```
> #case 021,00001 (below we checked the rules, and we checked the
  equations if they generate the same numbers)
> #---first twems of Aijk(x)
> restart: BA0:=0: BA00:=0: BA01:=0: BA000:=0: BA001:=0: BA002:=0:
  BA011:=0: BA0000:=0: BA0001:=0: BA0002:=0: BA0003:=0: BA0011:=0:
  BA0022:=0: BA0111:=0: BA00011:=0: BA00022:=0: BA00033:=0:
  BA00111:=0: BA00222:=0: BA01111:=0: BA000111:=0: BA000222:=0:
  BA000333:=0: BA001111:=0: BA0001111:=0: BA121:=0: BA122:=0:
  BA123:=0: BA131:=0: BA132:=0: BA133:=0: BA221:=0: BA222:=0:
  BA223:=0: BA231:=0: BA232:=0: BA233:=0: BA321:=0: BA322:=0:
  BA323:=0: BA331:=0: BA332:=0: BA333:=0:
> for i from 1 to 30 do BA0:=simplify(x+x*BA00+x*BA01):
  BA00:=simplify(x+x*BA000+x*BA001+x*BA002):
  BA01:=simplify(x+x*BA001+x*BA011+x*BA01):
  BA000:=simplify(x+x*BA0000+x*BA0001+x*BA0002+x*BA0003):
  BA001:=simplify(x+x*BA0001+x*BA0011+x*BA001+x*BA002):
  BA002:=simplify(x+x*BA0002+x*BA0022+x*BA002):
  BA011:=simplify(x+x*BA0011+x*BA0111+x*subs(v=0,BA121)+x*BA01):
  BA0000:=x+x*BA0000:
  BA0001:=simplify(x+x*BA0000+x*BA00011+x*BA0001+x*BA0002+x*BA0003):
  BA0002:=simplify(x+x*BA0000+x*BA00022+x*BA0002+x*BA0003):
  BA0003:=simplify(x+x*BA0000+x*BA00033+x*BA0003):
  BA0011:=simplify(x+x*BA00011+x*BA00111+x*subs(v=0,BA221)+x*BA001+x
  *BA002): BA0022:=simplify(x+x*BA00022+x*BA00222+x*BA001+x*BA002):
  BA0111:=simplify(x+x*BA00111+x*BA01111+x*subs(v=0,BA131)+x*subs(v=
  0,BA121)+x*BA01):
  BA00011:=simplify(x+x*BA0000+x*BA000111+x*subs(v=0,BA321)+x*BA0001
  +x*BA0002+x*BA0003):
  BA00022:=simplify(x+x*BA0000+x*BA000222+x*BA0001+x*BA0002+x*BA0003
  ): BA00033:=simplify(x+x*BA0000+x*BA000333+x*BA0002+x*BA0003):
  BA00111:=simplify(x+x*BA000111+x*BA001111+x*subs(v=0,BA231)+x*subs
  (v=0,BA221)+x*BA001+x*BA002):
  BA00222:=simplify(x+x*BA000222+x*BA001111+x*subs(v=0,BA221)+x*BA00
  1+x*BA002): BA01111:=simplify(x+x*BA001111+x*BA01111):
  BA000111:=simplify(x+x*BA0000+x*BA0001111+x*subs(v=0,BA331)+x*subs
  (v=0,BA321)+x*BA0001+x*BA0002+x*BA0003):
  BA000222:=simplify(x+x*BA0000+x*BA0001111+x*subs(v=0,BA321)+x*BA00
  01+x*BA0002+x*BA0003):
  BA000333:=simplify(x+x*BA0000+x*BA0001111+x*BA0001+x*BA0002+x*BA00
  03): BA001111:=simplify(x+x*BA0001111+x*BA001111):
  BA0001111:=simplify(x+x*BA0000+x*BA0001111):
  BA121:=simplify(x/(1-v)+x*BA221+x*BA122+x/(1-v)*BA121+x*v/(1-v)*BA
  131+x/(1-v)*BA01):
```

```
BA122:=simplify(x/(1-v)+x*BA222+x*BA123+x/(1-v)*(BA131+BA121)+x/(1
-v)*BA01):
BA123:=simplify(x/(1-v)+x*BA223+x/(1-v)*BA01111+x/v*(BA121-subs(v=
0,BA121))+x/(1-v)*(BA121+BA131)+x/(1-v)*BA01):
BA131:=simplify(x/(1-v)+x*BA231+x*BA132+x/(1-v)*(BA131+BA121)+x/(1
-v)*BA01):
BA132:=simplify(x/(1-v)+x*BA232+x*BA133+x/(1-v)*BA121+x/v*(BA121-s
ubs(v=0,BA121))+x/(1-v)*BA131+x/(1-v)*BA01):
BA133:=simplify(x/(1-v)+x*BA233+x/(1-v)*BA01111+x/v*(BA131+BA121-s
ubs(v=0,BA131)-subs(v=0,BA121))+x/(1-v)*(BA131+BA121)+x/(1-v)*BA01
):
BA221:=simplify(x/(1-v)+x*BA321+x*BA222+x/(1-v)*BA221+x*v/(1-v)*BA
231+x/(1-v)*BA001+x/(1-v)*BA002):
BA222:=simplify(x/(1-v)+x*BA322+x*BA223+x/(1-v)*BA231+x/(1-v)*BA22
1+x/(1-v)*BA001+x/(1-v)*BA002):
BA223:=simplify(x/(1-v)+x*BA323+x/(1-v)*BA001111+x/v*(BA221-subs(v
=0,BA221))+x/(1-v)*(BA221+BA231)+x/(1-v)*BA001+x/(1-v)*BA002):
BA231:=simplify(x/(1-v)+x*BA331+x*BA232+x/(1-v)*BA231+x/(1-v)*BA22
1+x/(1-v)*BA001+x/(1-v)*BA002):
BA232:=simplify(x/(1-v)+x*BA332+x*BA233+x/v*(BA221-subs(v=0,BA221)
)+x/(1-v)*(BA221+BA231)+x/(1-v)*BA001+x/(1-v)*BA002):
BA233:=simplify(x/(1-v)+x*BA333+x/(1-v)*BA001111+x/v*(BA231+BA221-
subs(v=0,BA231)-subs(v=0,BA221))+x/(1-v)*(BA231+BA221)+x/(1-v)*BA0
01+x/(1-v)*BA002):
BA321:=simplify(x/(1-v)+x/(1-v)*BA0000+x*BA322+x/(1-v)*BA321+x*v/(
1-v)*BA331+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA322:=simplify(x/(1-v)+x/(1-v)*BA0000+x*BA323+x/(1-v)*(BA331+BA32
1)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA323:=simplify(x/(1-v)+x/(1-v)*(BA0000+BA0001111)+x/v*(BA321-subs
(v=0,BA321))+x/(1-v)*(BA321+BA331)+x/(1-v)*BA0001+x/(1-v)*BA0002+x
/(1-v)*BA0003):
BA331:=simplify(x/(1-v)+x/(1-v)*BA0000+x*BA332+x/(1-v)*(BA331+BA32
1)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA332:=simplify(x/(1-v)+x/(1-v)*BA0000+x*BA333+x/v*(BA321-subs(v=0
,BA321))+x/(1-v)*(BA321+BA331)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-
v)*BA0003):
BA333:=simplify(x/(1-v)+x/(1-v)*(BA0000+BA0001111)+x/v*(BA331+BA32
1-subs(v=0,BA331)-subs(v=0,BA321))+x/(1-v)*(BA321+BA331)+x/(1-v)*B
A0001+x/(1-v)*BA0002+x/(1-v)*BA0003):  od:
> taylor(BA0,x,30);
```

$$x + 2x^2 + 6x^3 + 22x^4 + 86x^5 + 341x^6 + 1345x^7 + 5228x^8 + 20189x^9 + 77528x^{10} + 296107x^{11} + 1125894x^{12} + 4264107x^{13} + 16092759x^{14} + 60546106x^{15} + 227168798x^{16} + 850257918x^{17}$$

$$+ 3175465012\, x^{18} + 11836349331\, x^{19} + 44042065429\, x^{20} + 163618075048\, x^{21} + 606978238162\, x^{22} + 2248793328871\, x^{23} + 8321659356538\, x^{24} + 30760809228310\, x^{25} + 113592961758761\, x^{26} + 419087797809461\, x^{27} + 1544859145515076\, x^{28} + 5690234597391318\, x^{29} + O(x^{30})$$

```
> #-------------------------------------------------------------------
-----------------------------------------
> restart: #system1
> ss:=map(factor,solve({A0=(x+x*A00+x*A01),
A00=(x+x*A000+x*A001+x*A002), A01=(x+x*A001+x*A011+x*A01),
A000=(x+x*A0000+x*A0001+x*A0002+x*A0003),
A001=(x+x*A0001+x*A0011+x*A001+x*A002),
A002=(x+x*A0002+x*A0022+x*A002),
A011=(x+x*A0011+x*A0111+x*A121(0)+x*A01), A0000=x+x*A0000,
A0001=(x+x*A0000+x*A00011+x*A0001+x*A0002+x*A0003),
A0002=(x+x*A0000+x*A00022+x*A0002+x*A0003),
A0003=(x+x*A0000+x*A00033+x*A0003),
A0011=(x+x*A00011+x*A00111+x*A221(0)+x*A001+x*A002),
A0022=(x+x*A00022+x*A00222+x*A001+x*A002),
A0111=(x+x*A00111+x*A01111+x*A131(0)+x*A121(0)+x*A01),
A00011=(x+x*A0000+x*A000111+x*A321(0)+x*A0001+x*A0002+x*A0003),
A00022=(x+x*A0000+x*A000222+x*A0001+x*A0002+x*A0003),
A00033=(x+x*A0000+x*A000333+x*A0002+x*A0003),
A00111=(x+x*A000111+x*A001111+x*A231(0)+x*A221(0)+x*A001+x*A002),
A00222=(x+x*A000222+x*A001111+x*A221(0)+x*A001+x*A002),
A01111=(x+x*A001111+x*A01111),
A000111=(x+x*A0000+x*A0001111+x*A331(0)+x*A321(0)+x*A0001+x*A0002+
x*A0003),
A000222=(x+x*A0000+x*A0001111+x*A321(0)+x*A0001+x*A0002+x*A0003),
A000333=(x+x*A0000+x*A0001111+x*A0001+x*A0002+x*A0003),
A001111=(x+x*A0001111+x*A001111),
A0001111=(x+x*A0000+x*A0001111)},{A0,A00,A01,A000,A001,A002,A011,A
0000,A0001,A0002,A0003,A0011,A0022,A0111,A00011,A00022,A00033,A001
11,A00222,A01111,A000111,A000222,A000333,A001111,A0001111})):
> op(op(ss)[1])[1],op(op(ss)[2])[1],op(op(ss)[3])[1],op(op(ss)[4])[1
],op(op(ss)[5])[1],op(op(ss)[6])[1],op(op(ss)[7])[1],op(op(ss)[8])
[1],op(op(ss)[9])[1],op(op(ss)[10])[1],op(op(ss)[11])[1],op(op(ss)
[12])[1],op(op(ss)[13])[1],op(op(ss)[14])[1],op(op(ss)[15])[1],op(
op(ss)[16])[1],op(op(ss)[17])[1],op(op(ss)[18])[1],op(op(ss)[19])[
1],op(op(ss)[20])[1],op(op(ss)[21])[1],op(op(ss)[22])[1],op(op(ss)
[23])[1],op(op(ss)[24])[1],op(op(ss)[25])[1];
```

$$A0, A00, A01, A000, A0000, A0001, A00011, A000111, A0002, A00022, A000222, A0003, A00033, A000333, A001, A0011, A00111, A001111, A002, A0022, A00222, A011, A0111, A01111,$$

A0001111

```
> #the order: A0, A00, A01, A000, A0000, A0001, A00011, A000111,
  A0002, A00022, A000222, A0003, A00033, A000333, A001, A0011,
  A00111, A001111, A002, A0022, A00222, A011, A0111, A01111,
  A0001111
> A0:=op(op(ss)[1])[2]:A00:=op(op(ss)[2])[2]:A01:=op(op(ss)[3])[2]:A
  000:=op(op(ss)[4])[2]:A0000:=op(op(ss)[5])[2]:A0001:=op(op(ss)[6])
  [2]:A00011:=op(op(ss)[7])[2]:A000111:=op(op(ss)[8])[2]:A0002:=op(o
  p(ss)[9])[2]:A00022:=op(op(ss)[10])[2]:A000222:=op(op(ss)[11])[2]:
  A0003:=op(op(ss)[12])[2]:A00033:=op(op(ss)[13])[2]:A000333:=op(op(
  ss)[14])[2]:A001:=op(op(ss)[15])[2]:A0011:=op(op(ss)[16])[2]:A0011
  1:=op(op(ss)[17])[2]:A001111:=op(op(ss)[18])[2]:A002:=op(op(ss)[19
  ])[2]:A0022:=op(op(ss)[20])[2]:A00222:=op(op(ss)[21])[2]:A011:=op(
  op(ss)[22])[2]:A0111:=op(op(ss)[23])[2]:A01111:=op(op(ss)[24])[2]:
  A0001111:=op(op(ss)[25])[2]:
> #-----------------------------------------------------------------
  ----------------------------------------
> restart:
> #solving system of 18 eq system 1
> eq121:=-A121(v)+(x/(1-v)+x*A221(v)+x*A122(v)+x/(1-v)*A121(v)+x*v/(
  1-v)*A131(v)+x/(1-v)*A01):
  eq122:=-A122(v)+(x/(1-v)+x*A222(v)+x*A123(v)+x/(1-v)*(A131(v)+A121
  (v))+x/(1-v)*A01):
  eq131:=-A131(v)+(x/(1-v)+x*A231(v)+x*A132(v)+x/(1-v)*(A131(v)+A121
  (v))+x/(1-v)*A01):
  eq133:=-A133(v)+(x/(1-v)+x*A233(v)+x/(1-v)*A01111+x/v*(A131(v)+A12
  1(v)-A131(0)-A121(0))+x/(1-v)*(A131(v)+A121(v))+x/(1-v)*A01):
  eq221:=-A221(v)+(x/(1-v)+x*A321(v)+x*A222(v)+x/(1-v)*A221(v)+x*v/(
  1-v)*A231(v)+x/(1-v)*A001+x/(1-v)*A002):
  eq222:=-A222(v)+(x/(1-v)+x*A322(v)+x*A223(v)+x/(1-v)*A231(v)+x/(1-
  v)*A221(v)+x/(1-v)*A001+x/(1-v)*A002):
  eq231:=-A231(v)+(x/(1-v)+x*A331(v)+x*A232(v)+x/(1-v)*A231(v)+x/(1-
  v)*A221(v)+x/(1-v)*A001+x/(1-v)*A002):
  eq233:=-A233(v)+(x/(1-v)+x*A333(v)+x/(1-v)*A001111+x/v*(A231(v)+A2
  21(v)-A231(0)-A221(0))+x/(1-v)*(A231(v)+A221(v))+x/(1-v)*A001+x/(1
  -v)*A002):
  eq321:=-A321(v)+(x/(1-v)+x/(1-v)*A0000+x*A322(v)+x/(1-v)*A321(v)+x
  *v/(1-v)*A331(v)+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003):
  eq322:=-A322(v)+(x/(1-v)+x/(1-v)*A0000+x*A323(v)+x/(1-v)*(A331(v)+
  A321(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003):
  eq331:=-A331(v)+(x/(1-v)+x/(1-v)*A0000+x*A332(v)+x/(1-v)*(A331(v)+
  A321(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003):
  eq333:=-A333(v)+(x/(1-v)+x/(1-v)*(A0000+A0001111)+x/v*(A331(v)+A32
```

```
1(v)-A331(0)-A321(0))+x/(1-v)*(A321(v)+A331(v))+x/(1-v)*A0001+x/(1
-v)*A0002+x/(1-v)*A0003):
eq123:=-A123(v)+(x/(1-v)+x*A223(v)+x/(1-v)*A01111+x/v*(A121(v)-A12
1(0))+x/(1-v)*(A121(v)+A131(v))+x/(1-v)*A01):
eq132:=-A132(v)+(x/(1-v)+x*A232(v)+x*A133(v)+x/(1-v)*A121(v)+x/v*(
A121(v)-A121(0))+x/(1-v)*A131(v)+x/(1-v)*A01):
eq223:=-A223(v)+(x/(1-v)+x*A323(v)+x/(1-v)*A001111+x/v*(A221(v)-A2
21(0))+x/(1-v)*(A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002):
eq232:=-A232(v)+(x/(1-v)+x*A332(v)+x*A233(v)+x/v*(A221(v)-A221(0))
+x/(1-v)*(A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002):
eq323:=-A323(v)+(x/(1-v)+x/(1-v)*(A0000+A0001111)+x/v*(A321(v)-A32
1(0))+x/(1-v)*(A321(v)+A331(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v
)*A0003):
eq332:=-A332(v)+(x/(1-v)+x/(1-v)*A0000+x*A333(v)+x/v*(A321(v)-A321
(0))+x/(1-v)*(A321(v)+A331(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)
*A0003):
> EQ:=[eq121,eq122,eq131,eq133,eq221,eq222,eq231,eq233,eq321,eq322,e
q331,eq333,eq123,eq132,eq223,eq232,eq323,eq332]:
VQ:=[A121(v),A122(v),A131(v),A133(v),A221(v),A222(v),A231(v),A233(
v),A321(v),A322(v),A331(v),A333(v),A123(v),A132(v),A223(v),A232(v)
,A323(v),A332(v)]:
> with(linalg): T:=matrix(18,37,0):
> VQv:=VQ; VQ0:=subs(v=0,VQ);
```

$$VQv := [A121(v), A122(v), A131(v), A133(v), A221(v), A222(v), A231(v), A233(v), A321(v), A322(v), A331(v), A333(v), A123(v), A132(v), A223(v), A232(v), A323(v), A332(v)]$$

$$VQ0 := [A121(0), A122(0), A131(0), A133(0), A221(0), A222(0), A231(0), A233(0), A321(0), A322(0), A331(0), A333(0), A123(0), A132(0), A223(0), A232(0), A323(0), A332(0)]$$

```
> for i from 1 to 18 do T[i,37]:=EQ[i]: for j from 1 to 18 do
T[i,37]:=subs(VQv[j]=0,VQ0[j]=0,T[i,37]):
T[i,j]:=factor(coeff(EQ[i],VQv[j])):
T[i,j+18]:=factor(coeff(EQ[i],VQ0[j])): od:
T[i,37]:=factor(T[i,37]): od:
> T:=gausselim(T): evalm(T): gc():
> #starting solving the system:
> i:=18:
eq:=add(T[i,j]*VQv[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,3
7]: KK:=-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v^2*x-v^2:
alias(vv=RootOf(KK=0,v)): taylor(KK,v,10);
uu:=solve(u1+x^6/u1/u3=1-2*x-x^2-u3,u1):
ss:=map(factor,solve({subs(v=u1,subs(A332(v)=0,eq)),subs(v=u2,subs
```

```
(A332(v)=0,eq))},{A321(0),A331(0)})):
op(op(ss)[1])[1],op(op(ss)[2])[1];
A321x0:=simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],op(op
(ss)[1])[2]))+1)-1):
A331x0:=simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],op(op
(ss)[2])[2]))+1)-1):
A332xv:=subs(vv=u3,factor(simplify(subs(u3=vv,solve(subs(A321(0)=A
321x0,A331(0)=A331x0,eq),A332(v))))))):
```

$$-x^6+(-x^4+2x^3)v+(x^2+2x-1)v^2+v^3$$

$$\mathrm{A321}(0),\ \mathrm{A331}(0)$$

```
> series(-subs(v=0,BA331)+subs(u3=vv,A0001=BA0001,A0002=BA0002,A0000
=BA0000,A0003=BA0003,A0001111=BA0001111,A331x0),x,20);
series(-subs(v=0,BA321)+subs(u3=vv,A0001=BA0001,A0002=BA0002,A0000
=BA0000,A0003=BA0003,A0001111=BA0001111,A321x0),x,20);
>
>
> i:=17:
eq:=add(T[i,j]*VQ[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,37
]:
A323xv:=subs(vv=u3,factor(simplify(subs(u3=vv,solve(subs(A321(0)=A
321x0,A331(0)=A331x0,A332(v)=A332xv,eq)=0,A323(v))))))):
A323x0:=simplify(limit(A323xv,v=0)):
>
>
> i:=16:
eq:=add(T[i,j]*VQ[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,37
]:
eq:=simplify(subs(A321(0)=A321x0,A331(0)=A331x0,A323(0)=A323x0,A33
2(v)=A332xv,A323(v)=A323xv,eq)*(-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v
^2*x-v^2)): eq0:=diff(subs(A232(v)=0,eq),v):
ss:=map(factor,solve({subs(v=u1,eq0),subs(v=u2,eq0)},{A231(0),A221
(0)})):
> op(op(ss)[1])[1],op(op(ss)[2])[1];
A221x0:=simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],op(op
(ss)[1])[2]))+1)-1):
A231x0:=simplify(simplify(rationalize(subs(u1=uu[1],u2=uu[2],op(op
(ss)[2])[2]))+1)-1):
```

$$\mathrm{A221}(0),\ \mathrm{A231}(0)$$

```
> series(-subs(v=0,BA221)+subs(u3=vv,A001=BA001,A002=BA002,A001111=B
A001111,A0001=BA0001,A0002=BA0002,A0000=BA0000,A0003=BA0003,A00011
11=BA0001111,A221x0),x,25);
```

$$\mathrm{O}(x^{14})$$

```
> eq:=add(T[i,j]*VQ[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,37
  ]:
  eq:=subs(A321(0)=A321x0,A331(0)=A331x0,A323(0)=A323x0,A332(v)=A332
  xv,A323(v)=A323xv,A221(0)=A221x0,A231(0)=A231x0,eq):
  A232xv:=solve(eq=0,A232(v)): A232x0:=simplify(limit(A232xv,v=0)):
>
>
> i:=15:
  eq:=add(T[i,j]*VQ[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,37
  ]:
  A223xv:=solve(subs(A232(v)=A232xv,A323(v)=A323xv,A332(v)=A332xv,A2
  21(0)=A221x0,A231(0)=A231x0,A321(0)=A321x0,A331(0)=A331x0,eq),A223
  (v)): A223x0:=simplify(limit(A223xv,v=0)):
>
>
> i:=14:
  eq:=add(T[i,j]*VQ[j],j=1..18)+add(T[i,j+18]*VQ0[j],j=1..18)+T[i,37
  ]:
  eq:=simplify(subs(A321(0)=A321x0,A331(0)=A331x0,A323(0)=A323x0,A33
  2(v)=A332xv,A323(v)=A323xv,A221(0)=A221x0,A231(0)=A231x0,A223(0)=A
  223x0,A223(v)=A223xv,eq)*(-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v^2*x-v
  ^2)^2):
> eq0:=simplify(diff(subs(A132(v)=0,eq),v,v)):
> ss:=solve({subs(v=u1,eq0),subs(v=u2,eq0)},{A121(0),A131(0)}):
>
>
>
> restart: #system3
> read
  "C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
  4/testout00001.txt":
> ss:=map(factor,solve({A0=(x+x*A00+x*A01),
  A00=(x+x*A000+x*A001+x*A002), A01=(x+x*A001+x*A011+x*A01),
  A000=(x+x*A0000+x*A0001+x*A0002+x*A0003),
  A001=(x+x*A0001+x*A0011+x*A001+x*A002),
  A002=(x+x*A0002+x*A0022+x*A002),
  A011=(x+x*A0011+x*A0111+x*A121x0+x*A01), A0000=x+x*A0000,
  A0001=(x+x*A0000+x*A00011+x*A0001+x*A0002+x*A0003),
  A0002=(x+x*A0000+x*A00022+x*A0002+x*A0003),
  A0003=(x+x*A0000+x*A00033+x*A0003),
  A0011=(x+x*A00011+x*A00111+x*A221x0+x*A001+x*A002),
  A0022=(x+x*A00022+x*A00222+x*A001+x*A002),
  A0111=(x+x*A00111+x*A01111+x*A131x0+x*A121x0+x*A01),
```

```
A00011=(x+x*A0000+x*A000111+x*A321x0+x*A0001+x*A0002+x*A0003),
A00022=(x+x*A0000+x*A000222+x*A0001+x*A0002+x*A0003),
A00033=(x+x*A0000+x*A000333+x*A0002+x*A0003),
A00111=(x+x*A000111+x*A001111+x*A231x0+x*A221x0+x*A001+x*A002),
A00222=(x+x*A000222+x*A001111+x*A221x0+x*A001+x*A002),
A01111=(x+x*A001111+x*A01111),
A000111=(x+x*A0000+x*A0001111+x*A331x0+x*A321x0+x*A0001+x*A0002+x*
A0003),
A000222=(x+x*A0000+x*A0001111+x*A321x0+x*A0001+x*A0002+x*A0003),
A000333=(x+x*A0000+x*A0001111+x*A0001+x*A0002+x*A0003),
A001111=(x+x*A0001111+x*A001111),
A0001111=(x+x*A0000+x*A0001111)},{A0,A00,A01,A000,A001,A002,A011,A
0000,A0001,A0002,A0003,A0011,A0022,A0111,A00011,A00022,A00033,A001
11,A00222,A01111,A000111,A000222,A000333,A001111,A0001111})):
> op(op(ss)[1])[1];
```

$$A0$$

```
> FinA:=op(op(ss)[1])[2];
```

$$\begin{aligned}
FinA := &-x^2\,(256\,u3\,x^{22} + 768\,u3\,x^{21} - 512\,u3^3\,x^{18} - 768\,u3^2\,x^{19} - 608\,u3\,x^{20} - 1792\,u3^3\,x^{17} \\
&- 2560\,u3^2\,x^{18} - 1936\,u3\,x^{19} + 256\,x^{20} + 256\,u3^5\,x^{14} + 768\,u3^4\,x^{15} + 704\,u3^3\,x^{16} + 800\,u3^2\,x^{17} \\
&- 551\,u3\,x^{18} + 128\,x^{19} + 1024\,u3^5\,x^{13} + 2816\,u3^4\,x^{14} + 3712\,u3^3\,x^{15} + 1808\,u3^2\,x^{16} - 806\,u3\,x^{17} \\
&+ 576\,x^{18} + 416\,u3^5\,x^{12} + 224\,u3^4\,x^{13} + 382\,u3^3\,x^{14} + 613\,u3^2\,x^{15} + 1946\,u3\,x^{16} - 424\,x^{17} \\
&+ 16\,u3^5\,x^{11} - 112\,u3^4\,x^{12} - 989\,u3^3\,x^{13} - 3127\,u3^2\,x^{14} - 4829\,u3\,x^{15} + 99\,x^{16} - 23\,u3^5\,x^{10} \\
&- 213\,u3^4\,x^{11} - 5269\,u3^3\,x^{12} - 5777\,u3^2\,x^{13} - 1376\,u3\,x^{14} - 968\,x^{15} + 1411\,u3^5\,x^9 + 4512\,u3^4\,x^{10} \\
&+ 5300\,u3^3\,x^{11} + 2724\,u3^2\,x^{12} - 4287\,u3\,x^{13} + 375\,x^{14} + 3005\,u3^5\,x^8 + 7132\,u3^4\,x^9 + 4777\,u3^3\,x^{10} \\
&+ 2994\,u3^2\,x^{11} + 3152\,u3\,x^{12} - 304\,x^{13} + 1990\,u3^5\,x^7 + 3316\,u3^4\,x^8 + 5761\,u3^3\,x^9 + 5250\,u3^2\,x^{10} \\
&- 512\,u3\,x^{11} + 514\,x^{12} + 1030\,u3^5\,x^6 + 706\,u3^4\,x^7 - 2197\,u3^3\,x^8 - 1489\,u3^2\,x^9 + 1701\,u3\,x^{10} \\
&- 253\,x^{11} - 341\,u3^5\,x^5 - 1776\,u3^4\,x^6 - 993\,u3^3\,x^7 - 371\,u3^2\,x^8 - 1754\,u3\,x^9 + 81\,x^{10} - 111\,u3^5\,x^4 \\
&- 256\,u3^4\,x^5 - 1384\,u3^3\,x^6 - 1151\,u3^2\,x^7 + 340\,u3\,x^8 - 122\,x^9 - 40\,u3^5\,x^3 + 143\,u3^4\,x^4 \\
&+ 1215\,u3^3\,x^5 + 954\,u3^2\,x^6 - 423\,u3\,x^7 + 46\,x^8 + 16\,u3^5\,x^2 + 19\,u3^4\,x^3 - 254\,u3^3\,x^4 - 35\,u3^2\,x^5 \\
&+ 325\,u3\,x^6 - 4\,x^7 + 25\,u3^4\,x^2 + 301\,u3^3\,x^3 + 290\,u3^2\,x^4 - 53\,u3\,x^5 - 14\,u3^4\,x - 145\,u3^3\,x^2 \\
&- 161\,u3^2\,x^3 - 2\,u3\,x^4 + 8\,u3^4 + 40\,u3^3\,x - 8\,u3^3 + 6\,u3^2\,x)\Big/(u3\,(16\,x^3 + 8\,x^2 + 11\,x - 4)^2 \\
&(x^2 + x + 1)^3\,(x-1)^4\,(-x^4 + u3^2 + u3\,x)\,(-x^4 + x^3 + u3^2 + 2\,u3\,x - u3))
\end{aligned}$$

```
> KK:=-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v^2*x-v^2:
  alias(vv=RootOf(KK=0,v)):
> series(subs(u3=vv,FinA),x,20);
```

$$\begin{aligned}
&x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 86\,x^5 + 341\,x^6 + 1345\,x^7 + 5228\,x^8 + 20189\,x^9 + 77528\,x^{10} + 296107\,x^{11} \\
&+ 1125894\,x^{12} + 4264107\,x^{13} + 16092759\,x^{14} + 60546106\,x^{15} + 227168798\,x^{16} + 850257918\,x^{17}
\end{aligned}$$

$$+ O(x^{18})$$

```
> 021,00001 1, 2, 6, 22, 86, 341, 1345, 5228, 20189, 77528
>
> FinA:=subs(vv=u3,factor(simplify(subs(u3=vv,FinA))));
```

$$FinA := (-256\,x^{21} - 2560\,x^{20} + 608\,x^{19} + 1536\,u3^2\,x^{16} + 1536\,u3\,x^{17} - 3024\,x^{18} - 1024\,u3^2\,x^{15} - 4096\,u3\,x^{16} + 2039\,x^{17} + 1472\,u3^2\,x^{14} + 2240\,u3\,x^{15} - 6073\,x^{16} - 2016\,u3^2\,x^{13} - 4896\,u3\,x^{14} - 139\,x^{15} + 3318\,u3^2\,x^{12} + 5886\,u3\,x^{13} - 6412\,x^{14} - 1754\,u3^2\,x^{11} - 8188\,u3\,x^{12} + 839\,x^{13} + 782\,u3^2\,x^{10} + 602\,u3\,x^{11} - 4003\,x^{12} - 3604\,u3^2\,x^{9} - 6758\,u3\,x^{10} + 2417\,x^{11} + 1025\,u3^2\,x^{8} + 6010\,u3\,x^{9} - 1124\,x^{10} - 319\,u3^2\,x^{7} - 630\,u3\,x^{8} + 1857\,x^{9} + 1559\,u3^2\,x^{6} + 2547\,u3\,x^{7} - 1252\,x^{8} - 684\,u3^2\,x^{5} - 3177\,u3\,x^{6} + 13\,x^{7} - 97\,u3^2\,x^{4} + 231\,u3\,x^{5} - 392\,x^{6} - 302\,u3^2\,x^{3} - 388\,u3\,x^{4} + 143\,x^{5} + 76\,u3^2\,x^{2} + 486\,u3\,x^{3} + 31\,x^{4} + 42\,u3^2\,x - 2\,u3\,x^{2} - 10\,x^{3} - 10\,u3^2 - 62\,u3\,x + 10\,u3) \Big/ \left((x-1)^4\,(x^2+x+1)^3\,(16\,x^3+8\,x^2+11\,x-4)^2\,x^5\right)$$

```
> series(subs(u3=vv,FinA),x,20);
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 86\,x^5 + 341\,x^6 + 1345\,x^7 + 5228\,x^8 + 20189\,x^9 + 77528\,x^{10} + 296107\,x^{11} + 1125894\,x^{12} + 4264107\,x^{13} + 16092759\,x^{14} + O(x^{15})$$

```
> FinA :=
(-256*x^21-2560*x^20+608*x^19+1536*u3^2*x^16+1536*u3*x^17-3024*x^1
8-1024*u3^2*x^15-4096*u3*x^16+2039*x^17+1472*u3^2*x^14+2240*u3*x^1
5-6073*x^16-2016*u3^2*x^13-4896*u3*x^14-139*x^15+3318*u3^2*x^12+58
86*u3*x^13-6412*x^14-1754*u3^2*x^11-8188*u3*x^12+839*x^13+782*u3^2
*x^10+602*u3*x^11-4003*x^12-3604*u3^2*x^9-6758*u3*x^10+2417*x^11+1
025*u3^2*x^8+6010*u3*x^9-1124*x^10-319*u3^2*x^7-630*u3*x^8+1857*x^
9+1559*u3^2*x^6+2547*u3*x^7-1252*x^8-684*u3^2*x^5-3177*u3*x^6+13*x
^7-97*u3^2*x^4+231*u3*x^5-392*x^6-302*u3^2*x^3-388*u3*x^4+143*x^5+
76*u3^2*x^2+486*u3*x^3+31*x^4+42*u3^2*x-2*u3*x^2-10*x^3-10*u3^2-62
*u3*x+10*u3)/(x-1)^4/(x^2+x+1)^3/(16*x^3+8*x^2+11*x-4)^2/x^5;
```

$$FinA := (-256\,x^{21} - 2560\,x^{20} + 608\,x^{19} + 1536\,u3^2\,x^{16} + 1536\,u3\,x^{17} - 3024\,x^{18} - 1024\,u3^2\,x^{15} - 4096\,u3\,x^{16} + 2039\,x^{17} + 1472\,u3^2\,x^{14} + 2240\,u3\,x^{15} - 6073\,x^{16} - 2016\,u3^2\,x^{13} - 4896\,u3\,x^{14} - 139\,x^{15} + 3318\,u3^2\,x^{12} + 5886\,u3\,x^{13} - 6412\,x^{14} - 1754\,u3^2\,x^{11} - 8188\,u3\,x^{12} + 839\,x^{13} + 782\,u3^2\,x^{10} + 602\,u3\,x^{11} - 4003\,x^{12} - 3604\,u3^2\,x^{9} - 6758\,u3\,x^{10} + 2417\,x^{11} + 1025\,u3^2\,x^{8} + 6010\,u3\,x^{9} - 1124\,x^{10} - 319\,u3^2\,x^{7} - 630\,u3\,x^{8} + 1857\,x^{9} + 1559\,u3^2\,x^{6} + 2547\,u3\,x^{7} - 1252\,x^{8} - 684\,u3^2\,x^{5} - 3177\,u3\,x^{6} + 13\,x^{7} - 97\,u3^2\,x^{4} + 231\,u3\,x^{5} - 392\,x^{6} - 302\,u3^2\,x^{3} - 388\,u3\,x^{4} + 143\,x^{5} + 76\,u3^2\,x^{2} + 486\,u3\,x^{3} + 31\,x^{4} + 42\,u3^2\,x - 2\,u3\,x^{2} - 10\,x^{3} - 10\,u3^2 - 62\,u3\,x + 10\,u3) \Big/ \left((x-1)^4\,(x^2+x+1)^3\,(16\,x^3+8\,x^2+11\,x-4)^2\,x^5\right)$$

```
> series(subs(u3=vv,FinA),x,20);
```

$$x + 2\,x^2 + 6\,x^3 + 22\,x^4 + 86\,x^5 + 341\,x^6 + 1345\,x^7 + 5228\,x^8 + 20189\,x^9 + 77528\,x^{10} + 296107\,x^{11}$$

$$+ 1125894\, x^{12} + 4264107\, x^{13} + 16092759\, x^{14} + O(x^{15})$$

```
>
```