor(-(2*m3*x^4-2*m3*x^3-m2
  *x^4-m1*x^5+x^6+3*m3*x^2-2*m2*x^3+5*m1*x^4-3*x^5-3*m3*x+3*m2*x^2-4
  *m1*x^3+x^4+m3-m2*x+m1*x^2)/x^2/(x-1)/(-1+m1-m2+m3)/(-1+2*x)))));
```

$$FinA := -(u4^3\,x^4 - u4^2\,x^5 + u4\,x^6 + 2\,x^7 - 4\,u4^3\,x^3 + 3\,u4^2\,x^4 - 2\,u4\,x^5 - 5\,x^6 + 6\,u4^3\,x^2 - 2\,u4^2\,x^3$$

$$-2\,u4\,x^4+10\,x^5-4\,u4^3\,x-2\,u4^2\,x^2+8\,u4\,x^3-10\,x^4+u4^3+3\,u4^2\,x-7\,u4\,x^2+5\,x^3-u4^2+2\,u4\,x-x^2)\Big/(x^3\,(x-1)\,(x^2-x+1)\,(-1+2\,x))$$

```
> taylor(subs(u4=vv,FinA),x,14);
```

$$x+2\,x^2+6\,x^3+22\,x^4+90\,x^5+390\,x^6+1750\,x^7+8062\,x^8+37955\,x^9+181940\,x^{10}+\mathrm{O}(x^{11})$$

```
> #11002: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,0,1,1,2,:
0,0,0,0,0,1,1,0,2,: g=0113, h=01103, j=01104, k=011020 l=01100:
Reg6
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*(E+G):
eqD:=-DD+x/(1-v)*(1+L)+x/v*(DD-DD1)+x/(1-v)*(F+J+H):
eqE:=-E+x/(1-v)+x*F+x/v*(E-E1)+x/(1-v)*(E+G):
eqF:=-F+x/(1-v)*(1+K)+x/v*(F-F1)+x/(1-v)*(F+H+J):
eqG:=-GG+x*(1+HH+E1+GG): eqH:=-HH+x*(1+F1+HH+JJ+KK):
eqJ:=-JJ+x*(1+HH+JJ+KK): eqK:=-KK+x+x*KK: eqL:=-LL+x+2*x*LL:
> map(factor,solve({eqG,eqH,eqJ,eqK,eqL},{GG,HH,JJ,KK,LL}));
G:=-(F1*x^3+2*E1*x^2-2*F1*x^2-3*E1*x+F1*x+2*x^2+E1-2*x+1)*x/(x-1)^
2/(2*x-1): H:=(F1*x^2-2*F1*x+F1+1)*x/(2*x-1)/(x-1):
J:=-x*(F1*x^2-F1*x-1)/(2*x-1)/(x-1): K:=-x/(x-1): L:=x/(1-2*x):
```

$$\{GG=-\frac{(F1\,x^3+2\,E1\,x^2-2\,F1\,x^2-3\,E1\,x+F1\,x+2\,x^2+E1-2\,x+1)\,x}{(x-1)^2\,(2\,x-1)},$$

$$HH=\frac{(F1\,x^2-2\,F1\,x+F1+1)\,x}{(2\,x-1)\,(x-1)},\ JJ=-\frac{x\,(F1\,x^2-F1\,x-1)}{(2\,x-1)\,(x-1)},\ KK=-\frac{x}{x-1},\ LL=-\frac{x}{2\,x-1}\}$$

```
> Fxv:=SolveKE(F,F1,eqF,2): Fx0:=subs(v=0,Fxv);
Exv:=SolveKE(E,E1,subs(F=Fxv,F1=Fx0,eqE),2): Ex0:=subs(v=0,Exv):
Dxv:=SolveKE(DD,DD1,subs(F=Fxv,F1=Fx0,E=Exv,eqD),1):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,E1=Ex0,F=Fxv,F1=Fx0,E=Exv,eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),2):
FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
)))+1)-1)); taylor(FinA,x,15);
```

$$Fx0:=-\frac{-1+\sqrt{1-4\,x}}{(x\sqrt{1-4\,x}-1-\sqrt{1-4\,x}+3\,x)\,(x-1)}$$

$$FinA:=-(92\sqrt{1-4\,x}\,x^9+96\,x^{10}-468\sqrt{1-4\,x}\,x^8-688\,x^9+1150\sqrt{1-4\,x}\,x^7+1958\,x^8-1698\sqrt{1-4\,x}\,x^6-3416\,x^7+1612\sqrt{1-4\,x}\,x^5+3958\,x^6-1013\,x^4\sqrt{1-4\,x}-3128\,x^5$$

$$+419\,x^3\sqrt{1-4\,x}+1689\,x^4-109\,x^2\sqrt{1-4\,x}-609\,x^3+16\,x\sqrt{1-4\,x}+139\,x^2-\sqrt{1-4\,x}-18\,x+1)\Big/(2\,x^3\,(2\,x-1)\,(-1+4\,x)^2\,(x-1)^4)$$

$$x+2\,x^2+6\,x^3+22\,x^4+90\,x^5+390\,x^6+1729\,x^7+7697\,x^8+34133\,x^9+150366\,x^{10}+657627\,x^{11}+\mathrm{O}(x^{12})$$

```
> #11000:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
  0,0,0,0,0,0,1,1,0,: 0,0,0,0,0,1,1,0,0: f=0113, g=01103, h=01104,
  j=011003, k=011004, l=011005, Reg5:
> #10000:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
  0,0,0,0,0,0,1,0,0,: 0,0,0,0,0,1,0,0,0: f=0103, g=01003, h=01004,
  j=010003, k=010004, l=010005, Reg5:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x*DD+x/(1-v)*(C+F):
  eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*(DD+G+H):
  eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+J+K+L):
  eqF:=-FF+x*(1+FF+GG+C1): eqG:=-GG+x*(1+HH+DD1+GG+JJ):
  eqH:=-HH+x*(1+HH+GG+KK): eqJ:=-JJ+x*(1+E1+LL+JJ+KK):
  eqK:=-KK+x*(1+JJ+KK+LL): eqL:=-LL+x*(1+KK+LL):
> map(factor,solve({eqF,eqG,eqH,eqJ,eqK,eqL},{FF,GG,HH,JJ,KK,LL}));
  F:=-(E1*x^5+DD1*x^4-3*E1*x^4+2*C1*x^3-4*DD1*x^3+3*E1*x^3-7*C1*x^2+
  4*DD1*x^2-E1*x^2+x^3+5*C1*x-DD1*x-5*x^2-C1+4*x-1)*x/(x-1)/(x^2-3*x
  +1)/(2*x-1): G:=
  (x-1)*(E1*x^3+DD1*x^2-2*E1*x^2-3*DD1*x+E1*x+DD1-x+1)*x/(2*x-1)/(x^
  2-3*x+1):
  H:=-x*(E1*x^4+DD1*x^3-4*E1*x^3-3*DD1*x^2+2*E1*x^2+DD1*x+x^2-2*x+1)
  /(2*x-1)/(x^2-3*x+1): J:=-x*(2*E1*x-E1-1)/(x^2-3*x+1):
  K:=-x*(E1*x^2-E1*x-1)/(x^2-3*x+1): L:= x*(E1*x^2-x+1)/(x^2-3*x+1):
```

$$\{FF=-(E1\,x^5+DD1\,x^4-3\,E1\,x^4+2\,C1\,x^3-4\,DD1\,x^3+3\,E1\,x^3-7\,C1\,x^2+4\,DD1\,x^2-E1\,x^2+x^3+5\,C1\,x-DD1\,x-5\,x^2-C1+4\,x-1)\,x\Big/((x-1)\,(x^2-3\,x+1)\,(2\,x-1)),$$

$$GG=\frac{(x-1)\,(E1\,x^3+DD1\,x^2-2\,E1\,x^2-3\,DD1\,x+E1\,x+DD1-x+1)\,x}{(2\,x-1)\,(x^2-3\,x+1)},$$

$$HH=-\frac{x\,(E1\,x^4+DD1\,x^3-4\,E1\,x^3-3\,DD1\,x^2+2\,E1\,x^2+DD1\,x+x^2-2\,x+1)}{(2\,x-1)\,(x^2-3\,x+1)},$$

$$JJ=-\frac{x\,(2\,E1\,x-E1-1)}{x^2-3\,x+1},\ KK=-\frac{x\,(E1\,x^2-E1\,x-1)}{x^2-3\,x+1},\ LL=\frac{x\,(E1\,x^2-x+1)}{x^2-3\,x+1}\}$$

```
> Exv:=SolveKE(E,E1,eqE,2): Ex0:=subs(v=0,Exv):
  Dxv:=SolveKE(DD,DD1,subs(E=Exv,E1=Ex0,eqD),2): Dx0:=subs(v=0,Dxv):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=Dx0,E=Exv,E1=Ex0,eqC),2):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,DD1=Dx0,E=Exv,E1=Ex0,eqB),2):
  FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
  )))+1)-1)); taylor(FinA,x,15);
```

$$FinA := (14\sqrt{1-4x}\,x^7 - 316\sqrt{1-4x}\,x^6 - 320x^7 + 650\sqrt{1-4x}\,x^5 + 1072x^6 - 635x^4\sqrt{1-4x} - 1516x^5 + 343x^3\sqrt{1-4x} + 1169x^4 - 103x^2\sqrt{1-4x} - 521x^3 + 16x\sqrt{1-4x} + 133x^2 - \sqrt{1-4x} - 18x + 1) \Big/ (2x^4(-1+4x)^3)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 393x^6 + 1783x^7 + 8255x^8 + 38546x^9 + 180200x^{10} + O(x^{11})$$

```
> #10234:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,0,1,2,0,:
  g=0103, h=01023, Reg6:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+G):
  eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
  eqE:=-E+x/(1-v)+2*x/v*(E-E1)+x*sum((m+1)*v^(m-1)*H,m=1..infinity):
  eqF:=-F+x/(1-v)+x/v*(F-F1)+x/v*(E-E1)+x/(1-v)*(E+G):
  G:=x*(1+2*E1+H): H:=x/(1-2*x):
> Exv:=SolveKE(E,E1,eqE,1): Ex0:=subs(v=0,Exv):
  Fxv:=SolveKE(F,F1,subs(E=Exv,E1=Ex0,eqF),1):
  Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
  Cxv:=SolveKE(C,C1,subs(E=Exv,E1=Ex0,eqC),1):
  Bxv:=SolveKE(B,B1,subs(DD=Dxv,C=Cxv,eqB),1):
  FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
  )))+1)-1)); taylor(FinA,x,15);
```

$$FinA := (-64x^{12} + 24\sqrt{1-4x}\,x^{10} + 424x^{11} - 156\sqrt{1-4x}\,x^9 - 1500x^{10} + 474\sqrt{1-4x}\,x^8 + 3168x^9 - 889x^7\sqrt{1-4x} - 4638x^8 + 1132x^6\sqrt{1-4x} + 5025x^7 - 1010x^5\sqrt{1-4x} - 4132x^6 + 632x^4\sqrt{1-4x} + 2604x^5 - 272x^3\sqrt{1-4x} - 1242x^4 + 77x^2\sqrt{1-4x} + 432x^3 - 13x\sqrt{1-4x} - 103x^2 + \sqrt{1-4x} + 15x - 1) \Big/ (2x(-1+2x)^5(x-1)^6)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 390x^6 + 1707x^7 + 7278x^8 + 29714x^9 + 115911x^{10} + 434491x^{11} + 1578897x^{12} + 5614534x^{13} + O(x^{14})$$

```
> #10233:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,0,1,2,0,:
```

```
0,0,0,0,0,1,0,2,3,: h=01024: j=0103: Reg7:
> #10230:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,0,1,2,0,:
0,0,0,0,0,1,0,2,3,: h=01024: j=0103: Reg7:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+J):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+2*x/v*(E-E1)+x/(1-v)*(G+H):
eqF:=-F+x/(1-v)+x/v*(F-F1)+x/v*(E-E1)+x/(1-v)*(E+J):
eqG:=-G+x/(1-v)+x/v*(G-G1)+x/(1-v)*(G+H): H:=(x+x*G1)/(1-x):
J:=(x+2*x*E1+x*H):
> Gxv:=SolveKE(G,G1,eqG,2): Gx0:=subs(v=0,Gxv):
> Exv:=SolveKE(E,E1,subs(G=Gxv,G1=Gx0,eqE),1): Ex0:=subs(v=0,Exv):
Fxv:=SolveKE(F,F1,subs(G=Gxv,G1=Gx0,E=Exv,E1=Ex0,eqF),1):
> Dxv:=simplify(SolveKE(DD,DD1,subs(F=Fxv,eqD),2)):
Cx0:=simplify(solve(subs(v=2*x,subs(E=Exv,E1=Ex0,G1=Gx0,eqC)),C1))
:
Cxv:=simplify(solve(subs(C1=Cx0,subs(E=Exv,E1=Ex0,G1=Gx0,eqC)),C))
:
> Bx0:=simplify(solve(simplify(subs(v=x,subs(C=Cxv,DD=Dxv,eqB))),B1)
):
Bxv:=simplify(solve(simplify(subs(B1=Bx0,subs(C=Cxv,DD=Dxv,eqB))),
B)):
>
> FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
)))+1)-1)); taylor(FinA,x,15);
```

$$FinA := -\left(16x^8 + 15\sqrt{1-4x}\,x^6 - 40x^7 - 63\sqrt{1-4x}\,x^5 - x^6 + 118\sqrt{1-4x}\,x^4 + 149x^5 - 117\sqrt{1-4x}\,x^3 - 260x^4 + 64\sqrt{1-4x}\,x^2 + 215x^3 - 18\sqrt{1-4x}\,x - 96x^2 + 2\sqrt{1-4x} + 22x - 2\right) \Big/ \left(2x^2(-1+4x)(2x-1)(-1+x)^4\right)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 391x^6 + 1743x^7 + 7803x^8 + 34709x^9 + 152811x^{10} + 665542x^{11} + 2869954x^{12} + O(x^{13})$$

```
> #10223:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: h=01022: j=0103: Reg6:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
```

```
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+J):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+H+J):
eqF:=-F+x/(1-v)+x/v*(F+E-F1-E1)+x/(1-v)*(E+J): H:=x/(1-2*x):
J:=(x+x*E1+x*H)/(1-x):
```

```
> Exv:=SolveKE(E,E1,eqE,2): Ex0:=subs(v=0,Exv):
Fxv:=SolveKE(F,F1,subs(E=Exv,E1=Ex0,eqF),1):
Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,E1=Ex0,eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
FinA:=factor(simplify(simplify(rationalize(x/(1-x)*(1+subs(v=x,Bxv
)))+1)-1)); taylor(FinA,x,15);
```

$$FinA := -\left(32 x^{10} + 36 \sqrt{1-4x}\, x^8 - 96 x^9 - 236 \sqrt{1-4x}\, x^7 - 26 x^8 + 645 \sqrt{1-4x}\, x^6 + 712 x^7 - 962 \sqrt{1-4x}\, x^5 - 1711 x^6 + 857 \sqrt{1-4x}\, x^4 + 2116 x^5 - 467 \sqrt{1-4x}\, x^3 - 1573 x^4 + 152 \sqrt{1-4x}\, x^2 + 725 x^3 - 27 \sqrt{1-4x}\, x - 202 x^2 + 2 \sqrt{1-4x} + 31 x - 2\right) \Big/ \left(2 x^3 (-1+2x)^2 (-1+4x) (x-1)^4\right)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 90x^5 + 389x^6 + 1709x^7 + 7467x^8 + 32171x^9 + 136509x^{10} + 571466x^{11} + O(x^{12})$$

```
> #10222: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: g=0103, h=01033, Reg6
```

```
> #10202: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: g=0103, h=01033, Reg6
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+G):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x*F+x/(1-v)*(E+G):
eqF:=-F+x/(1-v)+x/v*(F+E-F1-E1)+x/(1-v)*(E+G):
eqG:=-GG+x*(1+E1+HH)/(1-x): eqH:=-HH+x*(1+E1+F1+GG):
```

```
> solve({eqH,eqG},{GG,HH}); G:= -x*(E1*x+F1*x+E1+x+1)/(x^2+x-1): H:=
-x*(-F1*x+E1+F1+1)/(x^2+x-1):
```

$$\{GG = -\frac{x\,(E1\,x + F1\,x + E1 + x + 1)}{x^2 + x - 1},\ HH = -\frac{x\,(-F1\,x + E1 + F1 + 1)}{x^2 + x - 1}\}$$

```
> eqE1:=subs(F=solve(eqE,F),eqF):
KK:=factor(v^2*(1-v)*x*coeff(eqE1,E)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
map(factor,solve({subs(v=u1,subs(E=0,eqE1)),subs(v=u2,subs(E=0,eqE
1))},{E1,F1})); Ex0:=u1*u2/x/(u1*u2-u1-u2-x+1):
Fx0:=-(u2*x+u2-x)*(u1*x+u1-x)/x^2/(u1*u2-u1-u2-x+1):
ss:=solve({subs(E1=Ex0,F1=Fx0,eqE),subs(E1=Ex0,F1=Fx0,eqF)},{E,F})
: op(op(ss)[1])[1],op(op(ss)[2])[1]; Exv:=op(op(ss)[1])[2]:
Fxv:=op(op(ss)[2])[2]: uu:=solve(u1+x^2/u1/u3=1+x-u3,u1):
uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2 + (x^2 + 2\,x)\,v + (-x - 1)\,v^2 + v^3$$

$$\{E1 = \frac{u1\,u2}{x\,(u1\,u2 - u1 - u2 - x + 1)},\ F1 = -\frac{(u2\,x + u2 - x)\,(u1\,x + u1 - x)}{x^2\,(u1\,u2 - u1 - u2 - x + 1)}\}$$

$$E, F$$

```
> Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,E1=Ex0,F1=Fx0,eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
FinA:=factor(subs(vv=u3,simplify(subs(u3=vv,simplify(simplify(rati
onalize(subs(u1=uu[1],u2=uu[2],1/(1-x)*(1+subs(v=x,Bxv))))+1)-1)))
)); taylor(subs(u3=vv,FinA),x,15);
```

$$FinA := (6\sqrt{1-4\,x}\,u3^2\,x^2 + 4\,u3^2\,x^3 - 6\,x^3\sqrt{1-4\,x}\,u3 - 8\,u3\,x^4 - 12\,x^5 - 7\sqrt{1-4\,x}\,u3^2\,x - 42\,u3^2\,x^2 + \sqrt{1-4\,x}\,u3\,x^2 + 28\,u3\,x^3 + 12\,x^3\sqrt{1-4\,x} + 4\,x^4 + \sqrt{1-4\,x}\,u3^2 + 13\,u3^2\,x + 6\sqrt{1-4\,x}\,u3\,x + 31\,u3\,x^2 - 14\,x^2\sqrt{1-4\,x} - 72\,x^3 - u3^2 - \sqrt{1-4\,x}\,u3 - 12\,u3\,x + 2\,x\sqrt{1-4\,x} + 24\,x^2 + u3 - 2\,x) \Big/ (2\,x^4\,(6\,x-1)\,(-1+x))$$

$$1 + 2\,x + 6\,x^2 + 22\,x^3 + 90\,x^4 + 392\,x^5 + 1771\,x^6 + 8180\,x^7 + 38320\,x^8 + 181259\,x^9 + 863498\,x^{10} + O(x^{11})$$

```
> #10220:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: 0,0,0,0,0,1,0,2,2,: h=0103, j=01033, k=010225,
Reg7
> #10200:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: 0,0,0,0,0,1,0,2,2,: h=0103, j=01030, k=010205,
Reg7
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
```

```
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+H):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x*G+x/(1-v)*(E+H):
eqF:=-F+x/(1-v)+x/v*(F+E-F1-E1)+x/(1-v)*(E+H):
eqG:=-G+x/(1-v)+x/v*(G-G1)+x/(1-v)*(G+J+K):
eqH:=-HH+x*(1+E1+JJ+HH): eqJ:=-JJ+x*(1+G1+JJ+KK):
eqK:=-KK+x*(1+JJ+KK):
```

```
> solve({eqH,eqJ,eqK},{HH,JJ,KK});
H:=-x*(G1*x^2+2*E1*x-G1*x-E1+x-1)/(2*x^2-3*x+1):J:=x*(G1*x-G1-1)/(
2*x-1):K:=-x*(G1*x+1)/(2*x-1):
```

$$\left\{HH=-\frac{x\,(G1\,x^2+2\,E1\,x-G1\,x-E1+x-1)}{2\,x^2-3\,x+1},\ JJ=\frac{x\,(G1\,x-G1-1)}{2\,x-1},\ KK=-\frac{x\,(G1\,x+1)}{2\,x-1}\right\}$$

```
> Gxv:=SolveKE(G,G1,eqG,2):
Exv:=SolveKE(E,E1,subs(G=Gxv,G1=subs(v=0,Gxv),eqE),2):
Fxv:=SolveKE(F,F1,subs(G=Gxv,G1=subs(v=0,Gxv),E=Exv,E1=subs(v=0,Ex
v),eqF),1): Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,E1=subs(v=0,Exv),G1=subs(v=0,Gxv),eqC
),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
```

```
> FinA:=simplify(simplify(rationalize(1/(1-x)*(1+subs(v=x,Bxv))+1)-1
)); taylor(FinA,x,15);
```

$$FinA := \left(-28\,x^7+23\sqrt{1-4\,x}\,x^6-245\,x^6-343\sqrt{1-4\,x}\,x^5+1247\,x^5+812\sqrt{1-4\,x}\,x^4 - 1872\,x^4-746\sqrt{1-4\,x}\,x^3+1278\,x^3+321\,x^2\sqrt{1-4\,x}-441\,x^2-65\,x\sqrt{1-4\,x}+75\,x + 5\sqrt{1-4\,x}-5\right) \Big/ \left(2\,x^5\,(16\,x^3-24\,x^2+9\,x-1)\right)$$

$$1+2\,x+6\,x^2+22\,x^3+90\,x^4+392\,x^5+1766\,x^6+8076\,x^7+37055\,x^8+169458\,x^9+O(x^{10})$$

```
> #10203: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:
0,0,0,0,0,0,1,2,0,: 0,0,0,0,0,1,0,2,3,: h=0103, j=01024, k=010230,
l=01020: Reg7
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(E+H):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)*(1+L)+x/v*(E-E1)+x/(1-v)*(G+J):
eqF:=-F+x/(1-v)+x/v*(F+E-F1-E1)+x/(1-v)*(E+H):
```

```
eqG:=-G+x/(1-v)+x/v*(G-G1)+x/(1-v)*(G+J+K):
eqH:=-HH+x*(1+LL+E1+JJ): eqJ:=-JJ+x*(1+G1+JJ+KK):
eqK:=-KK+x*(1+KK): eqL:=-LL+x*(1+2*LL):
```

```
> solve({eqH,eqJ,eqK,eqL},{HH,JJ,KK,LL});
H:=x*(2*E1*x^3-2*G1*x^3-5*E1*x^2+3*G1*x^2+x^3+4*E1*x-G1*x-x^2-E1+2
*x-1)/(2*x^3-5*x^2+4*x-1):J:=-x*(G1*x-G1-1)/(x^2-2*x+1):K:=-x/(x-1
):L:=-x/(2*x-1):
```

$$\{HH=\frac{x\,(2\,E1\,x^3-2\,G1\,x^3-5\,E1\,x^2+3\,G1\,x^2+x^3+4\,E1\,x-G1\,x-x^2-E1+2\,x-1)}{2\,x^3-5\,x^2+4\,x-1},$$

$$JJ=-\frac{x\,(G1\,x-G1-1)}{x^2-2\,x+1},\ KK=-\frac{x}{x-1},\ LL=-\frac{x}{2\,x-1}\}$$

```
> Gxv:=SolveKE(G,G1,eqG,2):
Exv:=simplify(SolveKE(E,E1,subs(G=Gxv,G1=subs(v=0,Gxv),eqE),1)):
Fxv:=simplify(SolveKE(F,F1,subs(E=Exv,E1=subs(v=0,Exv),G=Gxv,G1=su
bs(v=0,Gxv),eqF),1)):
Dxv:=simplify(SolveKE(DD,DD1,subs(F=Fxv,eqD),2)):
Cxv:=simplify(SolveKE(C,C1,subs(E=Exv,E1=subs(v=0,Exv),G=Gxv,G1=su
bs(v=0,Gxv),eqC),1)):
Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
```

```
> FinA:=simplify(simplify(rationalize(1/(1-x)*(1+subs(v=x,Bxv))+1)-1
)); taylor(FinA,x,15);
```

$$FinA := (2+32\sqrt{1-4\,x}\,x^{12}-24\sqrt{1-4\,x}\,x^{11}-848\sqrt{1-4\,x}\,x^{10}+373\,x^2-41\,x-1991\,x^3$$
$$+6937\,x^4-64\,x^{14}+448\,x^{13}-1564\,x^{12}+2404\,x^{11}+4146\sqrt{1-4\,x}\,x^9+706\,x^{10}-11056\,x^9$$
$$-9812\sqrt{1-4\,x}\,x^8+14368\,x^7\sqrt{1-4\,x}+24848\,x^8-32124\,x^7-14108\sqrt{1-4\,x}\,x^6$$
$$+9578\sqrt{1-4\,x}\,x^5+27678\,x^6-16574\,x^5-4515\sqrt{1-4\,x}\,x^4-303\sqrt{1-4\,x}\,x^2$$
$$+1451\sqrt{1-4\,x}\,x^3+37\sqrt{1-4\,x}\,x-2\sqrt{1-4\,x})\Big/(2\,x^4\,(32\,x^{11}-280\,x^{10}+1100\,x^9-2558\,x^8$$
$$+3907\,x^7-4109\,x^6+3031\,x^5-1565\,x^4+553\,x^3-127\,x^2+17\,x-1))$$

$$1+2\,x+6\,x^2+22\,x^3+90\,x^4+389\,x^5+1706\,x^6+7413\,x^7+31637\,x^8+132623\,x^9+548018\,x^{10}+O(x^{11})$$

```
> #10123: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: e=01012: Reg4
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*B:
eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
eqD:=-DD+x/(1-v)+2*x/v*(DD-DD1)+x*sum((m+2)*v^(m-1)*E,m=1..infinit
```

```
y): E:=x/(1-2*x):
> Dxv:=SolveKE(DD,DD1,eqD,1):
> eqB1:=subs(DD=Dxv,subs(C=solve(eqB,C),eqC)):
KK:=factor(coeff(eqB1,B)*x*v^2*(1-v)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
ss:=map(factor,solve({subs(v=u1,subs(B=0,eqB1)),subs(v=u2,subs(B=0
,eqB1))},{C1,B1})):op(op(ss)[1])[1],op(op(ss)[2])[1];
Bx0:=op(op(ss)[1])[2]: Cx0:=op(op(ss)[2])[2]:
Bxv:=solve(subs(B1=Bx0,C1=Cx0,eqB1),B):
Bxv:=subs(vv=u3,simplify(subs(u3=vv,factor(simplify(simplify(ratio
nalize(subs(u1=uu1,u2=uu2,Bxv)))+1)-1)))):
```

$$-x^2+(x^2+2x)v+(-x-1)v^2+v^3$$

$$B1, C1$$

```
> FinA:=factor(x/(1-x)*(1+subs(v=x,Bxv)));
taylor(subs(u3=vv,FinA),x,15);
```

$$FinA := -(2\,u3^2x^7-11\,u3^2x^6-2\,u3\,x^7+30\,u3^2x^5+11\,u3\,x^6+8\,x^7-49\,u3^2x^4-30\,u3\,x^5-12\,x^6+48\,u3^2x^3+49\,u3\,x^4-3\,x^5-27\,u3^2x^2-48\,u3\,x^3+21\,x^4+8\,u3^2x+27\,u3\,x^2-19\,x^3-u3^2-8\,u3\,x+7\,x^2+u3-x)\Big/(x^2(-1+x)^2(-1+2x)^3)$$

$$x+2x^2+6x^3+22x^4+90x^5+388x^6+1704x^7+7529x^8+33425x^9+149410x^{10}+673810x^{11}+3068824x^{12}+O(x^{13})$$

```
> #10112: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1,: e=01011: Reg4
> #10012: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,0,: Reg4
> #11012: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*B:
eqC:=-C+x/(1-v)+x/v*(C+B-C1-B1)+x*DD+x/(1-v)*B:
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*E+x/v^2*(B-B1-B2*v)+x/v*(B-B
1)+x/(1-v)*B: E:=x/(1-2*x):
> eqB1:=subs(DD=solve(subs(C=solve(eqB,C),eqC),DD),eqD):
factor(coeff(eqB1,B1)),factor(coeff(eqB1,C1)),factor(coeff(eqB1,DD
1));
```

$$-\frac{v^2 x^2 + v^2 x + v^2 - 2 v x + x^2}{x v^3}, -\frac{v - x}{v^2}, -\frac{x}{v}$$

```
> KK:=factor(coeff(eqB1,B)*x^2*v^3*(1-v)): taylor(KK,v,10);
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
```

$$x^3 - 3 x^2 v + (2 x^2 + 3 x) v^2 + (-2 x - 1) v^3 + v^4$$

```
> ss:=map(factor,solve({subs(v=u1,subs(B=0,eqB1)),subs(v=u2,subs(B=0
  ,eqB1)),subs(v=u3,subs(B=0,eqB1))},{B1,C1,DD1})):
  op(op(ss)[1])[1],op(op(ss)[2])[1],op(op(ss)[3])[1];
  Bx0:=op(op(ss)[1])[2]: Cx0:=op(op(ss)[2])[2]:
  Dx0:=op(op(ss)[3])[2]:
  Bxv:=solve(subs(B1=Bx0,C1=Cx0,DD1=Dx0,eqB1),B):
```

$$B1, C1, DD1$$

```
> FinA:=factor(x/(1-x)*(1+subs(v=x,Bxv)));
```

$$FinA := -(2\, u1\, u2\, u3\, x^4 - 2\, u1\, u2\, u3\, x^3 - u1\, u2\, x^4 - u1\, u3\, x^4 - u1\, x^5 - u2\, u3\, x^4 - u2\, x^5 - u3\, x^5 + x^6 + 3\, u1\, u2\, u3\, x^2 - 2\, u1\, u2\, x^3 - 2\, u1\, u3\, x^3 + 5\, u1\, x^4 - 2\, u2\, u3\, x^3 + 5\, u2\, x^4 + 5\, u3\, x^4 - 3\, x^5 - 3\, u1\, u2\, u3\, x + 3\, u1\, u2\, x^2 + 3\, u1\, u3\, x^2 - 4\, u1\, x^3 + 3\, u2\, u3\, x^2 - 4\, u2\, x^3 - 4\, u3\, x^3 + x^4 + u1\, u2\, u3 - u1\, u2\, x - u1\, u3\, x + u1\, x^2 - u2\, u3\, x + u2\, x^2 + u3\, x^2) \Big/ (x^2 (x-1)(-1+u2)(-1+2x)(-1+u1)(-1+u3))$$

```
>
> fu2:=simplify(series(VVV[2],x,13)) assuming x>0 and x<0.1:
  fu3:=simplify(series(VVV[3],x,13)) assuming x>0 and x<0.1:
  fu4:=simplify(series(VVV[4],x,13)) assuming x>0 and x<0.1:
> simplify(series(subs(u1=fu2,u2=fu3,u3=fu4,FinA),x,16)) assuming
  x>0 and x<0.1;
```

$$1750\, x^7 + O(x^{12}) + 390\, x^6 + 90\, x^5 + 6\, x^3 + 22\, x^4 + 2\, x^2 + 885298\, x^{11} + 181940\, x^{10} + 37955\, x^9 + 8062\, x^8 + x$$

```
> #10023:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,0,: 0,0,0,0,0,0,1,0,2,:
  0,0,0,0,0,1,0,2,0,: h=0103, j=01002, k=01030, reg7
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
  eqC:=-C+x/(1-v)+x*E+x/v*(C-C1)+x/(1-v)*(F+H):
  eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
  eqE:=-E+x/(1-v)+2*x/v*(E-E1)+x*sum((m+2)*v^(m-1)*J,m=1..infinity):
  eqF:=-F+x/(1-v)+x*G+x/v*(F-F1)+x/(1-v)*(F+H):
  eqG:=-G+x/(1-v)+x/v*(G-G1)+x*sum((m+3)*v^(m-1)*J,m=1..infinity):
```

```
eqH:=-HH+x*(1+KK+F1+HH): eqJ:=-JJ+x*(1+JJ+JJ):
eqK:=-KK+x*(1+G1+3*JJ):
> solve({eqH,eqJ,eqK},{HH,JJ,KK});
H:=-x*(2*G1*x^2+2*F1*x-G1*x-x^2-F1+x-1)/(2*x^2-3*x+1):
J:=-x/(2*x-1): K:=(2*G1*x-G1-x-1)*x/(2*x-1):
```

$$\left\{HH = -\frac{x\,(2\,G1\,x^2 + 2\,F1\,x - G1\,x - x^2 - F1 + x - 1)}{2\,x^2 - 3\,x + 1},\ JJ = -\frac{x}{2\,x - 1},\ KK = \frac{(2\,G1\,x - G1 - x - 1)\,x}{2\,x - 1}\right\}$$

```
> Gxv:=SolveKE(G,G1,eqG,1):
Fxv:=SolveKE(F,F1,subs(G=Gxv,G1=subs(v=0,Gxv),eqF),2):
Exv:=SolveKE(E,E1,eqE,1): Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,F=Fxv,F1=subs(v=0,Fxv),G1=subs(v=0,Gx
v),eqC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
> FinA:=factor(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x,Bxv)+1)
-1)); series(FinA,x,16);
```

$$FinA := -\left(1 - 18\,x + 1936\,x^{10} + 864\sqrt{1-4\,x}\,x^9 - 2183\,x^8\sqrt{1-4\,x} - 4768\,x^9 + 3464\,x^7\sqrt{1-4\,x} - 3683\,x^6\sqrt{1-4\,x} + 2708\sqrt{1-4\,x}\,x^5 + 8007\,x^8 - 9480\,x^7 + 8089\,x^6 - 5002\,x^5 - 1390\,x^4\sqrt{1-4\,x} + 492\,x^3\sqrt{1-4\,x} - 115\,x^2\sqrt{1-4\,x} + 16\,x\sqrt{1-4\,x} + 2226\,x^4 - \sqrt{1-4\,x} + 64\,x^{12} - 528\,x^{11} - 696\,x^3 + 145\,x^2 + 16\sqrt{1-4\,x}\,x^{11} - 188\sqrt{1-4\,x}\,x^{10}\right) \Big/ \left(2\,x^2\,(-1+4\,x)\,(2\,x-1)^3\,(x-1)^6\right)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 388\,x^6 + 1684\,x^7 + 7181\,x^8 + 29922\,x^9 + 122288\,x^{10} + 493124\,x^{11} + 1972118\,x^{12} + 7849484\,x^{13} + O(x^{14})$$

```
> #10022: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,0,:
0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,1,0,2,0,: h=0103, j=01002, k=01003,
l=01004, Reg7:
> #10020: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,0,:
0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,1,0,2,0,: h=0103, j=01002, k=01003,
l=01004, Reg7:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+x*E+x/v*(C-C1)+x/(1-v)*(F+H):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
```

```
eqE:=-E+x/(1-v)+2*x/v*(E-E1)+x*v/(1-v)*G+x/(1-v)*(J+K+L):
eqF:=-F+x/(1-v)+x*G+x/v*(F-F1)+x/(1-v)*(F+H):
eqG:=-G+x/(1-v)+x/v*(G-G1)+x/(1-v)*(G+J+K+L):
eqH:=-HH+x*(1+JJ+F1+HH): eqJ:=-JJ+x*(1+G1+JJ+KK+LL):
eqK:=-KK+x*(1+JJ+KK+LL): eqL:=-LL+x*(1+KK+LL):
```

```
> ss:=solve({eqH,eqJ,eqK,eqL},{HH,JJ,KK,LL}); H:=op(op(ss)[1])[2]:
J:=op(op(ss)[2])[2]: K:=op(op(ss)[3])[2]: L:= op(op(ss)[4])[2]:
```

$$ss := \{HH = -\frac{x\,(F1\,x^2 - 2\,G1\,x^2 - 3\,F1\,x + G1\,x + x^2 + F1 - 2\,x + 1)}{x^3 - 4\,x^2 + 4\,x - 1},$$

$$JJ = -\frac{x\,(2\,G1\,x - G1 - 1)}{x^2 - 3\,x + 1},\ KK = -\frac{x\,(G1\,x^2 - G1\,x - 1)}{x^2 - 3\,x + 1},\ LL = \frac{x\,(G1\,x^2 - x + 1)}{x^2 - 3\,x + 1}\}$$

```
> Gxv:=SolveKE(G,G1,eqG,2):
Fxv:=SolveKE(F,F1,(subs(G=Gxv,G1=subs(v=0,Gxv),eqF)),2):
Exv:=SolveKE(E,E1,subs(G=Gxv,G1=subs(v=0,Gxv),eqE),1):
Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,F=Fxv,G1=subs(v=0,Gxv),F1=subs(v=0,Fx
v),eqC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
```

```
> FinA:=factor(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x,Bxv)+1)
-1)); series(FinA,x,16);
```

$$FinA := -(44\,x^6 + 35\sqrt{1-4\,x}\,x^5 - 163\,x^5 - 116\sqrt{1-4\,x}\,x^4 + 314\,x^4 + 151\sqrt{1-4\,x}\,x^3 - 289\,x^3 - 87\,x^2\sqrt{1-4\,x} + 127\,x^2 + 22\,x\sqrt{1-4\,x} - 26\,x - 2\sqrt{1-4\,x} + 2) \Big/ (2\,x^3\,(x-1)\,(-1+4\,x)^2)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 391\,x^6 + 1748\,x^7 + 7889\,x^8 + 35573\,x^9 + 159476\,x^{10} + 709420\,x^{11} + 3130128\,x^{12} + O(x^{13})$$

```
> #10002: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,0,:
0,0,0,0,0,0,1,0,2,: 0,0,0,0,0,1,0,0,2,: h=0103, j=01003, k=01004,
l=01000, p=010020, Reg7
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x*C+x/(1-v)*DD:
eqC:=-C+x/(1-v)+x*E+x/v*(C-C1)+x/(1-v)*(F+H):
eqD:=-DD+x/(1-v)+x*F+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)*(1+L)+x/v*(E-E1)+x/(1-v)*G+x/(1-v)*(J+K):
eqF:=-F+x/(1-v)+x*G+x/v*(F-F1)+x/(1-v)*(F+H):
eqG:=-G+x/(1-v)*(1+P)+x/v*(G-G1)+x/(1-v)*(G+J+K):
eqH:=-HH+x*(1+JJ+F1+HH): eqJ:=-JJ+x*(1+P+G1+JJ+KK):
eqK:=-KK+x*(1+P+JJ+KK): L:=x/(1-2*x): P:=x/(1-x):
```

```
> ss:=solve({eqH,eqJ,eqK},{HH,JJ,KK}); H:=op(op(ss)[1])[2]:
  J:=op(op(ss)[2])[2]: K:=op(op(ss)[3])[2]:
```

$$ss := \{HH = -\frac{(G1\,x^3 + 2\,F1\,x^2 - 2\,G1\,x^2 - 3\,F1\,x + G1\,x + 2\,x^2 + F1 - 2\,x + 1)\,x}{(x^2 - 2\,x + 1)\,(2\,x - 1)},$$
$$JJ = \frac{(G1\,x^2 - 2\,G1\,x + G1 + 1)\,x}{(2\,x - 1)\,(x - 1)},\ KK = -\frac{x\,(G1\,x^2 - G1\,x - 1)}{2\,x^2 - 3\,x + 1}\}$$

```
> Gxv:=SolveKE(G,G1,eqG,2):
  Fxv:=SolveKE(F,F1,(subs(G=Gxv,G1=subs(v=0,Gxv),eqF)),2):
  Exv:=SolveKE(E,E1,subs(G=Gxv,G1=subs(v=0,Gxv),eqE),1):
  Dxv:=SolveKE(DD,DD1,subs(F=Fxv,eqD),2):
  Cxv:=SolveKE(C,C1,subs(E=Exv,F=Fxv,G1=subs(v=0,Gxv),F1=subs(v=0,Fx
  v),eqC),1): Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
> FinA:=factor(simplify(rationalize(x/(1-x)+x/(1-x)*subs(v=x,Bxv)+1)
  -1)); series(FinA,x,16);
```

$$FinA := -(1 + 3046\,x^4 - 905\,x^3 - 20\,x + 176\,x^2 + 96\,x^{12} - 800\,x^{11} + 3278\,x^{10} + 76\sqrt{1-4\,x}\,x^{11}$$
$$- 596\sqrt{1-4\,x}\,x^{10} + 2105\sqrt{1-4\,x}\,x^9 - 4423\sqrt{1-4\,x}\,x^8 - 7879\,x^9 + 6147\sqrt{1-4\,x}\,x^7$$
$$- 5934\sqrt{1-4\,x}\,x^6 + 4057\sqrt{1-4\,x}\,x^5 + 12685\,x^8 - 14447\,x^7 + 11902\,x^6 - 7115\,x^5$$
$$- 1960\sqrt{1-4\,x}\,x^4 + 653\sqrt{1-4\,x}\,x^3 - 142\,x^2\sqrt{1-4\,x} + 18\,x\sqrt{1-4\,x} - \sqrt{1-4\,x}) \Big/ (2\,x^3$$
$$(2\,x - 1)\,(-1 + 4\,x)^2\,(x - 1)^6)$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 90\,x^5 + 390\,x^6 + 1728\,x^7 + 7685\,x^8 + 34051\,x^9 + 149941\,x^{10} + 655751\,x^{11}$$
$$+ 2848469\,x^{12} + O(x^{13})$$

```
> #01234: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: Reg3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x*sum(m*v^(m-1)*DD,m=1..infinity):
  DD:=x/(1-2*x):
> Cxv:=SolveKE(C,C1,eqC,1): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{-x+1} + \frac{x^2\,(28\,x^6 - 78\,x^5 + 102\,x^4 - 80\,x^3 + 37\,x^2 - 9\,x + 1)}{(-x+1)\,(-1+2\,x)^5\,(x-1)^3}$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 89\,x^5 + 368\,x^6 + 1474\,x^7 + 5582\,x^8 + 19891\,x^9 + 66960\,x^{10} + 214372\,x^{11}$$
$$+ 657198\,x^{12} + 1941229\,x^{13} + 5553824\,x^{14} + 15457862\,x^{15} + O(x^{16})$$

```
> #01233: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
```

```
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: Reg4:
> #01230: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: Reg4:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*DD:
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*DD:
> Dxv:=SolveKE(DD,DD1,eqD,2): Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{1-x} + \frac{2\sqrt{1-4x}\,x^3 - 4x^4 + 6x^3 - 2x^2}{(1-x)(-1+x)(1+\sqrt{1-4x} - 6x - 6x\sqrt{1-4x} + 8x^2 + 8x^2\sqrt{1-4x})}$$

$$x + 2x^2 + 6x^3 + 22x^4 + 89x^5 + 374x^6 + 1583x^7 + 6668x^8 + 27866x^9 + 115570x^{10} + 476266x^{11} + 1952798x^{12} + 7975328x^{13} + 32471190x^{14} + 131883508x^{15} + O(x^{16})$$

```
> #01223: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,2,: d=0122: Reg3:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*(C+DD): DD:=x/(1-2*x):
> Cxv:=SolveKE(C,C1,eqC,2): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{1-x} + (2x^4\sqrt{1-4x} - 8x^5 + 4x^3\sqrt{1-4x} + 26x^4 - 4\sqrt{1-4x}\,x^2 - 22x^3 + \sqrt{1-4x}\,x + 8x^2 - x) \Big/ ((1-x)(-1+x)(-20x^2 + 8x - 1 + 16x^3 - 20\sqrt{1-4x}\,x^2 + 8\sqrt{1-4x}\,x - \sqrt{1-4x} + 16x^3\sqrt{1-4x}))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 88x^5 + 359x^6 + 1452x^7 + 5786x^8 + 22764x^9 + 88783x^{10} + 344512x^{11} + 1333633x^{12} + 5159326x^{13} + 19968173x^{14} + 77362136x^{15} + O(x^{16})$$

```
> #01222: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,2,: Reg4:
> #01202: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,0,: Reg4:
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*C+x*DD:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1+C-C1)+x/(1-v)*C:
> eqC1:=subs(DD=solve(eqC,DD),eqD):
  KK:=factor(x*v^2*(1-v)*coeff(eqC1,C)): taylor(KK,v,10);
  alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
  uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2\,x)\,v+(-x-1)\,v^2+v^3$$

```
> ss:=solve({subs(v=u1,subs(C=0,eqC1)),subs(v=u2,subs(C=0,eqC1))},{C
  1,DD1}): op(op(ss)[1])[1],op(op(ss)[2])[1]; Cx0:=op(op(ss)[1])[2]:
  Dx0:=op(op(ss)[2])[2]: Cxv:=solve(subs(C1=Cx0,DD1=Dx0,eqC1),C):
```

$$C1,\,DD1$$

```
> Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  Bxv:=subs(vv=u3,simplify(subs(u3=vv,simplify(simplify(rationalize(
  subs(u1=uu1,u2=uu2,Bxv))+1)-1)))):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv);
  series(subs(u3=vv,FinA),x,16);
```

$$FinA := \frac{x}{1-x}+\frac{x\,(4\,u3^2\,x^2-2\,u3\,x^3-u3^2\,x-4\,u3\,x^2+5\,x^3+u3\,x-x^2)}{(1-x)\,(6\,x^4-x^3)}$$

$$x+2\,x^2+6\,x^3+22\,x^4+89\,x^5+379\,x^6+1665\,x^7+7474\,x^8+34098\,x^9+157564\,x^{10}+735708\,x^{11}+3465116\,x^{12}+\mathrm{O}(x^{13})$$

```
> #01220:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,2,:
  0,0,0,0,0,0,1,2,2,: e=01224 Reg4
> #01200:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,2,:
  0,0,0,0,0,0,1,2,0,: e=01204 Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)+x/v*(C-C1)+x/(1-v)*C+x*DD:
  eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*(DD+E): E:=(x+x*DD1)/(1-x):
> Dxv:=SolveKE(DD,DD1,eqD,2): Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),2):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{1-x} - 4\,(10\sqrt{1-4x}\,x^7 + 8x^8 + 34x^7 - 33x^6 - 19\sqrt{1-4x}\,x^6 + 8\sqrt{1-4x}\,x^5 + 10x^5 - \sqrt{1-4x}\,x^4 - x^4) \Big/ (x^2(1-x)(12\sqrt{1-4x}\,x^2 - 16x^3 - 7\sqrt{1-4x}\,x + 24x^2 + \sqrt{1-4x} - 9x + 1)(1+\sqrt{1-4x} - 2x)(\sqrt{1-4x} + 1 - 4x))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 89x^5 + 377x^6 + 1628x^7 + 7072x^8 + 30707x^9 + 132891x^{10} + 572567x^{11} + 2455303x^{12} + 10480437x^{13} + O(x^{14})$$

```
> #01203:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,3,: e=0120, f=01230: Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+2*x/v*(B-B1)+x/(1-v)*C:
  eqC:=-C+x/(1-v)*(1+E)+x/v*(C-C1)+x/(1-v)*DD:
  eqD:=-DD+x/(1-v)*(1+F)+x/v*(DD-DD1)+x/(1-v)*DD: E:=x/(1-2*x):
  F:=x/(1-x):
> Dxv:=SolveKE(DD,DD1,eqD,2): Cxv:=SolveKE(C,C1,subs(DD=Dxv,eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),1):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{1-x} + (-x + 56x^8 + 24\sqrt{1-4x}\,x^8 - 46x^7\sqrt{1-4x} - 206x^7 - 354x^5 + 360x^6 + 24\sqrt{1-4x}\,x^6 + 30\sqrt{1-4x}\,x^5 + 205x^4 - 53\sqrt{1-4x}\,x^4 - 9\sqrt{1-4x}\,x^2 - 70x^3 + 13x^2 + 32\sqrt{1-4x}\,x^3 + \sqrt{1-4x}\,x) \Big/ ((1-x)(-1+x)^2(1 - 12x - 120x^5 + 32x^6 + 32\sqrt{1-4x}\,x^6 - 120\sqrt{1-4x}\,x^5 + 180x^4 + 180\sqrt{1-4x}\,x^4 + 57\sqrt{1-4x}\,x^2 - 138x^3 + 57x^2 - 138\sqrt{1-4x}\,x^3 - 12\sqrt{1-4x}\,x + \sqrt{1-4x}))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 88x^5 + 358x^6 + 1435x^7 + 5632x^8 + 21745x^9 + 83188x^{10} + 317244x^{11} + 1210801x^{12} + 4634360x^{13} + 17801713x^{14} + 68624309x^{15} + O(x^{16})$$

```
> #01123:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: d=0112 Reg3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)+2*x/v*(C-C1)+sum((m+1)*x*v^(m-1)*DD,m=1..infinity)
```

```
: DD:=x/(1-2*x):
> Cxv:=SolveKE(C,C1,eqC,1): Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{-x+1} - (2\sqrt{1-4x}\,x^5 - 10\sqrt{1-4x}\,x^4 + 2x^5 + 11x^3\sqrt{1-4x} + 4x^4 - 5x^2\sqrt{1-4x} - 9x^3 + x\sqrt{1-4x} + 5x^2 - x) \Big/ ((-x+1)(-1+2x)^3(\sqrt{1-4x} - 2x + 1)(x-1))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 87x^5 + 344x^6 + 1322x^7 + 4924x^8 + 17887x^9 + 63884x^{10} + 225971x^{11} + 796240x^{12} + 2806843x^{13} + 9927294x^{14} + 35290038x^{15} + O(x^{16})$$

```
> #01122: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: e=0113 reg4:
> #01120: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,2,: e=0113 reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x/(1-v)*(DD+E):
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*(DD+E): E:=(x+x*DD1)/(1-x):
> Dxv:=SolveKE(DD,DD1,eqD,2):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=subs(v=0,Dxv),eqC),1):
Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); series(FinA,x,16);
```

$$FinA := \frac{x}{1-x} + \frac{2(7\sqrt{1-4x}\,x^6 - 5\sqrt{1-4x}\,x^5 + \sqrt{1-4x}\,x^4 - 7x^5 + 11x^6 + 4x^7 + x^4)}{x^2(1-x)(-1-\sqrt{1-4x} + 6x + 4\sqrt{1-4x}\,x - 8x^2)(3\sqrt{1-4x}\,x - 4x^2 - \sqrt{1-4x} + 5x - 1)}$$

$$x + 2x^2 + 6x^3 + 22x^4 + 88x^5 + 364x^6 + 1522x^7 + 6374x^8 + 26640x^9 + 110980x^{10} + 460716x^{11} + 1906172x^{12} + 7862416x^{13} + O(x^{14})$$

```
> #01112:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
d=0111: Reg3
> #01012:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
d=0101: Reg3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
```

```
eqC:=-C+x/(1-v)+x/v*(C-C1+B-B1)+x/(1-v)*(B+DD): DD:=x/(1-2*x):
> eqB1:=subs(C=solve(eqB,C),eqC):
KK:=factor(x*v^2*(1-v)*coeff(eqB1,B)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2x)v+(-x-1)v^2+v^3$$

```
> ss:=solve({subs(v=u1,subs(B=0,eqB1)),subs(v=u2,subs(B=0,eqB1))},{B
1,C1}): op(op(ss)[1])[1],op(op(ss)[2])[1]; Bx0:=op(op(ss)[1])[2]:
Cx0:=op(op(ss)[2])[2]: Bxv:=solve(subs(C1=Cx0,B1=Bx0,eqB1),B):
```

$$B1, C1$$

```
> Bxv:=subs(vv=u3,simplify(subs(u3=vv,simplify(simplify(rationalize(
subs(u1=uu1,u2=uu2,Bxv))+1)-1)))):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv);
series(subs(u3=vv,FinA),x,16);
```

$$FinA := \frac{x}{1-x}-\frac{x\,(u3^2\,x^3-3\,u3^2\,x^2-u3\,x^3+3\,u3^2\,x+3\,u3\,x^2-x^3-u3^2-3\,u3\,x+2\,x^2+u3-x)}{(1-x)\,(2\,x^4-x^3)}$$

$$x+2\,x^2+6\,x^3+22\,x^4+87\,x^5+356\,x^6+1494\,x^7+6413\,x^8+28088\,x^9+125185\,x^{10}+566257\,x^{11}+2593591\,x^{12}+\mathrm{O}(x^{13})$$

```
> #01111:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
Reg3
> #01101:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
Reg3
> #01011:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
Reg3
> #01001:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
Ref3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+x/v*(C-C1+B-B1)+x/(1-v)*B+x/v^2*(A-A1-v*A2):
> eqA1:=subs(C=solve(subs(B=solve(eqA,B),eqB),C),B=solve(eqA,B),eqC)
: KK:=factor(v^3*x^2*coeff(eqA1,A)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
factor(coeff(eqA1,A2)),factor(coeff(eqA1,C1));
eqA2:=subs(A2=LA-C1,A=0,eqA1):
```

$$x^3-3\,x^2\,v+(2\,x^2+3\,x)\,v^2+(-2\,x-1)\,v^3+v^4$$

$$-\frac{x}{v}, -\frac{x}{v}$$

```
> ss:=solve({subs(v=u1,eqA2),subs(v=u2,eqA2),subs(v=u3,eqA2)},{A1,B1
,LA}): op(op(ss)[1])[1],op(op(ss)[2])[1],op(op(ss)[3])[1];
Ax0:=op(op(ss)[1])[2]; Ax0:=-(x^3/u4)/(x^3/u4-x^2):
```

$$A1, B1, LA$$

$$Ax0 := -\frac{u1\ u2\ u3}{u1\ u2\ u3 - x^2}$$

```
> simplify(series(subs(u4=vv,Ax0),x,15)) assuming x>0 and x<0.1;
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 89\,x^5 + 382\,x^6 + 1708\,x^7 + 7870\,x^8 + 37108\,x^9 + 178184\,x^{10} + 868318\,x^{11} + 4283402\,x^{12} + \mathrm{O}(x^{13})$$

```
> #01110: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,1, e=01113, f=01114: reg4
> #01010: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,1, e=01013, f=01014: reg4:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)+x/v*(C-C1+B-B1)+x/(1-v)*B+x*DD:
eqD:=-DD+x/(1-v)+x/v*(DD-DD1)+x/(1-v)*(DD+E+F):
eqE:=-EE+x*(1+EE+FF+DD1): eqF:=-FF+x*(1+EE+FF):
> solve({eqF,eqE},{EE,FF});E:= x*(DD1*x-DD1-1)/(2*x-1):F:=
-x*(DD1*x+1)/(2*x-1):
```

$$\{EE = \frac{x\,(DD1\,x - DD1 - 1)}{2\,x - 1}, FF = -\frac{x\,(DD1\,x + 1)}{2\,x - 1}\}$$

```
> Dxv:=SolveKE(DD,DD1,eqD,2):
```

$$-B + \frac{x}{1-v} + \frac{x\,(B - B1)}{v} + \frac{x\,B}{1-v} + x\,C, -C + \frac{x}{1-v} + \frac{x\,(C - C1 + B - B1)}{v} + \frac{x\,B}{1-v} + x\,DD$$

```
> eqB1:=subs(DD=Dxv,C=solve(subs(DD=Dxv,eqB),C),eqC):
KK:=factor(x*v^2*(1-v)*coeff(eqB1,B)): taylor(KK,v,10);
alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2 + (x^2 + 2\,x)\,v + (-x - 1)\,v^2 + v^3$$

```
> ss:=solve({subs(v=u1,subs(B=0,eqB1)),subs(v=u2,subs(B=0,eqB1))},{B
1,C1}): op(op(ss)[1])[1],op(op(ss)[2])[1]; Bx0:=op(op(ss)[1])[2]:
Cx0:=op(op(ss)[2])[2]: Bxv:=solve(subs(C1=Cx0,B1=Bx0,eqB1),B):
```

$$B1, C1$$

```
> Bxv:=subs(vv=u3,simplify(subs(u3=vv,simplify(simplify(rationalize(
  subs(u1=uu1,u2=uu2,Bxv))+1)-1)))):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv);
  series(subs(u3=vv,FinA),x,16);
```

$$FinA := \frac{x}{1-x} - (-x^3\sqrt{1-4x}\,u3 - x^2\sqrt{1-4x}\,u3^2 - \sqrt{1-4x}\,x^5 + \sqrt{1-4x}\,x^3\,u3^2 - x^2\,u3 - x^3\,u3^2 - 2\sqrt{1-4x}\,x^3 + x^3\,u3 + x^2\,u3^2 + x^2\sqrt{1-4x}\,u3 + 3\sqrt{1-4x}\,x^4 + 2x^3 - 5x^4 + 5x^5) \Big/ (2x^5(1-x))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 89x^5 + 379x^6 + 1662x^7 + 7426x^8 + 33624x^9 + 153821x^{10} + O(x^{11})$$

```
> #01102:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,1,:
  0,0,0,0,0,0,1,1,2, reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
  eqC:=-C+x/(1-v)*(1+F)+x/v*(C-C1)+x/(1-v)*(DD+E):
  eqD:=-DD+x/(1-v)*(1+G)+x/v*(DD-DD1)+x/(1-v)*(DD+E): G:=x/(1-x):
  F:=x/(1-2*x): E:=(x+x*DD1+x*G)/(1-x):
> Dxv:=SolveKE(DD,DD1,eqD,2):
  Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=subs(v=0,Dxv),eqC),1):
  Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
  FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); taylor(FinA,x,15);
```

$$FinA := \frac{x}{1-x} + 2(-11\sqrt{1-4x}\,x^7 + 2\sqrt{1-4x}\,x^8 + 13\sqrt{1-4x}\,x^6 - 6\sqrt{1-4x}\,x^5 - 11x^7 + 13x^6 - 6x^5 + \sqrt{1-4x}\,x^4 + x^4) \Big/ (x^2(1-x)(-13x^2 + 12x^3 + 6x - 1 - 5\sqrt{1-4x}\,x^2 + 2\sqrt{1-4x}\,x^3 + 4\sqrt{1-4x}\,x - \sqrt{1-4x} - 4x^4)(3\sqrt{1-4x}\,x - 4x^2 - \sqrt{1-4x} + 5x - 1))$$

$$x + 2x^2 + 6x^3 + 22x^4 + 87x^5 + 349x^6 + 1393x^7 + 5520x^8 + 21764x^9 + 85576x^{10} + 336083x^{11} + 1319461x^{12} + O(x^{13})$$

```
> #01100: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,1,: 0,0,0,0,0,0,1,1,0,: e=0113, f=01103, g=01104,
  Reg4
> #01000: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,1,0,0,: e=0103, f=01003, g=01004,
  Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
```

```
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x/v*(B-B1)+x/(1-v)*B+x*C:
eqC:=-C+x/(1-v)*(1+E)+x*DD+x/v*(C-C1)+x/(1-v)*C:
eqD:=-DD+x/(1-v)*(1+F+G)+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-EE+x*(1+C1+FF+EE): eqF:=-FF+x*(1+DD1+FF+GG):
eqG:=-GG+x*(1+FF+GG):
```

```
> solve({eqE,eqF,eqG},{EE,FF,GG});
E:=-x*(DD1*x^2+2*C1*x-DD1*x-C1+x-1)/(2*x^2-3*x+1): F:=
x*(DD1*x-DD1-1)/(2*x-1): G:= -x*(DD1*x+1)/(2*x-1):
```

$$\{EE = -\frac{x\,(DD1\,x^2 + 2\,C1\,x - DD1\,x - C1 + x - 1)}{2\,x^2 - 3\,x + 1},\ FF = \frac{x\,(DD1\,x - DD1 - 1)}{2\,x - 1},$$
$$GG = -\frac{x\,(DD1\,x + 1)}{2\,x - 1}\}$$

```
> Dxv:=SolveKE(DD,DD1,eqD,2):
Cxv:=SolveKE(C,C1,subs(DD=Dxv,DD1=subs(v=0,Dxv),eqC),2):
Bxv:=SolveKE(B,B1,subs(C=Cxv,eqB),2):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); taylor(FinA,x,15);
```

$$FinA := \frac{x}{-x+1} - 4\,(-226\,x^{12} - 28\sqrt{1-4\,x}\,x^{12} + 235\sqrt{1-4\,x}\,x^{11} - 381\sqrt{1-4\,x}\,x^{10}$$
$$+ 261\sqrt{1-4\,x}\,x^9 - 89\sqrt{1-4\,x}\,x^8 + 15\sqrt{1-4\,x}\,x^7 - \sqrt{1-4\,x}\,x^6 + 709\,x^{11} - 775\,x^{10} + 413\,x^9$$
$$- 117\,x^8 + 17\,x^7 - x^6 - 4\,x^{14} - 7\,x^{13} - 3\sqrt{1-4\,x}\,x^{13}) \Big/ (x^4\,(-x+1)$$
$$(x\sqrt{1-4\,x} - 1 - \sqrt{1-4\,x} + 3\,x)\,(-4\,x^2 + 5\,x - \sqrt{1-4\,x} - 1 + 3\,x\sqrt{1-4\,x})\,(28\sqrt{1-4\,x}\,x^4$$
$$- 16\,x^5 - 63\sqrt{1-4\,x}\,x^3 + 104\,x^4 + 42\,x^2\sqrt{1-4\,x} - 129\,x^3 - 11\,x\sqrt{1-4\,x} + 62\,x^2 + \sqrt{1-4\,x}$$
$$- 13\,x + 1))$$
$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 89\,x^5 + 377\,x^6 + 1628\,x^7 + 7072\,x^8 + 30706\,x^9 + 132872\,x^{10} + O(x^{11})$$

```
> #01023: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,0,:
f=0102: Reg5
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x*C+x/v*(B-B1)+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x*sum((m+1)*v^(m-1)*F,m=1..infinity):
eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x*sum((m+2)*v^(m-1)*F,m=1..infinity):
```

```
F:=x/(1-2*x):
> Exv:=SolveKE(E,E1,eqE,1): Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),2);
Cxv:=SolveKE(C,C1,eqC,1):
Bxv:=SolveKE(B,B1,subs(DD=Dxv,C=Cxv,eqB),1):
FinA:=x/(1-x)+x/(1-x)*subs(v=x,Bxv); taylor(FinA,x,15);
```

$$Dxv := x\,(3\sqrt{1-4x}\,v^2x^2 - 2v^2x^3 - 3\sqrt{1-4x}\,v^2x - 3\sqrt{1-4x}\,vx^2 + x^3\sqrt{1-4x} + 5v^2x^2 - 2vx^3 - 2x^4 + \sqrt{1-4x}\,v^2 + 3\sqrt{1-4x}\,vx - x^2\sqrt{1-4x} - 3v^2x - vx^2 + 7x^3 - \sqrt{1-4x}\,v + v^2 + xv - 5x^2 - v + 2x) \Big/ \left(\left(2x^3\sqrt{1-4x} - 4x^4 - 5x^2\sqrt{1-4x} + 12x^3 + 4x\sqrt{1-4x} - 13x^2 - \sqrt{1-4x} + 6x - 1\right)(v-1)(v^2 - v + x)\right)$$

$$FinA := \frac{x}{-x+1} + (x - 12x^9 + 26x^8 + 14\sqrt{1-4x}\,x^8 - 54\sqrt{1-4x}\,x^7 - 10x^7 + 104\sqrt{1-4x}\,x^6 - 120\sqrt{1-4x}\,x^5 + 84\sqrt{1-4x}\,x^4 - 36x^6 + 74x^5 - 68x^4 + 34x^3 - 36x^3\sqrt{1-4x} + 9x^2\sqrt{1-4x} - x\sqrt{1-4x} - 9x^2) \Big/ \left((-x+1)(-1+x)^2(4x^3 - 8x^2 + 5x - 1)\left(2x^3\sqrt{1-4x} - 4x^4 - 5x^2\sqrt{1-4x} + 12x^3 + 4x\sqrt{1-4x} - 13x^2 - \sqrt{1-4x} + 6x - 1\right)\right)$$

$$x + 2x^2 + 6x^3 + 22x^4 + 87x^5 + 341x^6 + 1280x^7 + 4595x^8 + 15961x^9 + 54418x^{10} + 184415x^{11} + 627008x^{12} + 2151350x^{13} + 7470265x^{14} + O(x^{15})$$

```
> #01022: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,0,:
f=0102, g=0103 Reg5
> #01020: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,2,0,:
f=0102, g=0103 Reg5
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
eqB:=-B+x/(1-v)+x*C+x/v*(B-B1)+x/(1-v)*DD:
eqC:=-C+x/(1-v)+2*x/v*(C-C1)+x*v/(1-v)*E+x/(1-v)*(F+G):
eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*DD:
eqE:=-E+x/(1-v)+x/v*(E-E1)+x/(1-v)*(E+F+G):
eqF:=-FF+x*(1+E1+FF+GG): eqG:=-GG+x*(1+FF+GG):
> solve({eqF,eqG},{FF,GG});F:= x*(E1*x-E1-1)/(2*x-1): G:=
-x*(E1*x+1)/(2*x-1):
```

$$\left\{FF = \frac{x\,(E1\,x - E1 - 1)}{2x-1},\ GG = -\frac{x\,(E1\,x + 1)}{2x-1}\right\}$$

```
> Exv:=SolveKE(E,E1,eqE,2): Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),2):
Cxv:=SolveKE(C,C1,subs(E=Exv,DD=Dxv,E1=subs(v=0,Exv),eqC),1): eqB;
```

```
Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
FinA:=simplify(x/(1-x)+x/(1-x)*subs(v=x,Bxv)); taylor(FinA,x,15);
```

$$-B+\frac{x}{1-v}+x\,C+\frac{x\,(B-B1)}{v}+\frac{x\,DD}{1-v}$$

$$FinA := 4\,x\,(-8\,x^7+24\sqrt{1-4\,x}\,x^6+130\,x^6-120\sqrt{1-4\,x}\,x^5-364\,x^5+205\sqrt{1-4\,x}\,x^4+443\,x^4-164\sqrt{1-4\,x}\,x^3-274\,x^3+66\sqrt{1-4\,x}\,x^2+90\,x^2-13\,x\sqrt{1-4\,x}-15\,x+\sqrt{1-4\,x}+1)\Big/((x-1)(2\sqrt{1-4\,x}\,x^3-9\sqrt{1-4\,x}\,x^2+12\,x^3+6\,x\sqrt{1-4\,x}-19\,x^2-\sqrt{1-4\,x}+8\,x-1)(x\sqrt{1-4\,x}-1-\sqrt{1-4\,x}+3\,x)(-4\,x^2+5\,x-\sqrt{1-4\,x}-1+3\,x\sqrt{1-4\,x}))$$

$$x+2\,x^2+6\,x^3+22\,x^4+88\,x^5+363\,x^6+1507\,x^7+6239\,x^8+25687\,x^9+105145\,x^{10}+428145\,x^{11}+1735649\,x^{12}+7010239\,x^{13}+28228774\,x^{14}+O(x^{15})$$

```
> #01002:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,0,:
  0,0,0,0,0,0,0,1,2,: 0,0,0,0,0,0,1,0,2,:  reg5:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA:=-A+x/(1-v)+x/v*(A-A1)+x/(1-v)*B:
  eqB:=-B+x/(1-v)+x*C+x/v*(B-B1)+x/(1-v)*DD:
  eqC:=-C+x/(1-v)*(1+G)+x/v*(C-C1)+x/(1-v)*E+x/(1-v)*F:
  eqD:=-DD+x/(1-v)+x*E+x/v*(DD-DD1)+x/(1-v)*DD:
  eqE:=-E+x/(1-v)*(1+H)+x/v*(E-E1)+x/(1-v)*(E+F): H:=x/(1-x):
  G:=x/(1-2*x): F:=(x+x*H+x*E1)/(1-x):
> Exv:=SolveKE(E,E1,eqE,2): Dxv:=SolveKE(DD,DD1,subs(E=Exv,eqD),2):
  Cxv:=SolveKE(C,C1,subs(E=Exv,DD=Dxv,E1=subs(v=0,Exv),eqC),1): eqB;
  Bxv:=SolveKE(B,B1,subs(C=Cxv,DD=Dxv,eqB),1):
  FinA:=simplify(x/(1-x)+x/(1-x)*subs(v=x,Bxv)); taylor(FinA,x,15);
```

$$-B+\frac{x}{1-v}+x\,C+\frac{x\,(B-B1)}{v}+\frac{x\,DD}{1-v}$$

$$FinA := -4\,x\,(-16\,x^{11}+32\sqrt{1-4\,x}\,x^{10}+236\,x^{10}-294\sqrt{1-4\,x}\,x^9-1282\,x^9+1092\sqrt{1-4\,x}\,x^8+3470\,x^8-2205\sqrt{1-4\,x}\,x^7-5655\,x^7+2814\sqrt{1-4\,x}\,x^6+6092\,x^6-2401\sqrt{1-4\,x}\,x^5-4471\,x^5+1379\sqrt{1-4\,x}\,x^4+2233\,x^4-523\sqrt{1-4\,x}\,x^3-743\,x^3+125\sqrt{1-4\,x}\,x^2+157\,x^2-17\sqrt{1-4\,x}\,x-19\,x+\sqrt{1-4\,x}+1)\Big/((-1+x)^2(10\sqrt{1-4\,x}\,x^5-8\,x^6-35\sqrt{1-4\,x}\,x^4+46\,x^5+47\sqrt{1-4\,x}\,x^3-95\,x^4-30\sqrt{1-4\,x}\,x^2+93\,x^3+9\sqrt{1-4\,x}\,x-46\,x^2-\sqrt{1-4\,x}+11\,x-1)(3\sqrt{1-4\,x}\,x^2-4\,x^3-4\sqrt{1-4\,x}\,x+9\,x^2+\sqrt{1-4\,x}-6\,x+1)(1+\sqrt{1-4\,x}-2\,x))$$

$$x+2\,x^2+6\,x^3+22\,x^4+87\,x^5+348\,x^6+1382\,x^7+5450\,x^8+21421\,x^9+84125\,x^{10}+330453\,x^{11}$$

$$+ 1298685\, x^{12} + 5106130\, x^{13} + 20083379\, x^{14} + O(x^{15})$$

```
> #00123: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,1,1,1,1,1,1,1,1,: a1=0, b1=01, d=002, e=0012  Reg3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*C2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+DD):
  eqB:=-B+x/(1-v)+2*x/v*(B-B2)+x*sum(m*v^(m-2)*E,m=2..infinity):
  eqC:=-C+x/(1-v)+x/v*(C-C2)+x/v^2*(A-A2-v*A3)+x/(1-v)*(B+DD):
  DD:=x+2*x*B2+x*E: E:=x/(1-2*x):
> Bxv:=SolveKE(B,B2,eqB,1):
  Axv:=SolveKE(A,A2,subs(B=Bxv,B2=subs(v=0,Bxv),eqA),1):
  Cxv:=SolveKE(C,C2,subs(A=Axv,A2=subs(v=0,Axv),B=Bxv,B2=subs(v=0,Bx
  v),A3=subs(v=0,diff(Axv,v)),eqC),1):
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),C2=subs(v=0,Cxv),eqB1),
  B1);
```

$$FinB1 := -\frac{x\,(4\,x^7 - 23\,x^6 + 58\,x^5 - 71\,x^4 + 58\,x^3 - 30\,x^2 + 8\,x - 1)}{(-1 + 2\,x)^3\,(x-1)^5}$$

```
> FinA1:=factor(solve(subs(A2=subs(v=0,Axv),B1=FinB1,eqA1)=0,A1));
  taylor(FinA1,x,14);
```

$$FinA1 := -\frac{x\,(8\,x^7 - 36\,x^6 + 86\,x^5 - 105\,x^4 + 78\,x^3 - 36\,x^2 + 9\,x - 1)}{(-1 + 2\,x)^3\,(x-1)^5}$$

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 86\,x^5 + 325\,x^6 + 1141\,x^7 + 3707\,x^8 + 11247\,x^9 + 32236\,x^{10} + 88208\,x^{11} + 232444\,x^{12} + 594004\,x^{13} + O(x^{14})$$

```
> #00122:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,1,1,1,1,1,1,1,1,: a1=0,a1=01, g=002, f=0013:
  Reg4:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*D2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+G):
  eqB:=-B+x/(1-v)+2*x/v*(B-B2)+x/(1-v)*(C+F):
  eqC:=-C+x/(1-v)+x/v*(C-C2)+x/(1-v)*(C+F):
  eqD:=-DD+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(DD-DD2)+x/(1-v)*(B+G):
  G:=x+2*x*B2+x*F: F:=(x+x*C2)/(1-x):
>
> Cxv:=SolveKE(C,C2,eqC,2):
```

```
Bxv:=SolveKE(B,B2,subs(C=Cxv,C2=subs(v=0,Cxv),eqB),1):
Axv:=SolveKE(A,A2,subs(B=Bxv,B2=subs(v=0,Bxv),C2=subs(v=0,Cxv),eqA
),1):
> Dxv:=SolveKE(DD,DD2,subs(A=Axv,B=Bxv,B2=subs(v=0,Bxv),A2=subs(v=0,
Axv),C2=subs(v=0,Cxv),A3=subs(v=0,diff(Axv,v)),eqD),1):
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),D2=subs(v=0,Dxv),C2=sub
s(v=0,Cxv),eqB1),B1):
FinA1:=factor(solve(subs(B1=FinB1,A2=subs(v=0,Axv),eqA1),A1));
```

$$FinA1 := -\frac{(4\sqrt{1-4x}\,x^3 - 8x^4 - 9\sqrt{1-4x}\,x^2 + 14x^3 + 8x\sqrt{1-4x} - 11x^2 - 2\sqrt{1-4x} + 2x)\,x}{(-1+x)^3(-1+4x)(1+\sqrt{1-4x}-2x)}$$

```
> taylor(FinA1,x,15);
```

$$x + 2x^2 + 6x^3 + 22x^4 + 87x^5 + 349x^6 + 1393x^7 + 5518x^8 + 21733x^9 + 85294x^{10} + 334105x^{11} + 1307576x^{12} + 5116189x^{13} + 20020992x^{14} + O(x^{15})$$

```
> #00120:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,0,0,0,0,0,0,1,2,:
0,1,1,1,1,1,1,1,1,: a1=0, b1=01, e=002, f=0013: Reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*DD2+x*B1:
eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
eqB:=-B+x/(1-v)+2*x/v*(B-B2)+x/(1-v)*(C+F):
eqC:=-C+x/(1-v)+x/v*(C-C2)+x/(1-v)*(C+F):
eqD:=-DD+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(DD-DD2)+x/(1-v)*(DD+B1):
E:=x+2*x*B2+x*F: F:=(x+x*C2)/(1-x):
> Cxv:=SolveKE(C,C2,eqC,2):
Bxv:=SolveKE(B,B2,subs(C=Cxv,C2=subs(v=0,Cxv),eqB),1):
Axv:=SolveKE(A,A2,subs(B=Bxv,B2=subs(v=0,Bxv),C2=subs(v=0,Cxv),eqA
),1):
> DDxv:=SolveKE(DD,DD2,subs(B1=FinB1,A2=subs(v=0,Axv),A3=subs(v=0,di
ff(Axv,v)),A=Axv,subs(B1=solve(eqB1,B1),eqD)),2):
> DDD2:=subs(v=0,DDxv):
FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),DD2=DDD2,eqB1),B1);
```

$$FinB1 := 2x(4\sqrt{1-4x}\,x^4 - 18\sqrt{1-4x}\,x^3 + 16x^4 + 21\sqrt{1-4x}\,x^2 - 48x^3 - 8\sqrt{1-4x}\,x + 35x^2 + \sqrt{1-4x} - 10x + 1) \Big/ ((-1+x)(1+\sqrt{1-4x}-2x)(8\sqrt{1-4x}\,x^3 - 18\sqrt{1-4x}\,x^2 + 32x^3 + 8\sqrt{1-4x}\,x - 32x^2 - \sqrt{1-4x} + 10x - 1))$$

```
> FinA1:=factor(solve(subs(B1=FinB1,A2=subs(v=0,Axv),A3=subs(v=0,dif
f(Axv,v)),eqA1),A1));
```

$$FinA1 := 2(7\sqrt{1-4x}\,x^4 - 4x^5 - 29\sqrt{1-4x}\,x^3 + 37x^4 + 27\sqrt{1-4x}\,x^2 - 69x^3 - 9\sqrt{1-4x}\,x$$

$$+43x^2+\sqrt{1-4x}-11x+1)x\Big/((-1+x)(1+\sqrt{1-4x}-2x)(-1+4x)(2\sqrt{1-4x}\,x^2-4\sqrt{1-4x}\,x+8x^2+\sqrt{1-4x}-6x+1))$$

```
>
> taylor(FinA1,x,14);
```

$$x+2x^2+6x^3+22x^4+87x^5+352x^6+1432x^7+5827x^8+23682x^9+96103x^{10}+389427x^{11}+1575983x^{12}+6370657x^{13}+O(x^{14})$$

```
> #00112: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,1,1,1,1,1,1,1,1,: a1=0,b1=01, e=002, f=0011, Reg3
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*C2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
  eqB:=-B+x/(1-v)*(1+F)+x/v*(B-B2)+x/(1-v)*(B+E):
  eqC:=-C+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(C-C2)+x/(1-v)*(B+E):
  E:=(x+x*B2+x*F)/(1-x): F:=x/(1-2*x):
> Bxv:=SolveKE(B,B2,eqB,2):
  Axv:=SolveKE(A,A2,subs(B=Bxv,B2=subs(v=0,Bxv),eqA),1):
> Cxv:=SolveKE(C,C2,subs(A2=subs(v=0,Axv),A3=subs(v=0,diff(Axv,v)),A
  =Axv,B=Bxv,B2=subs(v=0,Bxv),eqC),1):
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),C2=subs(v=0,Cxv),eqB1)=
  0,B1);
```

$$FinB1:=-(-2x^6+\sqrt{1-4x}\,x^4+8x^5-3\sqrt{1-4x}\,x^3-17x^4+6\sqrt{1-4x}\,x^2+19x^3-4\sqrt{1-4x}\,x-14x^2+\sqrt{1-4x}+6x-1)\Big/(x(-1+x)^3(1+\sqrt{1-4x}-2x)(-1+2x))$$

```
> FinA1:=solve(subs(B1=FinB1,A2=subs(v=0,Axv),eqA1),A1);
```

$$FinA1:=-(2\sqrt{1-4x}\,x^4-4x^5-5\sqrt{1-4x}\,x^3+8x^4+7\sqrt{1-4x}\,x^2-5x^3-4\sqrt{1-4x}\,x-3x^2+\sqrt{1-4x}+4x-1)\Big/((-1+x)^3(1+\sqrt{1-4x}-2x)(-1+2x))$$

```
> taylor(FinA1,x,14);
```

$$x+2x^2+6x^3+22x^4+85x^5+324x^6+1204x^7+4388x^8+15819x^9+56795x^{10}+203984x^{11}+734788x^{12}+2658149x^{13}+O(x^{14})$$

```
> #00111: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,1,1,1,1,1,1,1,1,: a1=1,b1=01,e=002, Reg3:
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*C2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
```

```
eqB:=-B+x/(1-v)+x/v*(B+A-B2-A2)+x/(1-v)*(B+E):
eqC:=-C+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(C-C2)+x/(1-v)*(B+E):
E:=(x+x*B2+x*A2)/(1-x):
```

```
> eqAA1:=subs(B=solve(eqA,B),eqB): KK:=factor(x*v^2*coeff(eqAA1,A)):
taylor(KK,v,10); alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2\,x)\,v+(-x-1)\,v^2+v^3$$

```
> ss:=map(factor,solve({subs(v=u1,subs(A=0,eqAA1)),subs(v=u2,subs(A=
0,eqAA1))},{B2,A2})); AA2:=-u1/x*u2/(u1+u2-1):
BB2:=(u2-x)*(u1-x)/x^2/(u1+u2-1):
```

$$ss := \{A2=-\frac{u1\,u2}{x\,(u1+u2-1)},\ B2=\frac{(u2-x)\,(u1-x)}{x^2\,(u1+u2-1)}\}$$

```
> ss:=solve({subs(A2=AA2,B2=BB2,eqB),subs(A2=AA2,B2=BB2,eqA)},{A,B})
; Axv:=op(op(ss)[1])[2]: Bxv:=op(op(ss)[2])[2]:
```

$$ss := \{A=x\,(u1\,u2-u1\,v-u2\,v+v^2)\Big/(u1\,v^3-u1\,v^2\,x+u1\,v\,x^2+u2\,v^3-u2\,v^2\,x+u2\,v\,x^2-u1\,v^2+2\,u1\,v\,x-u1\,x^2-u2\,v^2+2\,u2\,v\,x-u2\,x^2-v^3+v^2\,x-v\,x^2+v^2-2\,v\,x+x^2),\ B=-(u1\,u2\,v^2-u1\,v^2\,x+u1\,v\,x^2-u2\,v^2\,x+u2\,v\,x^2-v\,x^3-u1\,u2\,v+u1\,u2\,x+u1\,v\,x-u1\,x^2+u2\,v\,x-u2\,x^2-v\,x^2+x^3)\Big/((u1+u2-1)\,x\,(v^3-v^2\,x+v\,x^2-v^2+2\,v\,x-x^2))\}$$

```
> Cxv:=SolveKE(C,C2,subs(A2=subs(v=0,Axv),A3=subs(v=0,diff(Axv,v)),A
=Axv,B=Bxv,B2=subs(v=0,Bxv),eqC),1):
```

```
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),C2=subs(v=0,Cxv),eqB1)=
0,B1): FinA1:=solve(subs(B1=FinB1,A2=subs(v=0,Axv),eqA1),A1);
FinA1:=subs(vv=u3,simplify(subs(u3=vv,rationalize(subs(u1=uu1,u2=u
u2,FinA1)))));
```

$$FinA1 := -\frac{u1\,x^2+u2\,x^2+u1\,u2-u1\,x-u2\,x}{x\,(u1+u2-1)\,(-1+x)}$$

$$FinA1 := -\frac{x^3+2\,u3^2-u3\,x+x^2-2\,u3+3\,x}{x^2\,(-1+x)}$$

```
> taylor(subs(u3=vv,FinA1),x,15);
```

$$x+2\,x^2+6\,x^3+22\,x^4+88\,x^5+368\,x^6+1584\,x^7+6967\,x^8+31175\,x^9+141479\,x^{10}+649669\,x^{11}+3013173\,x^{12}+O(x^{13})$$

```
> #00110: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,1,: 0,1,1,1,1,1,1,1,1,: g=0022, e=002, f=00114
Reg4
```

```
> #00100: 0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
0,0,0,0,0,0,0,1,0,: 0,1,1,1,1,1,1,1,1,: g=0020, e=002, f=00104
Reg4
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
kk:=[solve(coeff(Eq,A)=0,v)]:
```

```
AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*DD2+x*B1:
eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
eqB:=-B+x/(1-v)+x/v*(B-B2)+x*C+x/(1-v)*(B+E):
eqC:=-C+x/(1-v)+x/v*(C-C2)+x/(1-v)*(C+G+F):
eqD:=-DD+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(DD-DD2)+x/(1-v)*(DD+B1):
eqG:=-GG+x+x*C2+x*FF+x*GG: eqF:=-FF+x+x*GG+x*FF:
E:=(x+x*B2+x*G)/(1-x):
```

```
> solve({eqG,eqF}, {GG,FF}); F:= -x*(C2*x+1)/(2*x-1):G:=
x*(C2*x-C2-1)/(2*x-1):
```

$$\{FF = -\frac{x\,(C2\,x+1)}{2\,x-1},\ GG = \frac{x\,(C2\,x-C2-1)}{2\,x-1}\}$$

```
>
```

```
> Cxv:=SolveKE(C,C2,eqC,2):
Bxv:=SolveKE(B,B2,subs(C=Cxv,C2=subs(v=0,Cxv),eqB),2):
Axv:=SolveKE(A,A2,subs(B=Bxv,B2=subs(v=0,Bxv),C2=subs(v=0,Cxv),eqA
),1):
Dxv:=SolveKE(DD,DD2,subs(A=Axv,A2=subs(v=0,Axv),A3=subs(v=0,diff(A
xv,v)),subs(B1=solve(eqB1=0,B1),eqD)),2):
FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),DD2=subs(v=0,Dxv),eqB1)
=0,B1):
```

```
> FinA1:=solve(subs(A2=subs(v=0,Axv),B1=FinB1,eqA1),A1);
```

$$
\begin{aligned}
FinA1 := {} & 2\,x\,(-4+132\,x-1972\,x^2+136\,(1-4\,x)^{(3/2)}\,x^{13}-76\,\sqrt{1-4\,x}\,x^{15} \\
& -3170\,(1-4\,x)^{(3/2)}\,x^{12}+816\,\sqrt{1-4\,x}\,x^{14}+25312\,(1-4\,x)^{(3/2)}\,x^{11}+4224\,\sqrt{1-4\,x}\,x^{13} \\
& -98132\,x^{10}\,(1-4\,x)^{(3/2)}-61586\,\sqrt{1-4\,x}\,x^{12}+217700\,x^9\,(1-4\,x)^{(3/2)} \\
& +257168\,\sqrt{1-4\,x}\,x^{11}-590288\,\sqrt{1-4\,x}\,x^{10}+16\,x^{16}-580\,x^{15}-5200\,x^{14}+123944\,x^{13} \\
& -742236\,x^{12}+2271416\,x^{11}-4256980\,x^{10}-303315\,(1-4\,x)^{(3/2)}\,x^8+56\,x\,(1-4\,x)^{(3/2)} \\
& +873152\,\sqrt{1-4\,x}\,x^9-698\,(1-4\,x)^{(3/2)}\,x^2-2\,(1-4\,x)^{(3/2)}+5317992\,x^9-4641412\,x^8 \\
& +2906988\,x^7-178297\,(1-4\,x)^{(3/2)}\,x^6-890025\,\sqrt{1-4\,x}\,x^8+78888\,(1-4\,x)^{(3/2)}\,x^5 \\
& +645424\,\sqrt{1-4\,x}\,x^7-24290\,(1-4\,x)^{(3/2)}\,x^4+5104\,(1-4\,x)^{(3/2)}\,x^3 \\
& +280720\,(1-4\,x)^{(3/2)}\,x^7-337655\,\sqrt{1-4\,x}\,x^6+127640\,\sqrt{1-4\,x}\,x^5-1323856\,x^6 \\
& -34506\,\sqrt{1-4\,x}\,x^4+6500\,\sqrt{1-4\,x}\,x^3-810\,\sqrt{1-4\,x}\,x^2+439120\,x^5-104996\,x^4+17628\,x^3 \\
& +60\,x\,\sqrt{1-4\,x}-2\,\sqrt{1-4\,x})\,\big/\,((-4\,x^2+5\,x-\sqrt{1-4\,x}-1+3\,x\,\sqrt{1-4\,x}) \\
& (x\,\sqrt{1-4\,x}-1-\sqrt{1-4\,x}+3\,x) \\
& (2\,\sqrt{1-4\,x}\,x^3-9\,\sqrt{1-4\,x}\,x^2+12\,x^3+6\,x\,\sqrt{1-4\,x}-19\,x^2-\sqrt{1-4\,x}+8\,x-1)\,(
\end{aligned}
$$

$$28\sqrt{1-4x}\,x^7-8x^8-231\sqrt{1-4x}\,x^6+198x^7+560\sqrt{1-4x}\,x^5-833x^6-606\sqrt{1-4x}\,x^4+1372x^5+340\sqrt{1-4x}\,x^3-1134x^4-103\sqrt{1-4x}\,x^2+518x^3+16x\sqrt{1-4x}-133x^2-\sqrt{1-4x}+18x-1)(x-1))$$

```
> taylor(FinA1,x,14);
```

$$x+2x^2+6x^3+22x^4+88x^5+365x^6+1535x^7+6481x^8+27354x^9+115201x^{10}+483788x^{11}+2025592x^{12}+8456376x^{13}+O(x^{14})$$

```
> #00102:  0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,:
  0,0,0,0,0,0,0,1,2,: 0,1,1,1,1,1,1,1,1,: a1=0,b1=01,
  e=002,f=0013,g=00120,h=0010, reg4
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*DD2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
  eqB:=-B+x/(1-v)*(1+H)+x/v*(B-B2)+x/(1-v)*(C+F):
  eqC:=-C+x/(1-v)*(1+G)+x/v*(C-C2)+x/(1-v)*(C+F):
  eqD:=-DD+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(DD-DD2)+x/(1-v)*(DD+B1):
  G:=x/(1-x): H:=x/(1-2*x): F:=(x+x*G+x*C2)/(1-x):
  E:=(x+x*B2+x*H+x*F):
> Cxv:=SolveKE(C,C2,eqC,2):
  Bxv:=SolveKE(B,B2,subs(C=Cxv,C2=subs(v=0,Cxv),eqB),1):
  Axv:=SolveKE(A,A2,subs(B=Bxv,C=Cxv,C2=subs(v=0,Cxv),B2=subs(v=0,Bx
  v),eqA),1):
> Dxv:=SolveKE(DD,DD2,subs(A=Axv,A2=subs(v=0,Axv),A3=subs(v=0,diff(A
  xv,v)),subs(B1=solve(eqB1,B1),eqD)),2):
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),DD2=subs(v=0,Dxv),eqB1)
  ,B1):
> FinA1:=solve(subs(B1=FinB1,A2=subs(v=0,Axv),eqA1),A1);
```

$$FinA1 := 2\,(13\sqrt{1-4x}\,x^7-4x^8-66\sqrt{1-4x}\,x^6+67x^7+138\sqrt{1-4x}\,x^5-214x^6-168\sqrt{1-4x}\,x^4+350x^5+121\sqrt{1-4x}\,x^3-344x^4-50\sqrt{1-4x}\,x^2+203x^3+11\sqrt{1-4x}\,x-70x^2-\sqrt{1-4x}+13x-1)\,x\Big/((-1+x)^2(10\sqrt{1-4x}\,x^5-8x^6-35\sqrt{1-4x}\,x^4+46x^5+47\sqrt{1-4x}\,x^3-95x^4-30\sqrt{1-4x}\,x^2+93x^3+9\sqrt{1-4x}\,x-46x^2-\sqrt{1-4x}+11x-1)(1+\sqrt{1-4x}-2x))$$

```
> taylor(FinA1,x,15);
```

$$x+2x^2+6x^3+22x^4+85x^5+326x^6+1232x^7+4622x^8+17340x^9+65316x^{10}+247368x^{11}+941886x^{12}+3603414x^{13}+13841132x^{14}+O(x^{15})$$

```
> #00101:0,0,0,0,0,0,0,0,0,: 0,0,0,0,0,0,0,0,1,: 0,1,1,1,1,1,1,1,1,:
  a1=0,b1=01, e=002: reg3
```

```
> restart: SolveKE:=proc(A,A0,Eq,d) local AA0,kk:
  kk:=[solve(coeff(Eq,A)=0,v)]:
  AA0:=limit(solve(subs(A=0,Eq)=0,A0),v=kk[d]):
  return(simplify(solve(subs(A0=AA0,Eq)=0,A))); end:
  eqA1:=-A1+x+x*A2+x*B1: eqB1:=-B1+x+x*A3+x*C2+x*B1:
  eqA:=-A+x/(1-v)+x/v*(A-A2)+x/(1-v)*(B+E):
  eqB:=-B+x/(1-v)+x/v*(B+A-B2-A2)+x/(1-v)*(B+E):
  eqC:=-C+x/(1-v)+x/v^2*(A-A2-A3*v)+x/v*(C-C2)+x/(1-v)*(C+B1):
  E:=(x+x*B2+x*A2)/(1-x):
> eqAA1:=subs(B=solve(eqA,B),eqB): KK:=factor(x*v^2*coeff(eqAA1,A)):
  taylor(KK,v,10); alias(vv=RootOf(KK=0,v)): VVV:=allvalues(vv):
  uu:=solve(u1+x^2/u1/u3=1+x-u3,u1): uu1:=uu[1]: uu2:=uu[2]:
```

$$-x^2+(x^2+2x)v+(-x-1)v^2+v^3$$

```
> solve({subs(A=0,v=u1,eqAA1),subs(A=0,v=u2,eqAA1)},{A2,B2}); AA2:=
  -u1/x*u2/(u1+u2-1): BB2:=(u1*u2-u1*x-u2*x+x^2)/x^2/(u1+u2-1):
  ss:=map(factor,solve({subs(A2=AA2,B2=BB2,eqB),subs(A2=AA2,B2=BB2,e
  qA)},{A,B})); Axv:=op(op(ss)[1])[2]: Bxv:=op(op(ss)[2])[2]:
```

$$\{A2=-\frac{u1\,u2}{x\,(u1+u2-1)},\ B2=\frac{u1\,u2-u1\,x-u2\,x+x^2}{x^2\,(u1+u2-1)}\}$$

$$ss:=\{A=\frac{x\,(u2-v)\,(u1-v)}{(v^3-v^2\,x+v\,x^2-v^2+2\,v\,x-x^2)\,(u1+u2-1)},\ B=-(u1\,u2\,v^2-u1\,v^2\,x+u1\,v\,x^2-u2\,v^2\,x+u2\,v\,x^2-v\,x^3-u1\,u2\,v+u1\,u2\,x+u1\,v\,x-u1\,x^2+u2\,v\,x-u2\,x^2-v\,x^2+x^3)\Big/((u1+u2-1)\,x\,(v^3-v^2\,x+v\,x^2-v^2+2\,v\,x-x^2))\}$$

```
> Cxv:=SolveKE(C,C2,subs(A=Axv,A2=subs(v=0,Axv),A3=subs(v=0,diff(Axv
  ,v)),subs(B1=solve(eqB1,B1),eqC)),2):
> FinB1:=solve(subs(A3=subs(v=0,diff(Axv,v)),C2=subs(v=0,Cxv),eqB1),
  B1):
> FinA1:=solve(subs(B1=FinB1,A2=subs(v=0,Axv),eqA1),A1):
  FinA1:=subs(vv=u3,simplify(subs(u3=vv,rationalize(subs(u1=uu1,u2=u
  u2,FinA1)))));
```

$$FinA1:=\frac{\sqrt{1-4\,x}\,u3^2-x\sqrt{1-4\,x}\,u3+u3^2-\sqrt{1-4\,x}\,u3-u3\,x+2\,x\sqrt{1-4\,x}-u3+2\,x}{2\,x^2}$$

```
> taylor(subs(u3=vv,FinA1),x,15);
```

$$x+2\,x^2+6\,x^3+22\,x^4+88\,x^5+370\,x^6+1607\,x^7+7141\,x^8+32279\,x^9+147868\,x^{10}+684715\,x^{11}+3199126\,x^{12}+O(x^{13})$$

```
> #00012: 0,0,0,0,0,0,0,0,0,: 0,0,1,1,1,1,1,1,1,:
  0,0,1,1,2,2,3,3,4,: 0,1,1,1,1,1,1,1,1,: 0,1,1,2,2,3,3,4,4,:
  a_m=0^m, b_m=01^m, c_m=001^m, d_m=0^21^2..m^2,
  e_m=0^21^2..(m-1)^2m, f_m=01^22^2..m^2, g_m=01^22^2..(m-1)^2m:
```

```
req5
> restart:
> ss:=solve({A0=x+x*A00+x*G1,A00=x+x*A3+x*E1+x*A002,A002=x+x*A3+x*A0
022+x*A002,A0022=x+x*A4+x*A00222+x*E1+x*A002,A00222=x+x*A5+x*C3+3*
x*A0001,A0001=x+x*A0001+x*A0001},{A0,A00,A002,A0022,A00222,A0001})
: op(op(ss)[1])[1],op(op(ss)[4])[1]; FA0:=op(op(ss)[1])[2];
FA002:=op(op(ss)[4])[2];
```

$$A0, A002$$

$$FA0 := x\,(-2\,A5\,x^5 - 2\,C3\,x^5 + 2\,A3\,x^4 - 2\,A4\,x^4 + A5\,x^4 + C3\,x^4 + x^5 - A3\,x^3 + A4\,x^3 + 2\,E1\,x^3 + 2\,G1\,x^3 + x^4 - 2\,A3\,x^2 - 3\,E1\,x^2 + G1\,x^2 + 2\,x^3 + A3\,x + E1\,x - 3\,G1\,x - x^2 + G1 - 2\,x + 1) \Big/ (2\,x^3 + x^2 - 3\,x + 1)$$

$$FA002 := -x\,(2\,A5\,x^3 + 2\,C3\,x^3 + 2\,A4\,x^2 - A5\,x^2 - C3\,x^2 + 2\,E1\,x^2 - x^3 + 2\,A3\,x - A4\,x - E1\,x + x^2 - A3 + x - 1) \Big/ (2\,x^3 + x^2 - 3\,x + 1)$$

```
> #a_m=a_{m+1}(0001)^m, m\ge3
> eqA:=A=x/(1-v)+x/v*(A-A3)+sum(m*x*v^(m-3)*x/(1-2*x),m=3..infinity)
: AA3:=solve(subs(v=x,eqA),A3): Axv:=solve(subs(A3=AA3,eqA),A):
AA4:=subs(v=0,diff(Axv,v)): AA5:=subs(v=0,diff(Axv,v,v)/2):
> #b_m=c_mb_{m+1}(0001)^m, m\ge3, c_m=a_{m+3}c_{m+1}(0001)^{m+1},
m\ge3
> eqB:=B=x/(1-v)+x*C+x/v*(B-B3)+sum(m*x*v^(m-3)*x/(1-2*x),m=3..infin
ity):eqC:=C=x/(1-v)+x/v^3*(A-A3-v*A4-v^2*A5)+x/v*(C-C3)+sum((m+1)*
x*v^(m-3)*x/(1-2*x),m=3..infinity):
> CC3:=simplify(solve(subs(v=x,subs(A3=AA3,A=Axv,A4=subs(v=0,diff(Ax
v,v)),A5=subs(v=0,diff(Axv,v,v)/2),eqC)),C3)):
Cxv:=simplify(solve(subs(C3=CC3,subs(A3=AA3,A=Axv,A4=subs(v=0,diff
(Axv,v)),A5=subs(v=0,diff(Axv,v,v)/2),eqC)),C)):
> BB3:=simplify(solve(subs(v=x,subs(C=Cxv,eqB)),B3)):
Bxv:=simplify(solve(subs(B3=BB3,subs(C=Cxv,eqB)),B)):
> #d_m->a_{m+4}c_{m+2}e_{m+1}...e_1,002:
e_m->a_{m+3}d_me_m...e_1,002, m\geq1
> eqD:=-DD+x/(1-v)+x/v^2*(A-A3-v*A4)+x*C+x/v*(E-E1)+x/(1-v)*(E+A002)
; eqE:=-E+x/(1-v)+x/v*(A-A3)+x*DD+x/(1-v)*(E+A002);
```

$$eqD := -DD + \frac{x}{1-v} + \frac{x\,(-A4\,v + A - A3)}{v^2} + x\,C + \frac{x\,(E - E1)}{v} + \frac{x\,(E + A002)}{1-v}$$

$$eqE := -E + \frac{x}{1-v} + \frac{x\,(A - A3)}{v} + x\,DD + \frac{x\,(E + A002)}{1-v}$$

```
> eqE1:=subs(DD=solve(eqE,DD),eqD):
v0:=solve(factor(coeff(eqE1,E))=0,v)[2];
```

$$v0 := -\frac{x}{2} + \frac{1}{2} - \frac{\sqrt{-3x^2 - 2x + 1}}{2}$$

```
> EE1:=factor(simplify(rationalize(solve(subs(v=v0,subs(C=Cxv,C3=CC3
  ,A3=AA3,subs(A4=AA4,subs(A5=AA5,subs(E=0,A=Axv,A002=FA002,eqE1))))
  ),E1))+1)-1);
```

$$EE1 := (-2x^9 + 4x^8 - 2x^6 + 3\sqrt{-(x+1)(3x-1)}\,x^4 + 3x^5 + \sqrt{-(x+1)(3x-1)}\,x^3 - 8x^4 - 4x^2\sqrt{-(x+1)(3x-1)} - x^3 + 3x\sqrt{-(x+1)(3x-1)} + 5x^2 - \sqrt{-(x+1)(3x-1)} - 4x + 1) \Big/ (2x^4(x+1)(2x-1)(x-1)^3)$$

```
> Exv:=solve(subs(E1=EE1,subs(C=Cxv,C3=CC3,A3=AA3,subs(A4=AA4,subs(A
  5=AA5,subs(A=Axv,A002=FA002,eqE1))))),E);
```

$$Exv := (-1 + 2v - 3x^5 + 4x - 5x^2 + x^3 + 8x^4 + \sqrt{-(x+1)(3x-1)}\,v^2 - 2\sqrt{-(x+1)(3x-1)}\,v + 2v^2x^7 + 2vx^8 - 4v^2x^6 + 3\sqrt{-(x+1)(3x-1)}\,vx^4 + 3\sqrt{-(x+1)(3x-1)}\,v^2x^2 + \sqrt{-(x+1)(3x-1)}\,vx^3 - 2\sqrt{-(x+1)(3x-1)}\,v^2x - 7\sqrt{-(x+1)(3x-1)}\,vx^2 + 5\sqrt{-(x+1)(3x-1)}\,vx - 3\sqrt{-(x+1)(3x-1)}\,x^4 + 4x^2\sqrt{-(x+1)(3x-1)} - 3x\sqrt{-(x+1)(3x-1)} + \sqrt{-(x+1)(3x-1)} - \sqrt{-(x+1)(3x-1)}\,x^3 + 2x^9 - 4x^8 + 2x^6 - 6vx^7 + 4vx^6 + 8v^2x^4 + 5vx^5 - v^2x^3 - 16vx^4 - 3v^2x^2 + 8vx^2 - 7vx - v^2 + 3xv^2) \Big/ (2x^2(2x-1)(x+1)(x-1)^3(v-1)(v^2 + vx + x^2 - v))$$

```
> Dxv:=factor(solve(subs(A5=AA5,A4=AA4,A3=AA3,C3=CC3,E=Exv,E1=EE1,A=
  Axv,C=Cxv,subs(A002=FA002,eqD)),DD));
```

$$Dxv := -(-1 + 3v - 6x^5 + 5x - 9x^2 + 7x^3 + 4x^4 - \sqrt{-(x+1)(3x-1)}\,v^3 + 3\sqrt{-(x+1)(3x-1)}\,v^2 - 3\sqrt{-(x+1)(3x-1)}\,v - 2v^2x^9 + 8v^2x^8 - 4v^2x^7 - 2vx^8 - 6v^2x^6 - 2v^3x^8 + 4v^3x^7 + 2v^3x^6 - 3v^3x^4 - 4v^3x^3 - 2vx^{10} + 6vx^9 + 6v^3x^2 - 4v^3x + 3\sqrt{-(x+1)(3x-1)}\,v^3x^3 - 5\sqrt{-(x+1)(3x-1)}\,v^3x^2 - 12\sqrt{-(x+1)(3x-1)}\,v^2x^3 + 3\sqrt{-(x+1)(3x-1)}\,v^3x + 17\sqrt{-(x+1)(3x-1)}\,v^2x^2 + 15\sqrt{-(x+1)(3x-1)}\,vx^3 - 10\sqrt{-(x+1)(3x-1)}\,v^2x - 19\sqrt{-(x+1)(3x-1)}\,vx^2 + 11\sqrt{-(x+1)(3x-1)}\,vx + 7x^2\sqrt{-(x+1)(3x-1)} - 4x\sqrt{-(x+1)(3x-1)} + \sqrt{-(x+1)(3x-1)} - 6\sqrt{-(x+1)(3x-1)}\,x^3 - 2x^9 - 4x^8 + 2x^{10} - 2vx^7 - 6v^2x^5 + 4vx^6 + 10v^2x^4 + 12vx^5 + 15v^2x^3 - 11vx^4 - 21v^2x^2 - 18vx^3 + 24vx^2 - 14vx - 3v^2 + 13xv^2 + v^3) \Big/ (2x^3(2x-1)(v^2 + vx + x^2 - v)(v-1)^2(x-1)^3(x+1))$$

```
> #f_m->d_mb_{m+2}g_{m+1}...g_1, g_m->e_mf_mg_m...g_1 m>=1
> eqF:=-F+x/(1-v)+x*DD+x*B+x/v*(G-G1)+x/(1-v)*G;
  eqG:=-G+x/(1-v)+x*(E+F)+x/(1-v)*G;
```

$$eqF := -F + \frac{x}{1-v} + x\,DD + x\,B + \frac{x(G - G1)}{v} + \frac{x\,G}{1-v}$$

$$eqG := -G + \frac{x}{1-v} + x\,(E+F) + \frac{x\,G}{1-v}$$

```
> eqG1:=subs(F=solve(eqF,F),eqG): v0:=solve(coeff(eqG1,G),v)[2];
```

$$v0 := -\frac{x}{2} + \frac{1}{2} - \frac{\sqrt{-3\,x^2 - 2\,x + 1}}{2}$$

```
> GG1:=simplify(limit(solve(subs(G=0,subs(subs(E=Exv,B=Bxv,DD=Dxv,eq
  G1))),G1),v=v0));
  Gxv:=solve(subs(G1=GG1,subs(subs(E=Exv,B=Bxv,DD=Dxv,eqG1))),G):
```

$$\begin{aligned} GG1 := -\,(&-6\,x^{10} + 11\,x^9 + 3\sqrt{-3\,x^2-2\,x+1}\;x^8 + 30\,x^8 - 7\sqrt{-3\,x^2-2\,x+1}\;x^7 - 23\,x^7 \\ &+ 8\sqrt{-3\,x^2-2\,x+1}\;x^6 + 19\,x^6 - 11\sqrt{-3\,x^2-2\,x+1}\;x^5 + 64\,x^5 + 9\sqrt{-3\,x^2-2\,x+1}\;x^4 - 44\,x^4 \\ &+ 3\sqrt{-3\,x^2-2\,x+1}\;x^3 - 17\,x^3 - 10\,x^2\sqrt{-3\,x^2-2\,x+1} + 23\,x^2 + 7\,x\sqrt{-3\,x^2-2\,x+1} - 11\,x \\ &- 2\sqrt{-3\,x^2-2\,x+1} + 2\,) \Big/ (-12\,x^{10} + 6\sqrt{-3\,x^2-2\,x+1}\;x^8 + 28\,x^9 - 17\sqrt{-3\,x^2-2\,x+1}\;x^7 \\ &+ 13\,x^8 + 5\sqrt{-3\,x^2-2\,x+1}\;x^6 - 76\,x^7 + 27\sqrt{-3\,x^2-2\,x+1}\;x^5 + 32\,x^6 - 27\sqrt{-3\,x^2-2\,x+1}\;x^4 \\ &+ 60\,x^5 - 3\sqrt{-3\,x^2-2\,x+1}\;x^3 - 54\,x^4 + 15\,x^2\sqrt{-3\,x^2-2\,x+1} - 4\,x^3 - 7\,x\sqrt{-3\,x^2-2\,x+1} \\ &+ 20\,x^2 + \sqrt{-3\,x^2-2\,x+1} - 8\,x + 1\,) \end{aligned}$$

```
> FinA:=simplify(simplify(rationalize(x+x*(x+x*AA3+x*EE1+x*subs(A3=A
  A3,C3=CC3,A4=AA4,A5=AA5,E1=EE1,FA002))+x*GG1)+1)-1);
```

$$\begin{aligned} FinA := (&-12\,x^{10} + 28\,x^9 + 18\sqrt{-3\,x^2-2\,x+1}\;x^7 + 16\,x^8 - 24\sqrt{-3\,x^2-2\,x+1}\;x^6 - 38\,x^7 \\ &+ 24\sqrt{-3\,x^2-2\,x+1}\;x^5 + 64\,x^6 + 8\sqrt{-3\,x^2-2\,x+1}\;x^4 - 36\,x^5 - 26\sqrt{-3\,x^2-2\,x+1}\;x^3 - 18\,x^4 \\ &+ 17\,x^2\sqrt{-3\,x^2-2\,x+1} + 35\,x^3 - 6\,x\sqrt{-3\,x^2-2\,x+1} - 21\,x^2 + \sqrt{-3\,x^2-2\,x+1} + 7\,x - 1\,) \Big/ \\ &(2\,x^2\,(6\,x^8 - 17\,x^7 + 5\,x^6 + 27\,x^5 - 27\,x^4 - 3\,x^3 + 15\,x^2 - 7\,x + 1)) \end{aligned}$$

```
> taylor(FinA,x,13);
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 84\,x^5 + 312\,x^6 + 1089\,x^7 + 3608\,x^8 + 11475\,x^9 + 35473\,x^{10} + O(x^{11})$$

```
> #00011: 0,0,0,0,0,0,0,0,0,: 0,0,1,1,1,1,1,1,1,:
  0,0,1,1,2,2,3,3,4,: 0,1,1,1,1,1,1,1,1,: 0,1,1,2,2,3,3,4,4,:
  a_m=0^m, b_m=01^m, c_m=001^m, d_m=0^21^2..m^2,
  e_m=0^21^2..(m-1)^2m, f_m=01^22^2..m^2, g_m=01^22^2..(m-1)^2m:
  Reg5
> #a_m->a_{m+1}...a_3 0002,0003 m>=3:
  b_m->c_mb_{m+1}a_m...a_3,0002,0003, m>=3:
  c_m->a_{m+3}c_{m+1}a_{m+1}...a_3,0002,0003 m>=1:
> #d_m->a_{m+4}c_{m+2}e_{m+1}...e_1,002:
  e_m->a_{m+3}d_me_m...e_1,002, f_m->d_mb_{m+2}g_{m+1}...g_1,
  g_m->e_mf_mg_m...g_1, m\geq1
> restart:
> eqA:=-A+x/(1-v)+x/v*(A-A3)+x/(1-v)*(A+A0002+A0003);
  va:=solve(coeff(eqA,A),v)[2];
```

$$eqA := -A + \frac{x}{1-v} + \frac{x\,(A - A3)}{v} + \frac{x\,(A + A0002 + A0003)}{1-v}$$

$$va := \frac{1}{2} - \frac{\sqrt{1-4\,x}}{2}$$

```
> solve({simplify(limit(eqA,v=va)),A0002=x+x*A3+x*A0002+x*A0003,A000
3=x+x*A0002+x*A0003},{A3,A0002,A0003});
AA3:=1/2*(2*x*(1-4*x)^(1/2)+2*x^2-(1-4*x)^(1/2)-4*x+1)/x^3:AA0002:
=1/2*(x*(1-4*x)^(1/2)-(1-4*x)^(1/2)-3*x+1)/x^2:AA0003:=-1/2/x*((1-
4*x)^(1/2)+2*x-1):
Axv:=solve(subs(A3=AA3,A0002=AA0002,A0003=AA0003,eqA),A);
AA4:=subs(v=0,diff(Axv,v)):AA5:=subs(v=0,diff(Axv,v,v)/2):
```

$$\left\{A3 = \frac{2\,x\sqrt{1-4\,x} + 2\,x^2 - \sqrt{1-4\,x} - 4\,x + 1}{2\,x^3},\ A0002 = \frac{x\sqrt{1-4\,x} - \sqrt{1-4\,x} - 3\,x + 1}{2\,x^2},\ A0003 = -\frac{\sqrt{1-4\,x} + 2\,x - 1}{2\,x}\right\}$$

$$Axv := -\frac{\sqrt{1-4\,x}\,v\,x - \sqrt{1-4\,x}\,v - 2\,x\sqrt{1-4\,x} - 3\,x\,v - 2\,x^2 + \sqrt{1-4\,x} + v + 4\,x - 1}{2\,x^2\,(v^2 - v + x)}$$

```
> eqC:=-C+x/(1-v)+x/v^3*(Axv-AA3-AA4*v-AA5*v^2)+x/v*(C-C3)+x/v*(Axv-
AA3)+x/(1-v)*(Axv+AA0002+AA0003):
CC3:=factor(solve(limit(eqC,v=x),C3)):
Cxv:=solve(subs(C3=CC3,eqC),C);
```

$$Cxv := -\left(3\sqrt{1-4\,x}\,v\,x^3 + \sqrt{1-4\,x}\,x^4 + 2\,v\,x^4 - 7\sqrt{1-4\,x}\,v\,x^2 - 7\sqrt{1-4\,x}\,x^3 - 11\,x^3\,v - 7\,x^4 + 5\sqrt{1-4\,x}\,v\,x + 11\sqrt{1-4\,x}\,x^2 + 15\,x^2\,v + 21\,x^3 - \sqrt{1-4\,x}\,v - 6\,x\sqrt{1-4\,x} - 7\,x\,v - 21\,x^2 + \sqrt{1-4\,x} + v + 8\,x - 1\right) \Big/ \left(2\,x^5\,(v^2 - v + x)\right)$$

```
> eqB:=-B+x/(1-v)+x*Cxv+x/v*(B-B3)+x/(1-v)*(Axv+AA0002+AA0003):
BB3:=factor(solve(limit(eqB,v=x),B3)):
Bxv:=solve(subs(B3=BB3,eqB),B);
```

$$Bxv := -\left(6\sqrt{1-4\,x}\,v\,x^3 + 2\sqrt{1-4\,x}\,x^4 + 4\,v\,x^4 - 11\sqrt{1-4\,x}\,v\,x^2 - 13\sqrt{1-4\,x}\,x^3 - 20\,x^3\,v - 14\,x^4 + 6\sqrt{1-4\,x}\,v\,x + 16\sqrt{1-4\,x}\,x^2 + 21\,x^2\,v + 35\,x^3 - \sqrt{1-4\,x}\,v - 7\,x\sqrt{1-4\,x} - 8\,x\,v - 28\,x^2 + \sqrt{1-4\,x} + v + 9\,x - 1\right) \Big/ \left(2\,x^5\,(v^2 - v + x)\right)$$

```
> solve({A002=x+x*AA3+x*A0022+x*A002,A0022=x+x*AA4+x*A00222+x*E1+x*A
002,A00222=x+x*AA5+x*CC3+x*AA3+x*AA0002+x*AA0003},{A002,A0022,A002
22});
AA002:=-1/2*(2*x^5*E1+2*x^5+2*(1-4*x)^(1/2)*x^3+4*x^4-2*(1-4*x)^(1
/2)*x^2-4*x^3+3*x*(1-4*x)^(1/2)+6*x^2-(1-4*x)^(1/2)-5*x+1)/x^3/(x^
2+x-1):
```

$$\{A002 = -\left(2\,x^5\,E1 + 2\,x^5 + 2\sqrt{1-4\,x}\,x^3 + 4\,x^4 - 2\sqrt{1-4\,x}\,x^2 - 4\,x^3 + 3\,x\sqrt{1-4\,x} + 6\,x^2\right.$$

$$-\sqrt{1-4x}-5x+1)\Big/(2x^3(x^2+x-1)),\ A0022=(2x^6\,E1-2x^5\,E1-2x^5-5\sqrt{1-4x}\,x^3$$
$$-4x^4+8\sqrt{1-4x}\,x^2+15x^3-5x\sqrt{1-4x}-16x^2+\sqrt{1-4x}+7x-1)\Big/(2x^4(x^2+x-1)),$$
$$A00222=$$
$$\frac{3\sqrt{1-4x}\,x^3+2x^4-7\sqrt{1-4x}\,x^2-11x^3+5x\sqrt{1-4x}+15x^2-\sqrt{1-4x}-7x+1}{2x^5}\}$$

```
> eqD:=-DD+x/(1-v)+x/v^2*(Axv-AA3-v*AA4)+x*Cxv+x/v*(E-E1)+x/(1-v)*(E
  +AA002):eqE:=-E+x/(1-v)+x/v*(Axv-AA3)+x*DD+x/(1-v)*(E+AA002):
  eqM1:=eqD*x+eqE: ve:=solve(coeff(eqM1,E)=0,v)[2];
  EE1:=simplify(limit(solve(subs(E=0,eqM1),E1),v=ve)):
  Exv:=solve(subs(E1=EE1,eqM1),E):
  Dxv:=solve(subs(E=Exv,E1=EE1,eqD),DD):
```

$$ve := -\frac{x}{2}+\frac{1}{2}-\frac{\sqrt{-3x^2-2x+1}}{2}$$

```
> eqF:=-F+x/(1-v)+x*Dxv+x*Bxv+x/v*(G-G1)+x/(1-v)*G:
  eqG:=-G+x/(1-v)+x*Exv+x*F+x/(1-v)*G:
  eqM2:=subs(F=solve(eqF=0,F),eqG):
  vg:=solve(simplify(coeff(eqM2,G)),v)[2];
```

$$vg := -\frac{x}{2}+\frac{1}{2}-\frac{\sqrt{-3x^2-2x+1}}{2}$$

```
> GG1:=solve(limit(eqM2,v=vg)=0,G1):
> taylor(x+x*(x+x*AA3+x*EE1+x*subs(E1=EE1,AA002))+x*GG1,x,15);
```

$$x+2x^2+6x^3+22x^4+87x^5+352x^6+1418x^7+5651x^8+22259x^9+86790x^{10}+335624x^{11}+O(x^{12})$$

```
> FinA:=factor(simplify(simplify(rationalize(x+x*(x+x*AA3+x*EE1+x*su
  bs(E1=EE1,AA002))+x*GG1)+1)-1)); taylor(FinA,x,15);
```

$$FinA := (-9\sqrt{1-4x}\,x^4-6x^5+3\sqrt{1-4x}\sqrt{-(1+x)(3x-1)}\,x^2+9x^3\sqrt{-(1+x)(3x-1)}$$
$$+20x^4+\sqrt{1-4x}\,x\sqrt{-(1+x)(3x-1)}+3x^2\sqrt{-(1+x)(3x-1)}+\sqrt{1-4x}\,x^2-3x^3$$
$$-\sqrt{1-4x}\sqrt{-(1+x)(3x-1)}-4x\sqrt{-(1+x)(3x-1)}-6x\sqrt{1-4x}-16x^2$$
$$+\sqrt{-(1+x)(3x-1)}+2\sqrt{1-4x}+11x-2)\Big/(2x^3(1+x)(3x-1))$$
$$x+2x^2+6x^3+22x^4+87x^5+352x^6+1418x^7+5651x^8+22259x^9+86790x^{10}+335624x^{11}+O(x^{12})$$

```
> #00010: 0,0,0,0,0,0,0,0,0,: 0,0,1,1,1,1,1,1,1,:
  0,1,1,1,1,1,1,1,1,:  Reg3
> restart:
  eqA:=-A+x/(1-v)+x/v*(A-A3)+x*A/(1-v)+x/(1-v)*(A0002+A0003);
  va:=solve(coeff(eqA,A)=0,v)[2];
```

$$eqA := -A + \frac{x}{1-v} + \frac{x\,(A - A3)}{v} + \frac{x\,A}{1-v} + \frac{x\,(A0002 + A0003)}{1-v}$$

$$va := \frac{1}{2} - \frac{\sqrt{1-4\,x}}{2}$$

```
> solve({subs(v=va,eqA),A0002=x+x*A3+x*A0002+x*A0003,A0003=x+x*A0002
  +x*A0003},{A3,A0002,A0003});;
  AA3:=1/2*(2*x*(1-4*x)^(1/2)+2*x^2-(1-4*x)^(1/2)-4*x+1)/x^3:
  AA0002:=1/2*(x*(1-4*x)^(1/2)-(1-4*x)^(1/2)-3*x+1)/x^2:
  AA0003:=-1/2/x*((1-4*x)^(1/2)+2*x-1):
```

$$\left\{A3 = \frac{2\,x\sqrt{1-4\,x} + 2\,x^2 - \sqrt{1-4\,x} - 4\,x + 1}{2\,x^3},\ A0002 = \frac{x\sqrt{1-4\,x} - \sqrt{1-4\,x} - 3\,x + 1}{2\,x^2},\ A0003 = -\frac{\sqrt{1-4\,x} + 2\,x - 1}{2\,x}\right\}$$

```
> Axv:=solve(subs(A3=AA3,A0002=AA0002,A0003=AA0003,eqA),A);
  AA4:=subs(v=0,diff(Axv,v)): AA5:=subs(v=0,diff(Axv,v,v)/2):
```

$$Axv := -\frac{\sqrt{1-4\,x}\,v\,x - \sqrt{1-4\,x}\,v - 2\,x\sqrt{1-4\,x} - 3\,x\,v - 2\,x^2 + \sqrt{1-4\,x} + v + 4\,x - 1}{2\,x^2\,(v^2 - v + x)}$$

```
> eqB:=(v^2-v+x)*(-B+x/(1-v)+x/v^3*(Axv-AA3-v*AA4-v^2*AA5)+x/v*(B-B3
  )+x/(1-v)*(B+A0011+A001+A00)):
  BB3:=solve(simplify(limit(diff(eqB,v),v=va)),B3):
  Bxv:=simplify(solve(subs(B3=BB3,eqB),B));
```

$$\begin{aligned}
Bxv := -\big(&-1 + 3\,v - 3\,v^2 - 3\,x^3\,v - 16\,x\,v + 6\,x - \sqrt{1-4\,x}\,v^3 + 3\sqrt{1-4\,x}\,v^2 - 4\,A00\,v^2\,x^4 \\
&- 4\,v^2\,x^4\,A001 - 4\,v^2\,x^4\,A0011 + A00\,v^2\,x^3 + 4\,A00\,v\,x^4 + v^2\,x^3\,A001 + v^2\,x^3\,A0011 + 4\,v\,x^4\,A001 \\
&+ 4\,v\,x^4\,A0011 - A00\,v\,x^3 - v\,x^3\,A001 - v\,x^3\,A0011 + 2\sqrt{1-4\,x}\,v^3\,x - 8\sqrt{1-4\,x}\,v^2\,x \\
&+ 2\sqrt{1-4\,x}\,v^3\,x^3 - 3\sqrt{1-4\,x}\,v^2\,x^3 + 2\sqrt{1-4\,x}\,v\,x^4 + 2\sqrt{1-4\,x}\,v^2\,x^2 + \sqrt{1-4\,x}\,v\,x^3 \\
&- 5\sqrt{1-4\,x}\,v\,x^2 - 4\,v^2\,x^4 + 2\,v^3\,x^2 + v^2\,x^3 - 4\,v^3\,x - 12\,v^2\,x^2 + 14\,v^2\,x + 4\,v\,x^4 + A00\,x^4 \\
&+ x^4\,A001 + x^4\,A0011 + 2\,A00\sqrt{1-4\,x}\,v^3\,x^3 + 2\sqrt{1-4\,x}\,v^3\,x^3\,A001 + 2\sqrt{1-4\,x}\,v^3\,x^3\,A0011 \\
&- 3\,A00\sqrt{1-4\,x}\,v^2\,x^3 + 2\,A00\sqrt{1-4\,x}\,v\,x^4 - 3\sqrt{1-4\,x}\,v^2\,x^3\,A001 - 3\sqrt{1-4\,x}\,v^2\,x^3\,A0011 \\
&+ 2\sqrt{1-4\,x}\,v\,x^4\,A001 + 2\sqrt{1-4\,x}\,v\,x^4\,A0011 + A00\sqrt{1-4\,x}\,v\,x^3 + \sqrt{1-4\,x}\,v\,x^3\,A001 \\
&+ \sqrt{1-4\,x}\,v\,x^3\,A0011 - A00\,x^4\sqrt{1-4\,x} - x^4\sqrt{1-4\,x}\,A001 - x^4\sqrt{1-4\,x}\,A0011 - \sqrt{1-4\,x}\,x^4 \\
&+ 3\sqrt{1-4\,x}\,x^2 - 4\,x^5 - 4\,A00\,x^5 - 4\,x^5\,A001 - 4\,x^5\,A0011 + v^3 - 3\sqrt{1-4\,x}\,v - 4\,x\sqrt{1-4\,x} \\
&+ x^4 + 2\,x^3 - 9\,x^2 + 10\sqrt{1-4\,x}\,v\,x + 19\,x^2\,v + \sqrt{1-4\,x}\big) \Big/ \big(x^2\,(-4\,x + \sqrt{1-4\,x} + 1)\,(v^2 - v + x)^2\big)
\end{aligned}$$

```
> eqC:=(v^2-v+x)^2*(-C+x/(1-v)+x*Bxv+x/v*(C-C3)+x/(1-v)*C+x/(1-v)*(A
  011+A01)):
  CC3:=factor(solve(simplify(limit(diff(eqC,v,v),v=va))=0,C3));
```

$$CC3 := -\left(8\,x^3\,A00 + 8\,A01\sqrt{1-4\,x}\,x^2 + 8\sqrt{1-4\,x}\,x^2\,A011 + 8\,x^3\,A001 + 8\,x^3\,A0011 - 2\,x^2\,A00 - 2\,x\,A01\sqrt{1-4\,x} + 8\sqrt{1-4\,x}\,x^2 - 2\,x\sqrt{1-4\,x}\,A011 + 8\,x^3 - 2\,x^2\,A001 - 2\,x^2\,A0011 - 3\,x\sqrt{1-4\,x} - 2\,x^2 + \sqrt{1-4\,x} + 3\,x - 1\right) \Big/ \left((-1+4\,x)\left(2\,x\sqrt{1-4\,x} - \sqrt{1-4\,x} + 4\,x - 1\right)\right)$$

```
> ss:=solve({A0=x+x*A00+x*A01,A00=x+x*AA3+x*A001+x*A00,A01=x+x*A001+
  x*A011+x*A01,A001=x+x*AA4+x*A0011+x*A001+x*A00,A011=x+x*A0011+x*CC
  3+x*A011+x*A01,A0011=x+x*AA5+x*BB3+x*A0011+x*A001+x*A00},{A01,A0,A
  00,A001,A011,A0011}):
> finA:=simplify(op(op(ss)[1])[2]);
```

$$finA := -\frac{19\sqrt{1-4\,x}\,x^2 + 12\,x^3 - 9\,x\sqrt{1-4\,x} - 23\,x^2 + \sqrt{1-4\,x} + 9\,x - 1}{2\,(-1+4\,x)^2}$$

```
> taylor(op(op(ss)[1])[2],x,11);
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 87\,x^5 + 355\,x^6 + 1468\,x^7 + 6103\,x^8 + 25413\,x^9 + 105799\,x^{10} + O(x^{11})$$

```
> #cases 00001 and 00000 in other two files.
```