

```

[ > restart: #021 and 00000
> BA0:=0: BA00:=0: BB11:=0: BA000:=0: BB21:=0: BB22:=0: BA011:=0:
BA0000:=0: BA0011:=0: BA0111:=0: BA0022:=0: BA00222:=0:
BA00111:=0: BA001111:=0: BA002222:=0: BB31:=0: BB32:=0: BB33:=0:
BA00011:=0: BA00022:=0: BA00033:=0: BA000333:=0: BA000111:=0:
BA000222:=0: BA01111:=0: BA000011:=0: BA000022:=0: BA000033:=0:
BA000044:=0: BA0000111:=0: BA0000222:=0: BA0000333:=0:
BA0000444:=0: BB41:=0: BB42:=0: BB43:=0: BB44:=0: BA121:=0:
BA122:=0: BA123:=0: BA124:=0: BA131:=0: BA132:=0: BA133:=0:
BA134:=0: BA141:=0: BA142:=0: BA143:=0: BA144:=0: BA221:=0:
BA222:=0: BA223:=0: BA224:=0: BA231:=0: BA232:=0: BA233:=0:
BA234:=0: BA241:=0: BA242:=0: BA243:=0: BA244:=0: BA321:=0:
BA322:=0: BA323:=0: BA324:=0: BA331:=0: BA332:=0: BA333:=0:
BA334:=0: BA341:=0: BA342:=0: BA343:=0: BA344:=0: BA421:=0:
BA422:=0: BA423:=0: BA424:=0: BA431:=0: BA432:=0: BA433:=0:
BA434:=0: BA441:=0: BA442:=0: BA143:=0: BA444:=0:
> N:=45: for i from 1 to N do BA0:=simplify(x+x*BA00+x*BB11):
BA00:=simplify(x+x*BA000+x*BB21+x*BB22):
BB11:=simplify(x+x*BB21+x*BA011+x*BB11):
BA000:=simplify(x+x*BA0000+x*BB31+x*BB32+x*BB33):
BB21:=simplify(x+x*BB31+x*BA0011+x*BB21+x*BB22):
BB22:=simplify(x+x*BB32+x*BA0022+x*BB22):
BA011:=simplify(x+x*BA0011+x*BA0111+x*subs(v=0,BA121)+x*BB11):
BA0000:=simplify(x+x*BB41+x*BB42+x*BB43+x*BB44):
BB31:=simplify(x+x*BB41+x*BA00011+x*BB31+x*BB32+x*BB33):
BB32:=simplify(x+x*BB42+x*BA00022+x*BB32+x*BB33):
BB33:=simplify(x+x*BB43+x*BA00033+x*BB33):
BA0011:=simplify(x+x*BA00011+x*BA00111+x*subs(v=0,BA221)+x*BB21+x*
BB22):
BA0022:=simplify(x+x*BA00022+x*BA00222+x*BB21+x*BB22):
BA0111:=simplify(x+x*BA00111+x*BA01111+x*subs(v=0,BA131+BA121)+x*B
B11):
BB41:=simplify(x+x*BA000011+x*BB41+x*BB42+x*BB43+x*BB44):
BB42:=simplify(x+x*BA000022+x*BB42+x*BB43+x*BB44):
BB43:=simplify(x+x*BA000033+x*BB43+x*BB44):
BB44:=simplify(x+x*BA000044+x*BB44):
BA00011:=simplify(x+x*BA000011+x*BA000111+x*subs(v=0,BA321)+x*BB31
+x*BB32+x*BB33):
BA00022:=simplify(x+x*BA000022+x*BA000222+x*BB31+x*BB32+x*BB33):
BA00033:=simplify(x+x*BA000033+x*BA000333+x*BB32+x*BB33):
BA00111:=simplify(x+x*BA000111+x*BA001111+x*subs(v=0,BA231+BA221)+
x*BB21+x*BB22):
BA00222:=simplify(x+x*BA000222+x*BA002222+x*subs(v=0,BA221)+x*BB21

```

```

+x*BB22) :
BA01111:=simplify(x+x*BA001111+x*subs(v=0,BA141+BA131+BA121)+x*BB1
1) :
BA000011:=simplify(x+x*BA0000111+x*subs(v=0,BA421)+x*BB41+x*BB42+x
*BB43+x*BB44) :
BA000022:=simplify(x+x*BA0000222+x*BB41+x*BB42+x*BB43+x*BB44) :
BA000033:=simplify(x+x*BA0000333+x*BB42+x*BB43+x*BB44) :
BA000044:=simplify(x+x*BA0000444+x*BB43+x*BB44) :
BA000111:=simplify(x+x*BA0000111+x*BA0001111+x*subs(v=0,BA331+BA32
1)+x*BB31+x*BB32+x*BB33) :
BA000222:=simplify(x+x*BA0000222+x*BA0002222+x*subs(v=0,BA321)+x*B
B31+x*BB32+x*BB33) :
BA000333:=simplify(x+x*BA0000333+x*BA0003333+x*BB31+x*BB32+x*BB33)
:
BA001111:=simplify(x+x*BA0001111+x*subs(v=0,BA241+BA231+BA221)+x*B
B21+x*BB22) :
BA002222:=simplify(x+x*BA0002222+x*subs(v=0,BA231+BA221)+x*BB21+x*
BB22) :
BA0000111:=simplify(x+x*BA00001111+x*subs(v=0,BA431+BA421)+x*BB41+
x*BB42+x*BB43+x*BB44) :
BA0000222:=simplify(x+x*BA00002222+x*subs(v=0,BA421)+x*BB41+x*BB42
+x*BB43+x*BB44) :
BA0000333:=simplify(x+x*BA00003333+x*BB41+x*BB42+x*BB43+x*BB44) :
BA0000444:=simplify(x+x*BA0000+x*BB42+x*BB43+x*BB44) :
BA0001111:=simplify(x+x*BA00001111+x*subs(v=0,BA341+BA331+BA321)+x
*BB31+x*BB32+x*BB33) :
BA0002222:=simplify(x+x*BA00002222+x*subs(v=0,BA331+BA321)+x*BB31+
x*BB32+x*BB33) :
BA0003333:=simplify(x+x*BA00003333+x*subs(v=0,BA321)+x*BB31+x*BB32
+x*BB33) :
BA00001111:=simplify(x+x*subs(v=0,BA441+BA431+BA421)+x*BB41+x*BB42
+x*BB43+x*BB44) :
BA00002222:=simplify(x+x*subs(v=0,BA431+BA421)+x*BB41+x*BB42+x*BB4
3+x*BB44) :
BA00003333:=simplify(x+x*subs(v=0,BA421)+x*BB41+x*BB42+x*BB43+x*BB
44) :
BA121:=simplify(x/(1-v)+x*BA221+x*BA122+x/(1-v)*(BA121+v*BA141+v*B
A131+BB11)) :
BA122:=simplify(x/(1-v)+x*BA222+x*BA123+x/(1-v)*(BA131+BA121+v*BA1
41+BB11)) :
BA123:=simplify(x/(1-v)+x*BA223+x*BA124+x/(1-v)*(BA141+BA131+BA121
+BB11)) :
BA124:=simplify(x/(1-v)+x*BA224+x/v*(BA121-subs(v=0,BA121))+x/(1-v

```

```

) * (BA121+BA141+BA131+BB11)) :
BA131:=simplify(x/(1-v)+x*BA231+x*BA132+x/(1-v)*(BA131+BA121+v*BA1
41+BB11)) :
BA132:=simplify(x/(1-v)+x*BA232+x*BA133+x/(1-v)*(BA141+BA131+BA121
+BB11)) :
BA133:=simplify(x/(1-v)+x*BA233+x*BA134+x/v*(BA121-subs(v=0,BA121)
)+x/(1-v)*(BA121+BA141+BA131+BB11)) :
BA134:=simplify(x/(1-v)+x*BA234+x/v*(BA131+BA121-subs(v=0,BA131+BA
121))+x/(1-v)*(BA131+BA121+BA141+BB11)) :
BA141:=simplify(x/(1-v)+x*BA241+x*BA142+x/(1-v)*(BA141+BA131+BA121
+BB11)) :
BA142:=simplify(x/(1-v)+x*BA242+x*BA143+x/v*(BA121-subs(v=0,BA121)
)+x/(1-v)*(BA121+BA141+BA131+BB11)) :
BA143:=simplify(x/(1-v)+x*BA243+x*BA144+x/v*(BA131+BA121-subs(v=0,
BA131+BA121))+x/(1-v)*(BA131+BA121+BA141+BB11)) :
BA144:=simplify(x/(1-v)+x*BA244+x/v*(BA141+BA131+BA121-subs(v=0,BA
141+BA131+BA121))+x/(1-v)*(BA141+BA131+BA121+BB11)) :
BA221:=simplify(x/(1-v)+x*BA321+x*BA222+x/(1-v)*(BA221+v*BA241+v*B
A231+BB21+BB22)) :
BA222:=simplify(x/(1-v)+x*BA322+x*BA223+x/(1-v)*(BA231+BA221+v*BA2
41+BB21+BB22)) :
BA223:=simplify(x/(1-v)+x*BA323+x*BA224+x/(1-v)*(BA241+BA231+BA221
+BB21+BB22)) :
BA224:=simplify(x/(1-v)+x*BA324+x/v*(BA221-subs(v=0,BA221))+x/(1-v)
)*(BA221+BA241+BA231+BB21+BB22)) :
BA231:=simplify(x/(1-v)+x*BA331+x*BA232+x/(1-v)*(BA231+BA221+v*BA2
41+BB21+BB22)) :
BA232:=simplify(x/(1-v)+x*BA332+x*BA233+x/(1-v)*(BA241+BA231+BA221
+BB21+BB22)) :
BA233:=simplify(x/(1-v)+x*BA333+x*BA234+x/v*(BA221-subs(v=0,BA221)
)+x/(1-v)*(BA221+BA241+BA231+BB21+BB22)) :
BA234:=simplify(x/(1-v)+x*BA334+x/v*(BA231+BA221-subs(v=0,BA231+BA
221))+x/(1-v)*(BA231+BA221+BA241+BB21+BB22)) :
BA241:=simplify(x/(1-v)+x*BA341+x*BA242+x/(1-v)*(BA241+BA231+BA221
+BB21+BB22)) :
BA242:=simplify(x/(1-v)+x*BA342+x*BA243+x/v*(BA221-subs(v=0,BA221)
)+x/(1-v)*(BA221+BA241+BA231+BB21+BB22)) :
BA243:=simplify(x/(1-v)+x*BA343+x*BA244+x/v*(BA231+BA221-subs(v=0,
BA231+BA221))+x/(1-v)*(BA231+BA221+BA241+BB21+BB22)) :
BA244:=simplify(x/(1-v)+x*BA344+x/v*(BA241+BA231+BA221-subs(v=0,BA
241+BA231+BA221))+x/(1-v)*(BA241+BA231+BA221+BB21+BB22)) :
BA321:=simplify(x/(1-v)+x*BA421+x*BA322+x/(1-v)*(BA321+v*BA341+v*B
A331+BB31+BB32+BB33)) :

```

```

BA322:=simplify(x/(1-v)+x*BA422+x*BA323+x/(1-v)*(BA331+BA321+v*BA3
41+BB31+BB32+BB33)):
BA323:=simplify(x/(1-v)+x*BA423+x*BA324+x/(1-v)*(BA341+BA331+BA321
+BB31+BB32+BB33)):
BA324:=simplify(x/(1-v)+x*BA424+x/v*(BA321-subs(v=0,BA321))+x/(1-v
)*(BA321+BA341+BA331+BB31+BB32+BB33)):
BA331:=simplify(x/(1-v)+x*BA431+x*BA332+x/(1-v)*(BA331+BA321+v*BA3
41+BB31+BB32+BB33)):
BA332:=simplify(x/(1-v)+x*BA432+x*BA333+x/(1-v)*(BA341+BA331+BA321
+BB31+BB32+BB33)):
BA333:=simplify(x/(1-v)+x*BA433+x*BA334+x/v*(BA321-subs(v=0,BA321)
)+x/(1-v)*(BA321+BA341+BA331+BB31+BB32+BB33)):
BA334:=simplify(x/(1-v)+x*BA434+x/v*(BA331+BA321-subs(v=0,BA331+BA
321))+x/(1-v)*(BA331+BA321+BA341+BB31+BB32+BB33)):
BA341:=simplify(x/(1-v)+x*BA441+x*BA342+x/(1-v)*(BA341+BA331+BA321
+BB31+BB32+BB33)):
BA342:=simplify(x/(1-v)+x*BA442+x*BA343+x/v*(BA321-subs(v=0,BA321)
)+x/(1-v)*(BA321+BA341+BA331+BB31+BB32+BB33)):
BA343:=simplify(x/(1-v)+x*BA443+x*BA344+x/v*(BA331+BA321-subs(v=0,
BA331+BA321))+x/(1-v)*(BA331+BA321+BA341+BB31+BB32+BB33)):
BA344:=simplify(x/(1-v)+x*BA444+x/v*(BA341+BA331+BA321-subs(v=0,BA
341+BA331+BA321))+x/(1-v)*(BA341+BA331+BA321+BB31+BB32+BB33)):
BA421:=simplify(x/(1-v)+x*BA422+x/(1-v)*(BA421+v*BA441+v*BA431+BB4
1+BB42+BB43+BB44)):
BA422:=simplify(x/(1-v)+x*BA423+x/(1-v)*(BA431+BA421+v*BA441+BB41+
BB42+BB43+BB44)):
BA423:=simplify(x/(1-v)+x*BA424+x/(1-v)*(BA441+BA431+BA421+BB41+BB
42+BB43+BB44)):
BA424:=simplify(x/(1-v)+x/v*(BA421-subs(v=0,BA421))+x/(1-v)*(BA421
+BA441+BA431+BB41+BB42+BB43+BB44)):
BA431:=simplify(x/(1-v)+x*BA432+x/(1-v)*(BA431+BA421+v*BA441+BB41+
BB42+BB43+BB44)):
BA432:=simplify(x/(1-v)+x*BA433+x/(1-v)*(BA441+BA431+BA421+BB41+BB
42+BB43+BB44)):
BA433:=simplify(x/(1-v)+x*BA434+x/v*(BA421-subs(v=0,BA421))+x/(1-v
)*(BA421+BA441+BA431+BB41+BB42+BB43+BB44)):
BA434:=simplify(x/(1-v)+x/v*(BA431+BA421-subs(v=0,BA431+BA421))+x/
(1-v)*(BA421+BA441+BA431+BB41+BB42+BB43+BB44)):
BA441:=simplify(x/(1-v)+x*BA442+x/(1-v)*(BA441+BA431+BA421+BB41+BB
42+BB43+BB44)):
BA442:=simplify(x/(1-v)+x*BA443+x/v*(BA421-subs(v=0,BA421))+x/(1-v
)*(BA421+BA441+BA431+BB41+BB42+BB43+BB44)):
BA443:=simplify(x/(1-v)+x*BA444+x/v*(BA431+BA421-subs(v=0,BA431+BA

```

```

421)) + x / (1 - v) * (BA431 + BA421 + BA441 + BB41 + BB42 + BB43 + BB44)) :
BA444 := simplify(x / (1 - v) + x / v * (BA441 + BA431 + BA421 - subs(v = 0, BA441 + BA43
1 + BA421)) + x / (1 - v) * (BA441 + BA431 + BA421 + BB41 + BB42 + BB43 + BB44)) : od:

```

```

> taylor(subs(v = 0, BA0), x, 45);

```

```

x + 2 x^2 + 6 x^3 + 22 x^4 + 89 x^5 + 377 x^6 + 1630 x^7 + 7099 x^8 + 30911 x^9 + 134122 x^10 + 579055 x^11
+ 2486122 x^12 + 10613550 x^13 + 45061160 x^14 + 190314295 x^15 + 799860726 x^16 + 3346439323
x^17 + 13941999966 x^18 + 57860268300 x^19 + 239264677555 x^20 + 986139459335 x^21 +
4051986668110 x^22 + 16602214224885 x^23 + 67845590135527 x^24 + 276576825849574 x^25 +
1124920771473566 x^26 + 4565693415516002 x^27 + 18493999452648271 x^28 +
74773795711942922 x^29 + 301795571949692439 x^30 + 1216094059524950197 x^31 +
4892766241235145695 x^32 + 19656826517742615155 x^33 + 78864124509747661270 x^34 +
315998774926173592790 x^35 + 1264623613587434277900 x^36 + 5055161090667858845315 x^37
+ 20185165287741374949425 x^38 + 80515057805155850159600 x^39 +
320842269162414713346825 x^40 + 1277310107879055414071239 x^41 +
5080548839456028658413392 x^42 + 20190745014057430759450524 x^43 +
80174863254254118244596305 x^44 + O(x^45)

```

```

> # 021,00000 1, 2, 6, 22, 89, 377, 1630, 7099, 30911, 134122,
579055, 2486122

```

```

> taylor(BB43, x, 20); ##BB44 =
1 + x*BB44 + x^2*BB44^2 + x^3*BB44^3 + x^4*BB44^4

```

```

x + 3 x^2 + 9 x^3 + 28 x^4 + 89 x^5 + 288 x^6 + 947 x^7 + 3157 x^8 + 10647 x^9 + 36263 x^10 + 124564 x^11 +
431055 x^12 + 1501370 x^13 + 5259307 x^14 + 18517374 x^15 + 65494898 x^16 + 232602483 x^17 +
829143585 x^18 + 2965558475 x^19 + O(x^20)

```

```

>

```

```

> #-----

```

```

> restart: gc(): #equations

```

```

> eq121 := -A121(v) + (x / (1 - v) + x * A221(v) + x * A122(v) + x / (1 - v) * (A121(v) + v * A1
41(v) + v * A131(v) + B11)) :
eq122 := -A122(v) + (x / (1 - v) + x * A222(v) + x * A123(v) + x / (1 - v) * (A131(v) + A121
(v) + v * A141(v) + B11)) :
eq123 := -A123(v) + (x / (1 - v) + x * A223(v) + x * A124(v) + x / (1 - v) * (A141(v) + A131
(v) + A121(v) + B11)) :
eq124 := -A124(v) + (x / (1 - v) + x * A224(v) + x / v * (A121(v) - A121(0)) + x / (1 - v) * (
A121(v) + A141(v) + A131(v) + B11)) :
eq131 := -A131(v) + (x / (1 - v) + x * A231(v) + x * A132(v) + x / (1 - v) * (A131(v) + A121
(v) + v * A141(v) + B11)) :
eq132 := -A132(v) + (x / (1 - v) + x * A232(v) + x * A133(v) + x / (1 - v) * (A141(v) + A131
(v) + A121(v) + B11)) :

```

```

eq133:=-A133(v)+(x/(1-v)+x*A233(v)+x*A134(v)+x/v*(A121(v)-A121(0))
+x/(1-v)*(A121(v)+A141(v)+A131(v)+B11)):
eq134:=-A134(v)+(x/(1-v)+x*A234(v)+x/v*(A131(v)+A121(v)-A131(0)-A1
21(0))+x/(1-v)*(A131(v)+A121(v)+A141(v)+B11)):
eq141:=-A141(v)+(x/(1-v)+x*A241(v)+x*A142(v)+x/(1-v)*(A141(v)+A131
(v)+A121(v)+B11)):
eq142:=-A142(v)+(x/(1-v)+x*A242(v)+x*A143(v)+x/v*(A121(v)-A121(0))
+x/(1-v)*(A121(v)+A141(v)+A131(v)+B11)):
eq143:=-A143(v)+(x/(1-v)+x*A243(v)+x*A144(v)+x/v*(A131(v)+A121(v)-
A131(0)-A121(0))+x/(1-v)*(A131(v)+A121(v)+A141(v)+B11)):
eq144:=-A144(v)+(x/(1-v)+x*A244(v)+x/v*(A141(v)+A131(v)+A121(v)-A1
41(0)-A131(0)-A121(0))+x/(1-v)*(A141(v)+A131(v)+A121(v)+B11)):
eq221:=-A221(v)+(x/(1-v)+x*A321(v)+x*A222(v)+x/(1-v)*(A221(v)+v*A2
41(v)+v*A231(v)+B21+B22)):
eq222:=-A222(v)+(x/(1-v)+x*A322(v)+x*A223(v)+x/(1-v)*(A231(v)+A221
(v)+v*A241(v)+B21+B22)):
eq223:=-A223(v)+(x/(1-v)+x*A323(v)+x*A224(v)+x/(1-v)*(A241(v)+A231
(v)+A221(v)+B21+B22)):
eq224:=-A224(v)+(x/(1-v)+x*A324(v)+x/v*(A221(v)-A221(0))+x/(1-v)*(
A221(v)+A241(v)+A231(v)+B21+B22)):
eq231:=-A231(v)+(x/(1-v)+x*A331(v)+x*A232(v)+x/(1-v)*(A231(v)+A221
(v)+v*A241(v)+B21+B22)):
eq232:=-A232(v)+(x/(1-v)+x*A332(v)+x*A233(v)+x/(1-v)*(A241(v)+A231
(v)+A221(v)+B21+B22)):
eq233:=-A233(v)+(x/(1-v)+x*A333(v)+x*A234(v)+x/v*(A221(v)-A221(0))
+x/(1-v)*(A221(v)+A241(v)+A231(v)+B21+B22)):
eq234:=-A234(v)+(x/(1-v)+x*A334(v)+x/v*(A231(v)+A221(v)-A231(0)-A2
21(0))+x/(1-v)*(A231(v)+A221(v)+A241(v)+B21+B22)):
eq241:=-A241(v)+(x/(1-v)+x*A341(v)+x*A242(v)+x/(1-v)*(A241(v)+A231
(v)+A221(v)+B21+B22)):
eq242:=-A242(v)+(x/(1-v)+x*A342(v)+x*A243(v)+x/v*(A221(v)-A221(0))
+x/(1-v)*(A221(v)+A241(v)+A231(v)+B21+B22)):
eq243:=-A243(v)+(x/(1-v)+x*A343(v)+x*A244(v)+x/v*(A231(v)+A221(v)-
A231(0)-A221(0))+x/(1-v)*(A231(v)+A221(v)+A241(v)+B21+B22)):
eq244:=-A244(v)+(x/(1-v)+x*A344(v)+x/v*(A241(v)+A231(v)+A221(v)-A2
41(0)-A231(0)-A221(0))+x/(1-v)*(A241(v)+A231(v)+A221(v)+B21+B22)):
eq321:=-A321(v)+(x/(1-v)+x*A421(v)+x*A322(v)+x/(1-v)*(A321(v)+v*A3
41(v)+v*A331(v)+B31+B32+B33)):
eq322:=-A322(v)+(x/(1-v)+x*A422(v)+x*A323(v)+x/(1-v)*(A331(v)+A321
(v)+v*A341(v)+B31+B32+B33)):
eq323:=-A323(v)+(x/(1-v)+x*A423(v)+x*A324(v)+x/(1-v)*(A341(v)+A331
(v)+A321(v)+B31+B32+B33)):
eq324:=-A324(v)+(x/(1-v)+x*A424(v)+x/v*(A321(v)-A321(0))+x/(1-v)*(

```

$A321(v) + A341(v) + A331(v) + B31 + B32 + B33) :$
 $eq331 := -A331(v) + (x/(1-v) + x*A431(v) + x*A332(v) + x/(1-v) * (A331(v) + A321(v) + v*A341(v) + B31 + B32 + B33)) :$
 $eq332 := -A332(v) + (x/(1-v) + x*A432(v) + x*A333(v) + x/(1-v) * (A341(v) + A331(v) + A321(v) + B31 + B32 + B33)) :$
 $eq333 := -A333(v) + (x/(1-v) + x*A433(v) + x*A334(v) + x/v * (A321(v) - A321(0)) + x/(1-v) * (A321(v) + A341(v) + A331(v) + B31 + B32 + B33)) :$
 $eq334 := -A334(v) + (x/(1-v) + x*A434(v) + x/v * (A331(v) + A321(v) - A331(0) - A321(0)) + x/(1-v) * (A331(v) + A321(v) + A341(v) + B31 + B32 + B33)) :$
 $eq341 := -A341(v) + (x/(1-v) + x*A441(v) + x*A342(v) + x/(1-v) * (A341(v) + A331(v) + A321(v) + B31 + B32 + B33)) :$
 $eq342 := -A342(v) + (x/(1-v) + x*A442(v) + x*A343(v) + x/v * (A321(v) - A321(0)) + x/(1-v) * (A321(v) + A341(v) + A331(v) + B31 + B32 + B33)) :$
 $eq343 := -A343(v) + (x/(1-v) + x*A443(v) + x*A344(v) + x/v * (A331(v) + A321(v) - A331(0) - A321(0)) + x/(1-v) * (A331(v) + A321(v) + A341(v) + B31 + B32 + B33)) :$
 $eq344 := -A344(v) + (x/(1-v) + x*A444(v) + x/v * (A341(v) + A331(v) + A321(v) - A341(0) - A331(0) - A321(0)) + x/(1-v) * (A341(v) + A331(v) + A321(v) + B31 + B32 + B33)) :$
 $eq421 := -A421(v) + (x/(1-v) + x*A422(v) + x/(1-v) * (A421(v) + v*A441(v) + v*A431(v) + B41 + B42 + B43 + B44)) :$
 $eq422 := -A422(v) + (x/(1-v) + x*A423(v) + x/(1-v) * (A431(v) + A421(v) + v*A441(v) + B41 + B42 + B43 + B44)) :$
 $eq423 := -A423(v) + (x/(1-v) + x*A424(v) + x/(1-v) * (A441(v) + A431(v) + A421(v) + B41 + B42 + B43 + B44)) :$
 $eq424 := -A424(v) + (x/(1-v) + x/v * (A421(v) - A421(0)) + x/(1-v) * (A421(v) + A441(v) + A431(v) + B41 + B42 + B43 + B44)) :$
 $eq431 := -A431(v) + (x/(1-v) + x*A432(v) + x/(1-v) * (A431(v) + A421(v) + v*A441(v) + B41 + B42 + B43 + B44)) :$
 $eq432 := -A432(v) + (x/(1-v) + x*A433(v) + x/(1-v) * (A441(v) + A431(v) + A421(v) + B41 + B42 + B43 + B44)) :$
 $eq433 := -A433(v) + (x/(1-v) + x*A434(v) + x/v * (A421(v) - A421(0)) + x/(1-v) * (A421(v) + A441(v) + A431(v) + B41 + B42 + B43 + B44)) :$
 $eq434 := -A434(v) + (x/(1-v) + x/v * (A431(v) + A421(v) - A431(0) - A421(0)) + x/(1-v) * (A421(v) + A441(v) + A431(v) + B41 + B42 + B43 + B44)) :$
 $eq441 := -A441(v) + (x/(1-v) + x*A442(v) + x/(1-v) * (A441(v) + A431(v) + A421(v) + B41 + B42 + B43 + B44)) :$
 $eq442 := -A442(v) + (x/(1-v) + x*A443(v) + x/v * (A421(v) - A421(0)) + x/(1-v) * (A421(v) + A441(v) + A431(v) + B41 + B42 + B43 + B44)) :$
 $eq443 := -A443(v) + (x/(1-v) + x*A444(v) + x/v * (A431(v) + A421(v) - A431(0) - A421(0)) + x/(1-v) * (A431(v) + A421(v) + A441(v) + B41 + B42 + B43 + B44)) :$
 $eq444 := -A444(v) + (x/(1-v) + x/v * (A441(v) + A431(v) + A421(v) - A441(0) - A431(0) - A421(0)) + x/(1-v) * (A441(v) + A431(v) + A421(v) + B41 + B42 + B43 + B44)) :$

```

> ss4:=solve({eq421,eq422,eq423,eq424,eq431,eq432,eq433,eq434,eq441,
eq442,eq443,eq444},{A421(v),A422(v),A423(v),A424(v),A431(v),A432(v)
),A433(v),A434(v),A441(v),A442(v),A443(v),A444(v)}):
> eq:=(-op(ss4[1])[1]+op(ss4[1])[2])*(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v
^2*x^5+v^3*x^3+3*v^2*x^4+v^4+3*v^3*x-v^3);
eq:=( -A421(v)+x(-A421(0)v x^11-A431(0)v x^11-A441(0)v x^11-A421(0)v^2 x^8
-A431(0)v^2 x^8-A441(0)v^2 x^8-A441(0)v x^10-A421(0)v x^10-A431(0)v x^10
-A421(0)v x^9-A431(0)v x^9+A421(0)v^2 x^7+A431(0)v^2 x^7+A441(0)v^2 x^7
+2 A421(0)v^3 x^4+2 A431(0)v^3 x^4+A441(0)v^3 x^4+A421(0)x^11-v x^11+v^2 x^7-v x^10
-2 A421(0)v x^7+2 A421(0)v^2 x^5+2 A431(0)v^2 x^5+A421(0)v^3 x^2-B41 v x^11+B41 v^2 x^7
-B42 v x^11+B42 v^2 x^7-B43 v x^11+B43 v^2 x^7-B44 v x^11+B44 v^2 x^7-B41 v^2 x^8+2 B41 v^3 x^4
-B42 v^2 x^8+2 B42 v^3 x^4-B43 v^2 x^8+2 B43 v^3 x^4-B44 v^2 x^8+2 B44 v^3 x^4-B41 v x^10
-B42 v x^10-B43 v x^10-B44 v x^10+A421(0)v^2 x^6+2 A431(0)v^2 x^6+A441(0)v^2 x^6
+A421(0)v^3 x^3+A431(0)v^3 x^3-B41 v x^9-B42 v x^9-B43 v x^9-B44 v x^9-B41 v x^8
-B42 v x^8-B43 v x^8-B44 v x^8+B41 v^2 x^6+B42 v^2 x^6+B43 v^2 x^6+B44 v^2 x^6+B41 v^2 x^5
+B42 v^2 x^5+B43 v^2 x^5+B44 v^2 x^5+B41 v^3 x^3+2 B41 v^2 x^4+B42 v^3 x^3+2 B42 v^2 x^4+B43 v^3 x^3
+2 B43 v^2 x^4+B44 v^3 x^3+2 B44 v^2 x^4+B41 v^3 x^2+B42 v^3 x^2+B43 v^3 x^2+B44 v^3 x^2+B41 v^3 x
+B42 v^3 x+B43 v^3 x+B44 v^3 x+v^3 x^2-v^2 x^8-v x^9-v x^8+v^2 x^6+2 v^3 x^4+v^2 x^5+v^3 x^3
+2 v^2 x^4+v^3 x+A421(0)v^2 x^3-v^3-B41 v^3-B42 v^3-B43 v^3-B44 v^3)/((
x^12+v x^9-3 v x^8+v^2 x^6-3 v^2 x^5+v^3 x^3+3 v^2 x^4+v^4+3 v^3 x-v^3))
(x^12+v x^9-3 v x^8+v^2 x^6-3 v^2 x^5+v^3 x^3+3 v^2 x^4+v^4+3 v^3 x-v^3)
> taylor(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4+
3*v^3*x-v^3,v,10); mp4:=x^12: mp3:=3*x^8-x^9:
mp2:=x^6-3*x^5+3*x^4: mp1:=1-x^3-3*x:
alias(vv=RootOf(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2
*x^4+v^4+3*v^3*x-v^3=0,v)):
x^12+(x^9-3 x^8)v+(x^6-3 x^5+3 x^4)v^2+(x^3+3 x-1)v^3+v^4
> ##u1u2u3=x^12/u4: u1+u2+u3=1-3*x-x^3-u4:
u1u2+u1u3+u2u3=x^6-3*x^5+3*x^4-(1-3*x-x^3-u4)*u4:
>
> ss40:=map(factor,solve({subs(v=u1,subs(A421(v)=0,eq)),subs(v=u2,su
bs(A421(v)=0,eq)),subs(v=u3,subs(A421(v)=0,eq))},{A441(0),A431(0),
A421(0)})):
> #ss40[3];
> A421x0:=factor(simplify(-x^12/vv*(x+1)*(x^2+1)*(x^3+x^2-1)*(B41+B4
2+B43+B44+1)/x/(2*x^12+(1-3*x-x^3-vv)*(2*x^9+x^10)+2*x^11+x^10+(x^
6-3*x^5+3*x^4-(1-3*x-x^3-vv)*vv)*(x^4-x^5+x^7)+x^12/vv*x^5+2*x^12/

```



```

vv*x^4+x^12/vv*x^2+x^12/vv*x-2*x^12/vv) ) ) ;
##series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,A421x0)-subs(v=0
,BA421),x,19);

```

$$A421x0 := (B41 + B42 + B43 + B44 + 1) (-x^{13} - x^{12} - 2x^{11} + 3x^{10} - 4x^8 vv + x^9 + 3x^7 vv - x^8 - 2vv^2 x^5 - x^6 vv + vv^3 x^3 + 3vv^2 x^4 - 2x^5 vv - x^2 vv^3 - 3x^3 vv^2 + 2vvx^4 - vv^3 x - 2vv^2 x^2 + vv^3 + 4vv^2 x - vv^2) / (x^{10} (x^3 + x^2 - 1))$$

```

> A431x0:=factor(simplify((x+1)*(x^2+1)*(-x^12-(1-3*x-x^3-vv)*(x^10+
x^9-x^8)-(x^6-3*x^5+3*x^4-(1-3*x-x^3-vv)*vv)*(x^7-x^6-x^5+x^4)+2*x
^12/vv*x^3-2*x^12/vv*x^2-x^12/vv*x+x^12/vv)*(B41+B42+B43+B44+1)/x^
3/(2*x^12+(1-3*x-x^3-vv)*(x^10+2*x^9)+2*x^11+x^10+(x^6-3*x^5+3*x^4
-(1-3*x-x^3-vv)*vv)*(x^7-x^5+x^4)+x^12/vv*(x^5+2*x^4+x^2+x-2)))));
##series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,A431x0)-subs(v=0
,BA431),x,21);

```

$$A431x0 := -(B41 + B42 + B43 + B44 + 1) (-2x^9 + 3x^8 - 5x^6 vv + 6x^5 vv - 2x^3 vv^2 - 5vvx^4 + vv^3 x + 3vv^2 x^2 - 2vv^3 - 7vv^2 x + 2vv^2) / x^{11}$$

```

> A441x0:=factor(simplify(-(x+1)*(x^2+1)*(x^13-(1-3*x-x^3-vv)*(x^9-x
^8)+(x^6-3*x^5+3*x^4-(1-3*x-x^3-vv)*vv)*(x^6+2*x^5-2*x^4)+x^12/vv*
x^3-2*x^12/vv*x^2-3*x^12/vv*x+3*x^12/vv)*(B41+B42+B43+B44+1)/(2*x^
12+(1-3*x-x^3-vv)*(x^10+2*x^9)+2*x^11+x^10+(x^6-3*x^5+3*x^4-(1-3*x
-x^3-vv)*vv)*(x^7-x^5+x^4)+x^12/vv*(x^5+2*x^4+x^2+x-2))/x^4));
##series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,A441x0)-subs(v=0
,BA441),x,21);

```

$$A441x0 := -(B41 + B42 + B43 + B44 + 1) (2x^{12} + x^{10} vv - 4x^{11} + 3vvx^9 - 6x^{10} - 4x^8 vv + 2x^9 + 3vv^2 x^6 - 2x^7 vv + 3x^8 + 2vv^2 x^5 + 10x^6 vv + 2vv^3 x^3 + 4vv^2 x^4 - 2x^5 vv + 3x^2 vv^3 + 6x^3 vv^2 - 3vvx^4 - 2vv^3 x - 9vv^2 x^2 - vv^3 - vv^2 x + vv^2) / (x^{12} (x^3 + x^2 - 1))$$

```

> A421x0:=subs(vv=u4,A421x0):A431x0:=subs(vv=u4,A431x0):A441x0:=subs
(vv=u4,A441x0):

```

```

> op(ss4[1])[1];
A421xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[1])[2])))):

```

A421(v)

```

> op(ss4[2])[1];
A422xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[2])[2])))):

```

A422(v)

```

> op(ss4[3])[1];
A423xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[3])[2])))):

```

```

[
A423(v)
> op(ss4[4])[1];
A424xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[4])[2])))):
A424(v)
> op(ss4[5])[1];
A431xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[5])[2])))):
A431(v)
> op(ss4[6])[1];
A432xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[6])[2])))):
A432(v)
> op(ss4[7])[1];
A433xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[7])[2])))):
A433(v)
> op(ss4[8])[1];
A434xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[8])[2])))):
A434(v)
> op(ss4[9])[1];
A441xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[9])[2])))):
A441(v)
> op(ss4[10])[1];
A442xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[10])[2])))):
A442(v)
> op(ss4[11])[1];
A443xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[11])[2])))):
A443(v)
> op(ss4[12])[1];
A444xv:=factor(simplify(subs(vv=u4,subs(A421(0)=A421x0,A431(0)=A43
1x0,A441(0)=A441x0,op(ss4[12])[2])))):
A444(v)
> save(A421x0,A431x0,A441x0,A421xv,A422xv,A423xv,A424xv,A431xv,A432x
v,A433xv,A434xv,A441xv,A442xv,A443xv,A444xv,"C:/DriveData_Mansour2
=201805/Toufik_data/Projects_2022/InvSeq3and4/avoid021x00000/A021x
00000Step1.txt");

```

```

> #simplify(taylor(subs(u4=vv,B41=BB41,B42=BB42,B43=BB43,B44=BB44,A4
41xv)-BA441,x,20));
>
> #----step 2
> read
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/A021x00000Step1.txt":
> alias(vv=RootOf(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2
*x^4+v^4+3*v^3*x-v^3=0,v)):
> feq321:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(
v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,eq321): feq322:=subs(A421(0
)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422
xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A43
3(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A
443xv,A444(v)=A444xv,eq322): feq323:=subs(A421(0)=A421x0,A431(0)=A4
31x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A
424(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)
=A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444x
v,eq323): feq324:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0
,A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(
v)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A44
1xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,eq324): feq331:=su
bs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A42
2(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A
432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,
A443(v)=A443xv,A444(v)=A444xv,eq331): feq332:=subs(A421(0)=A421x0,A
431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422xv,A423(v)
=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A433(v)=A433x
v,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A443xv,A444
(v)=A444xv,eq332): feq333:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(
0)=A441x0,A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A42
4xv,A431(v)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A4
41(v)=A441xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,eq333): f
eq334:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A
421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,
A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)
=A442xv,A443(v)=A443xv,A444(v)=A444xv,eq334): feq341:=subs(A421(0)
=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422x
v,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A433
(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A4
43xv,A444(v)=A444xv,eq341): feq342:=subs(A421(0)=A421x0,A431(0)=A43

```

```

1x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A4
24(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)=
A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444xv
,eq342):feq343:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,
A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)
)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441
xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,eq343):feq344:=sub
s(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A422
(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A4
32xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,A
443(v)=A443xv,A444(v)=A444xv,eq344):

```

- > ##feq321:=subs(u4=vv,feq321):feq322:=subs(u4=vv,feq322):feq323:=su
bs(u4=vv,feq323):feq324:=subs(u4=vv,feq324):
feq331:=subs(u4=vv,feq331):feq332:=subs(u4=vv,feq332):feq333:=subs
(u4=vv,feq333):feq334:=subs(u4=vv,feq334):
feq341:=subs(u4=vv,feq341):feq342:=subs(u4=vv,feq342):feq343:=subs
(u4=vv,feq343):feq344:=subs(u4=vv,feq344):
- > ss3:=solve({feq321,feq322,feq323,feq324,feq331,feq332,feq333,feq33
4,feq341,feq342,feq343,feq344},{A321(v),A322(v),A323(v),A324(v),A3
31(v),A332(v),A333(v),A334(v),A341(v),A342(v),A343(v),A344(v)}):
- > op(ss3[1])[1];eq:=simplify(diff(subs(A321(v)=0,factor(op(ss3[1])[2
]*(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4+3*v^
3*x-v^3)^2)),v)):

A321(v)

- > eqc1:=factor(coeff(eq,A321(0),1)):
eqc2:=factor(coeff(eq,A331(0),1)):
eqc3:=factor(coeff(eq,A341(0),1)):
eqc4:=factor(simplify(coeff(coeff(coeff(eq,A321(0),0),A331(0),0),A
341(0),0))):
 - > map(factor,solve({c11*v1+c12*v2+c13*v3+c14=0,c21*v1+c22*v2+c23*v3+
c24=0,c31*v1+c32*v2+c33*v3+c34=0},{v1,v2,v3}));
- $$\left\{ \begin{aligned} v1 &= -\frac{c12\,c23\,c34 - c12\,c24\,c33 - c13\,c22\,c34 + c13\,c24\,c32 + c14\,c22\,c33 - c14\,c23\,c32}{c11\,c22\,c33 - c11\,c23\,c32 - c12\,c21\,c33 + c12\,c23\,c31 + c13\,c21\,c32 - c13\,c22\,c31}, \\ v2 &= \frac{c11\,c23\,c34 - c11\,c24\,c33 - c13\,c21\,c34 + c13\,c24\,c31 + c14\,c21\,c33 - c14\,c23\,c31}{c11\,c22\,c33 - c11\,c23\,c32 - c12\,c21\,c33 + c12\,c23\,c31 + c13\,c21\,c32 - c13\,c22\,c31}, \\ v3 &= -\frac{c11\,c22\,c34 - c11\,c24\,c32 - c12\,c21\,c34 + c12\,c24\,c31 + c14\,c21\,c32 - c14\,c22\,c31}{c11\,c22\,c33 - c11\,c23\,c32 - c12\,c21\,c33 + c12\,c23\,c31 + c13\,c21\,c32 - c13\,c22\,c31} \end{aligned} \right\}$$
- > vv1:=- (c12*c23*c34-c12*c24*c33-c13*c22*c34+c13*c24*c32+c14*c22*c33
-c14*c23*c32)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13
*c21*c32-c13*c22*c31):
vv2:=(c11*c23*c34-c11*c24*c33-c13*c21*c34+c13*c24*c31+c14*c21*c33-

```
c14*c23*c31)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13*
c21*c32-c13*c22*c31):
```

```
vv3:=(c11*c22*c34-c11*c24*c32-c12*c21*c34+c12*c24*c31+c14*c21*c32
-c14*c22*c31)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13
*c21*c32-c13*c22*c31):
```

```
> ww1:=simplify(subs(c11=subs(v=u1,eqc1),c12=subs(v=u1,eqc2),c13=sub
s(v=u1,eqc3),c14=subs(v=u1,eqc4),c21=subs(v=u2,eqc1),c22=subs(v=u2
,eqc2),c23=subs(v=u2,eqc3),c24=subs(v=u2,eqc4),c31=subs(v=u3,eqc1)
,c32=subs(v=u3,eqc2),c33=subs(v=u3,eqc3),c34=subs(v=u3,eqc4),vv1))
:
```

```
A321x0:=factor(simplify(subs(u4=vv,op(solve({f=ww1,u1*u2*u3=x^12/u
4,u1*u2+u1*u3+u2*u3=x^6-3*x^5+3*x^4-(1-3*x-x^3-u4)*u4,u1+u2+u3=1-3
*x-x^3-u4},{f,u1,u2,u3})[1])[2]))):
```

```
> ww2:=simplify(subs(c11=subs(v=u1,eqc1),c12=subs(v=u1,eqc2),c13=sub
s(v=u1,eqc3),c14=subs(v=u1,eqc4),c21=subs(v=u2,eqc1),c22=subs(v=u2
,eqc2),c23=subs(v=u2,eqc3),c24=subs(v=u2,eqc4),c31=subs(v=u3,eqc1)
,c32=subs(v=u3,eqc2),c33=subs(v=u3,eqc3),c34=subs(v=u3,eqc4),vv2))
:
```

```
A331x0:=factor(simplify(subs(u4=vv,op(solve({f=ww2,u1*u2*u3=x^12/u
4,u1*u2+u1*u3+u2*u3=x^6-3*x^5+3*x^4-(1-3*x-x^3-u4)*u4,u1+u2+u3=1-3
*x-x^3-u4},{f,u1,u2,u3})[1])[2]))):
```

```
> ww3:=simplify(subs(c11=subs(v=u1,eqc1),c12=subs(v=u1,eqc2),c13=sub
s(v=u1,eqc3),c14=subs(v=u1,eqc4),c21=subs(v=u2,eqc1),c22=subs(v=u2
,eqc2),c23=subs(v=u2,eqc3),c24=subs(v=u2,eqc4),c31=subs(v=u3,eqc1)
,c32=subs(v=u3,eqc2),c33=subs(v=u3,eqc3),c34=subs(v=u3,eqc4),vv3))
:
```

```
A341x0:=factor(simplify(subs(u4=vv,op(solve({f=ww3,u1*u2*u3=x^12/u
4,u1*u2+u1*u3+u2*u3=x^6-3*x^5+3*x^4-(1-3*x-x^3-u4)*u4,u1+u2+u3=1-3
*x-x^3-u4},{f,u1,u2,u3})[1])[2]))):
```

```
> #series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31=BB31,B32=BB32
,B33=BB33,A341x0)-subs(v=0,BA341),x,28);
```

$O(x^{16})$

```
> A321x0:=subs(vv=u4,A321x0):A331x0:=subs(vv=u4,A331x0):A341x0:=subs
(vv=u4,A341x0):
```

```
> op(ss3[1])[1];
```

```
A321xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[1])[2])))):
```

$A321(v)$

```
> op(ss3[2])[1];
```

```
A322xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[2])[2])))):
```

$A322(v)$

```

> op(ss3[3])[1];
A323xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[3])[2])))):
A323(v)
> op(ss3[4])[1];
A324xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[4])[2])))):
A324(v)
> op(ss3[5])[1];
A331xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[5])[2])))):
A331(v)
> op(ss3[6])[1];
A332xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[6])[2])))):
A332(v)
> op(ss3[7])[1];
A333xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[7])[2])))):
A333(v)
> op(ss3[8])[1];
A334xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[8])[2])))):
A334(v)
> op(ss3[9])[1];
A341xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[9])[2])))):
A341(v)
> op(ss3[10])[1];
A342xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[10])[2])))):
A342(v)
> op(ss3[11])[1];
A343xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[11])[2])))):
A343(v)
> op(ss3[12])[1];
A344xv:=factor(simplify(subs(vv=u4,subs(A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,op(ss3[12])[2])))):
A344(v)
> save(A421x0,A431x0,A441x0,A421xv,A422xv,A423xv,A424xv,A431xv,A432x

```

```
v,A433xv,A434xv,A441xv,A442xv,A443xv,A444xv,
A321x0,A331x0,A341x0,A321xv,A322xv,A323xv,A324xv,A331xv,A332xv,A33
3xv,A334xv,A341xv,A342xv,A343xv,A344xv,"C:/DriveData_Mansour2=2018
05/Toufik_data/Projects_2022/InvSeq3and4/avoid021x00000/A021x00000
Step2.txt");
```

```
[ >
```

```
[ > #-----step 3-----
```

```
[ > read
```

```
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/A021x00000Step2.txt":
```

```
[ > alias (vv=RootOf (x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2
*x^4+v^4+3*v^3*x-v^3=0,v) ) :
```

```
[ > feq221:=subs (A421 (0)=A421x0,A431 (0)=A431x0,A441 (0)=A441x0,A421 (v)=
A421xv,A422 (v)=A422xv,A423 (v)=A423xv,A424 (v)=A424xv,A431 (v)=A431xv
,A432 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (
v)=A442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A33
1x0,A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A3
24 (v)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=
A334xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv
,eq221) :
```

```
feq222:=subs (A421 (0)=A421x0,A431 (0)=A431x0,A441 (0)=A441x0,A421 (v)=
A421xv,A422 (v)=A422xv,A423 (v)=A423xv,A424 (v)=A424xv,A431 (v)=A431xv
,A432 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (
v)=A442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A33
1x0,A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A3
24 (v)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=
A334xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv
,eq222) :
```

```
feq223:=subs (A421 (0)=A421x0,A431 (0)=A431x0,A441 (0)=A441x0,A421 (v)=
A421xv,A422 (v)=A422xv,A423 (v)=A423xv,A424 (v)=A424xv,A431 (v)=A431xv
,A432 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (
v)=A442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A33
1x0,A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A3
24 (v)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=
A334xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv
,eq223) :
```

```
feq224:=subs (A421 (0)=A421x0,A431 (0)=A431x0,A441 (0)=A441x0,A421 (v)=
A421xv,A422 (v)=A422xv,A423 (v)=A423xv,A424 (v)=A424xv,A431 (v)=A431xv
,A432 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (
v)=A442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A33
1x0,A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A3
24 (v)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=
A334xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv
```

```
feq231:=subs (A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(
v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,A321(v)=A321xv,A322(v)=A322xv,A323(v)=A323xv,A3
24(v)=A324xv,A331(v)=A331xv,A332(v)=A332xv,A333(v)=A333xv,A334(v)=
A334xv,A341(v)=A341xv,A342(v)=A342xv,A343(v)=A343xv,A344(v)=A344xv
,eq231):
```

```

feq232:=subs (A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(
v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,A321(v)=A321xv,A322(v)=A322xv,A323(v)=A323xv,A3
24(v)=A324xv,A331(v)=A331xv,A332(v)=A332xv,A333(v)=A333xv,A334(v)=
A334xv,A341(v)=A341xv,A342(v)=A342xv,A343(v)=A343xv,A344(v)=A344xv
,eq221):

```

```

feq221:=subs (A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(
v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,A321(v)=A321xv,A322(v)=A322xv,A323(v)=A323xv,A3
24(v)=A324xv,A331(v)=A331xv,A332(v)=A332xv,A333(v)=A333xv,A334(v)=
A334xv,A341(v)=A341xv,A342(v)=A342xv,A343(v)=A343xv,A344(v)=A344xv
,eq232):

```

```

feq233:=subs (A421 (0)=A421x0,A431 (0)=A431x0,A441 (0)=A441x0,A421 (v)=
A421xv,A422 (v)=A422xv,A423 (v)=A423xv,A424 (v)=A424xv,A431 (v)=A431xv
,A432 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (
v)=A442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A33
1x0,A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A3
24 (v)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=
A334xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv
,eq233) :

```

```

feq234:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(
v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,A321(0)=A321x0,A331(0)=A33
1x0,A341(0)=A341x0,A321(v)=A321xv,A322(v)=A322xv,A323(v)=A323xv,A3
24(v)=A324xv,A331(v)=A331xv,A332(v)=A332xv,A333(v)=A333xv,A334(v)=
A334xv,A341(v)=A341xv,A342(v)=A342xv,A343(v)=A343xv,A344(v)=A344xv
,eq234):

```

```
feq241:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=
A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv
```



```
, A432 (v) = A432xv, A433 (v) = A433xv, A434 (v) = A434xv, A441 (v) = A441xv, A442 (v) = A442xv, A443 (v) = A443xv, A444 (v) = A444xv, A321 (0) = A321x0, A331 (0) = A331x0, A341 (0) = A341x0, A321 (v) = A321xv, A322 (v) = A322xv, A323 (v) = A323xv, A324 (v) = A324xv, A331 (v) = A331xv, A332 (v) = A332xv, A333 (v) = A333xv, A334 (v) = A334xv, A341 (v) = A341xv, A342 (v) = A342xv, A343 (v) = A343xv, A344 (v) = A344xv, eq241) :
```

```
feq242:=subs (A421 (0)=A421x0, A431 (0)=A431x0, A441 (0)=A441x0, A421 (v)=A421xv, A422 (v)=A422xv, A423 (v)=A423xv, A424 (v)=A424xv, A431 (v)=A431xv, A432 (v)=A432xv, A433 (v)=A433xv, A434 (v)=A434xv, A441 (v)=A441xv, A442 (v)=A442xv, A443 (v)=A443xv, A444 (v)=A444xv, A321 (0)=A321x0, A331 (0)=A331x0, A341 (0)=A341x0, A321 (v)=A321xv, A322 (v)=A322xv, A323 (v)=A323xv, A324 (v)=A324xv, A331 (v)=A331xv, A332 (v)=A332xv, A333 (v)=A333xv, A334 (v)=A334xv, A341 (v)=A341xv, A342 (v)=A342xv, A343 (v)=A343xv, A344 (v)=A344xv, eq242) :
```

```
feq243:=subs (A421 (0)=A421x0, A431 (0)=A431x0, A441 (0)=A441x0, A421 (v)=A421xv, A422 (v)=A422xv, A423 (v)=A423xv, A424 (v)=A424xv, A431 (v)=A431xv, A432 (v)=A432xv, A433 (v)=A433xv, A434 (v)=A434xv, A441 (v)=A441xv, A442 (v)=A442xv, A443 (v)=A443xv, A444 (v)=A444xv, A321 (0)=A321x0, A331 (0)=A331x0, A341 (0)=A341x0, A321 (v)=A321xv, A322 (v)=A322xv, A323 (v)=A323xv, A324 (v)=A324xv, A331 (v)=A331xv, A332 (v)=A332xv, A333 (v)=A333xv, A334 (v)=A334xv, A341 (v)=A341xv, A342 (v)=A342xv, A343 (v)=A343xv, A344 (v)=A344xv, eq243) :
```

```
feq244:=subs (A421 (0)=A421x0, A431 (0)=A431x0, A441 (0)=A441x0, A421 (v)=A421xv, A422 (v)=A422xv, A423 (v)=A423xv, A424 (v)=A424xv, A431 (v)=A431xv, A432 (v)=A432xv, A433 (v)=A433xv, A434 (v)=A434xv, A441 (v)=A441xv, A442 (v)=A442xv, A443 (v)=A443xv, A444 (v)=A444xv, A321 (0)=A321x0, A331 (0)=A331x0, A341 (0)=A341x0, A321 (v)=A321xv, A322 (v)=A322xv, A323 (v)=A323xv, A324 (v)=A324xv, A331 (v)=A331xv, A332 (v)=A332xv, A333 (v)=A333xv, A334 (v)=A334xv, A341 (v)=A341xv, A342 (v)=A342xv, A343 (v)=A343xv, A344 (v)=A344xv, eq244) :
```

```
> ss2:=map (factor, solve ({ feq221, feq222, feq223, feq224, feq231, feq232, feq233, feq234, feq241, feq242, feq243, feq244 }, { A221 (v), A222 (v), A223 (v), A224 (v), A231 (v), A232 (v), A233 (v), A234 (v), A241 (v), A242 (v), A243 (v), A244 (v) } ) ) :
```

```
> ##take care for A221(0) :
```

```
> #op (op (ss2) [1]) [1] ; eq:=simplify (diff (subs (A221 (v)=0, factor (op (ss2) [1]) [2]) * (x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4+3*v^3*x-v^3) ^3) , v, v) ) :
```

```
> #eqb1:=factor (coeff (eq, A221 (0), 1)) :
```

```
eqb2:=factor (coeff (eq, A231 (0), 1)) :
```

```
eqb3:=factor (coeff (eq, A241 (0), 1)) :
```

```
eqb4:=factor (simplify (coeff (coeff (coeff (eq, A221 (0), 0), A231 (0), 0), A241 (0), 0) ) ) :
```

```

> #vv1:=- (c12*c23*c34-c12*c24*c33-c13*c22*c34+c13*c24*c32+c14*c22*c3
3-c14*c23*c32) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c1
3*c21*c32-c13*c22*c31) :
vv2:=(c11*c23*c34-c11*c24*c33-c13*c21*c34+c13*c24*c31+c14*c21*c33-
c14*c23*c31) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13*
c21*c32-c13*c22*c31) :
vv3:=- (c11*c22*c34-c11*c24*c32-c12*c21*c34+c12*c24*c31+c14*c21*c32
-c14*c22*c31) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13
*c21*c32-c13*c22*c31) :
> #ww1:=simplify(subs(c11=subs(v=u1,eqb1),c12=subs(v=u1,eqb2),c13=su
bs(v=u1,eqb3),c14=subs(v=u1,eqb4),c21=subs(v=u2,eqb1),c22=subs(v=u
2,eqb2),c23=subs(v=u2,eqb3),c24=subs(v=u2,eqb4),c31=subs(v=u3,eqb1
),c32=subs(v=u3,eqb2),c33=subs(v=u3,eqb3),c34=subs(v=u3,eqb4),vv1)
):
> #save(ww1,"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/
InvSeq3and4/avoid021x00000/testA221x0ww.txt"):
> #see file testA221x0.mws
> #read
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/FA221x0.txt": A221x0:=subs(u4=vv,www):
> #series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31=BB31,B32=BB32
,B33=BB33,B21=BB21,B22=BB22,A221x0)-subs(v=0,BA221),x,28);
>
> ##take care for A231(0):
> #op(op(ss2)[1])[1];eq:=simplify(diff(subs(A221(v)=0,factor(op(ss2[
1])[2]*(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4
+3*v^3*x-v^3)^3)),v,v)):
> #eqb1:=factor(coeff(eq,A221(0),1)):
eqb2:=factor(coeff(eq,A231(0),1)):
eqb3:=factor(coeff(eq,A241(0),1)):
eqb4:=factor(simplify(coeff(coeff(coeff(eq,A221(0),0),A231(0),0),A
241(0),0))):
> #vv1:=- (c12*c23*c34-c12*c24*c33-c13*c22*c34+c13*c24*c32+c14*c22*c3
3-c14*c23*c32) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c1
3*c21*c32-c13*c22*c31) :
vv2:=(c11*c23*c34-c11*c24*c33-c13*c21*c34+c13*c24*c31+c14*c21*c33-
c14*c23*c31) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13*
c21*c32-c13*c22*c31) :
vv3:=- (c11*c22*c34-c11*c24*c32-c12*c21*c34+c12*c24*c31+c14*c21*c32
-c14*c22*c31) / (c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13
*c21*c32-c13*c22*c31) :
> #ww1:=simplify(subs(c11=subs(v=u1,eqb1),c12=subs(v=u1,eqb2),c13=su
bs(v=u1,eqb3),c14=subs(v=u1,eqb4),c21=subs(v=u2,eqb1),c22=subs(v=u

```

```

2,eqb2),c23=subs(v=u2,eqb3),c24=subs(v=u2,eqb4),c31=subs(v=u3,eqb1)
),c32=subs(v=u3,eqb2),c33=subs(v=u3,eqb3),c34=subs(v=u3,eqb4),vv2)
):
> #save(ww1,"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/
InvSeq3and4/avoid021x00000/testA231x0ww.txt"):
> #see file testA231x0.mws
> #read
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/FA231x0.txt": A231x0:=subs(u4=vv,www):
> #series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31=BB31,B32=BB32
,B33=BB33,B21=BB21,B22=BB22,A231x0)-subs(v=0,BA231),x,31);
>
> ##take care for A241(0):
> #op(op(ss2)[1])[1];eq:=simplify(diff(subs(A221(v)=0,factor(op(ss2[
1])[2]*(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4
+3*v^3*x-v^3)^3)),v,v)):
> #eqb1:=factor(coeff(eq,A221(0),1)):
eqb2:=factor(coeff(eq,A231(0),1)):
eqb3:=factor(coeff(eq,A241(0),1)):
eqb4:=factor(simplify(coeff(coeff(coeff(eq,A221(0),0),A231(0),0),A
241(0),0))):
> #vv1:=(c12*c23*c34-c12*c24*c33-c13*c22*c34+c13*c24*c32+c14*c22*c3
3-c14*c23*c32)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c1
3*c21*c32-c13*c22*c31):
vv2:=(c11*c23*c34-c11*c24*c33-c13*c21*c34+c13*c24*c31+c14*c21*c33-
c14*c23*c31)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13*
c21*c32-c13*c22*c31):
vv3:=(c11*c22*c34-c11*c24*c32-c12*c21*c34+c12*c24*c31+c14*c21*c32
-c14*c22*c31)/(c11*c22*c33-c11*c23*c32-c12*c21*c33+c12*c23*c31+c13
*c21*c32-c13*c22*c31):
> #ww1:=simplify(subs(c11=subs(v=u1,eqb1),c12=subs(v=u1,eqb2),c13=su
bs(v=u1,eqb3),c14=subs(v=u1,eqb4),c21=subs(v=u2,eqb1),c22=subs(v=u
2,eqb2),c23=subs(v=u2,eqb3),c24=subs(v=u2,eqb4),c31=subs(v=u3,eqb1
),c32=subs(v=u3,eqb2),c33=subs(v=u3,eqb3),c34=subs(v=u3,eqb4),vv3)
):
> #save(ww1,"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/
InvSeq3and4/avoid021x00000/testA241x0ww.txt"):
> #see file testA241x0.mws
> #read
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/FA241x0.txt": A241x0:=subs(u4=vv,www):
> #series(subs(B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31=BB31,B32=BB32
,B33=BB33,B21=BB21,B22=BB22,A241x0)-subs(v=0,BA241),x,31);

```

```

> ##combine all files FA221x0.txt,FF231x0.txt,FA241x0.txt and
  A021x00000Step2.txt into a one file called A021x00000Step31.txt
> read
  "C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
  4/avoid021x00000/A021x00000Step31.txt":
> op(ss2[1])[1];
  A221xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[1])[2])))):
                                     A221(v)
> op(ss2[2])[1];
  A222xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[2])[2])))):
                                     A222(v)
> op(ss2[3])[1];
  A223xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[3])[2])))):
                                     A223(v)
> op(ss2[4])[1];
  A224xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[4])[2])))):
                                     A224(v)
> op(ss2[5])[1];
  A231xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[5])[2])))):
                                     A231(v)
> op(ss2[6])[1];
  A232xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[6])[2])))):
                                     A232(v)
> op(ss2[7])[1];
  A233xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[7])[2])))):
                                     A233(v)
> op(ss2[8])[1];
  A234xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[8])[2])))):
                                     A234(v)
> op(ss2[9])[1];
  A241xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A23
  1x0,A241(0)=A241x0,op(ss2[9])[2])))):
                                     A241(v)

```

```

> op(ss2[10])[1];
A242xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A231x0,A241(0)=A241x0,op(ss2[10])[2])))):
A242(v)
> op(ss2[11])[1];
A243xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A231x0,A241(0)=A241x0,op(ss2[11])[2])))):
A243(v)
> op(ss2[12])[1];
A244xv:=factor(simplify(subs(vv=u4,subs(A221(0)=A221x0,A231(0)=A231x0,A241(0)=A241x0,op(ss2[12])[2])))):
A244(v)
>
> A221xv:=subs(vv=u4,simplify(subs(u4=vv,A221xv))):A222xv:=subs(vv=u4,simplify(subs(u4=vv,A222xv))):A223xv:=subs(vv=u4,simplify(subs(u4=vv,A223xv))):A224xv:=subs(vv=u4,simplify(subs(u4=vv,A224xv))):A231xv:=subs(vv=u4,simplify(subs(u4=vv,A231xv))):A232xv:=subs(vv=u4,simplify(subs(u4=vv,A232xv))):A233xv:=subs(vv=u4,simplify(subs(u4=vv,A233xv))):A234xv:=subs(vv=u4,simplify(subs(u4=vv,A234xv))):A241xv:=subs(vv=u4,simplify(subs(u4=vv,A241xv))):A242xv:=subs(vv=u4,simplify(subs(u4=vv,A242xv))):A243xv:=subs(vv=u4,simplify(subs(u4=vv,A243xv))):A244xv:=subs(vv=u4,simplify(subs(u4=vv,A244xv))):
> save(A421x0,A431x0,A441x0,A421xv,A422xv,A423xv,A424xv,A431xv,A432xv,A433xv,A434xv,A441xv,A442xv,A443xv,A444xv,A321x0,A331x0,A341x0,A321xv,A322xv,A323xv,A324xv,A331xv,A332xv,A333xv,A334xv,A341xv,A342xv,A343xv,A344xv,A221x0,A231x0,A241x0,A221xv,A222xv,A223xv,A224xv,A231xv,A232xv,A233xv,A234xv,A241xv,A242xv,A243xv,A244xv,"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and4/avoid021x00000/A021x00000Step3.txt");
> #simplify(series(subs(u4=vv,B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31=BB31,B32=BB32,B33=BB33,B21=BB21,B22=BB22,A244xv)-BA244,x,20));
> #-----Step4-----
> read
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and4/avoid021x00000/A021x00000Step3.txt":
> alias(vv=RootOf(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2*x^4+v^4+3*v^3*x-v^3=0,v)):
> ssw1:=map(factor,solve({eq121,eq122,eq123,eq124,eq131,eq132,eq133,eq134,eq141,eq142,eq143,eq144},{A121(v),A122(v),A123(v),A124(v),A131(v),A132(v),A133(v),A134(v),A141(v),A142(v),A143(v),A144(v)})):
> eq:=subs(A421(0)=A421x0,A431(0)=A431x0,A441(0)=A441x0,A421(v)=A421xv,A422(v)=A422xv,A423(v)=A423xv,A424(v)=A424xv,A431(v)=A431xv,A432(v)=A432xv,A433(v)=A433xv,A434(v)=A434xv,A441(v)=A441xv,A442(v)=A442xv,A443(v)=A443xv,A444(v)=A444xv,A321(v)=A321xv,A322(v)=A322xv,A323(v)=A323xv,A324(v)=A324xv,A331(v)=A331xv,A332(v)=A332xv,A333(v)=A333xv,A334(v)=A334xv,A341(v)=A341xv,A342(v)=A342xv,A343(v)=A343xv,A344(v)=A344xv,A221(v)=A221xv,A222(v)=A222xv,A223(v)=A223xv,A224(v)=A224xv,A231(v)=A231xv,A232(v)=A232xv,A233(v)=A233xv,A234(v)=A234xv,A241(v)=A241xv,A242(v)=A242xv,A243(v)=A243xv,A244(v)=A244xv);

```

```

2 (v)=A432xv,A433 (v)=A433xv,A434 (v)=A434xv,A441 (v)=A441xv,A442 (v)=A
442xv,A443 (v)=A443xv,A444 (v)=A444xv,A321 (0)=A321x0,A331 (0)=A331x0,
A341 (0)=A341x0,A321 (v)=A321xv,A322 (v)=A322xv,A323 (v)=A323xv,A324 (v
)=A324xv,A331 (v)=A331xv,A332 (v)=A332xv,A333 (v)=A333xv,A334 (v)=A334
xv,A341 (v)=A341xv,A342 (v)=A342xv,A343 (v)=A343xv,A344 (v)=A344xv,A22
1 (0)=A221x0,A231 (0)=A231x0,A241 (0)=A241x0,A221 (v)=A221xv,A222 (v)=A
222xv,A223 (v)=A223xv,A224 (v)=A224xv,A231 (v)=A231xv,A232 (v)=A232xv,
A233 (v)=A233xv,A234 (v)=A234xv,A241 (v)=A241xv,A242 (v)=A242xv,A243 (v
)=A243xv,A244 (v)=A244xv,ssw1[1]):

```

```

> eq:=subs (A121 (v)=0,eq):

```

```

> save (eq,
"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/testA121x0sa.txt"):

```

```

>

```

```

> #in the file test121x0_sa.mws we find A121x0,A131x0,A141x0, and we
save all the g.f in one file A021x00000Step4.txt

```

```

>

```

```

> #checking A121x0,A131x0,A141x0:

```

```

> read

```

```

"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/A021x00000Step4.txt":

```

```

> alias (vv=RootOf (x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2
*x^4+v^4+3*v^3*x-v^3=0,v)):

```

```

> simplify (series (subs (u4=vv,B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31
=BB31,B32=BB32,B33=BB33,B21=BB21,B22=BB22,B11=BB11,A121x0)-subs (v=
0,BA121),x,40));

```

```

simplify (series (subs (u4=vv,B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31
=BB31,B32=BB32,B33=BB33,B21=BB21,B22=BB22,B11=BB11,A131x0)-subs (v=
0,BA131),x,41));

```

```

simplify (series (subs (u4=vv,B41=BB41,B42=BB42,B43=BB43,B44=BB44,B31
=BB31,B32=BB32,B33=BB33,B21=BB21,B22=BB22,B11=BB11,A141x0)-subs (v=
0,BA141),x,40));

```

$O(x^{30})$

$O(x^{30})$

$O(x^{28})$

```

> ##final step: to solve the system with Aij1(0)=Aij1x0 for
i=1,2,3,4 and j=2,3,4

```

```

> restart:

```

```

> read

```

```

"C:/DriveData_Mansour2=201805/Toufik_data/Projects_2022/InvSeq3and
4/avoid021x00000/A021x00000Step4.txt":

```

```

> A121x0:=subs (B11=BB11,B21=BB21,B22=BB22,B31=BB31,B32=BB32,B33=BB33

```

```

, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A121x0) :
A131x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A131x0) :
A141x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A141x0) :
A221x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A221x0) :
A231x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A231x0) :
A241x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A241x0) :
A321x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A321x0) :
A331x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A331x0) :
A341x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A341x0) :
A421x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A421x0) :
A431x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A431x0) :
A441x0:=subs (B11=BB11, B21=BB21, B22=BB22, B31=BB31, B32=BB32, B33=BB33
, B41=BB41, B42=BB42, B43=BB43, B44=BB44, A441x0) :

```

```

> SYS:={BA0=(x+x*BA00+x*BB11), BA00=(x+x*BA000+x*BB21+x*BB22),
BB11=(x+x*BB21+x*BA011+x*BB11),
BA000=(x+x*BA0000+x*BB31+x*BB32+x*BB33),
BB21=(x+x*BB31+x*BA0011+x*BB21+x*BB22),
BB22=(x+x*BB32+x*BA0022+x*BB22),
BA011=(x+x*BA0011+x*BA0111+x*A121x0+x*BB11),
BA0000=(x+x*BB41+x*BB42+x*BB43+x*BB44),
BB31=(x+x*BB41+x*BA00011+x*BB31+x*BB32+x*BB33),
BB32=(x+x*BB42+x*BA00022+x*BB32+x*BB33),
BB33=(x+x*BB43+x*BA00033+x*BB33),
BA0011=(x+x*BA00011+x*BA00111+x*A221x0+x*BB21+x*BB22),
BA0022=(x+x*BA00022+x*BA00222+x*BB21+x*BB22),
BA0111=(x+x*BA00111+x*BA01111+x*(A131x0+A121x0)+x*BB11),
BB41=(x+x*BA000011+x*BB41+x*BB42+x*BB43+x*BB44),
BB42=(x+x*BA000022+x*BB42+x*BB43+x*BB44),
BB43=(x+x*BA000033+x*BB43+x*BB44), BB44=(x+x*BA000044+x*BB44),
BA00011=(x+x*BA000011+x*BA000111+x*A321x0+x*BB31+x*BB32+x*BB33),
BA00022=(x+x*BA000022+x*BA000222+x*BB31+x*BB32+x*BB33),
BA00033=(x+x*BA000033+x*BA000333+x*BB32+x*BB33),
BA00111=(x+x*BA000111+x*BA001111+x*(A231x0+A221x0)+x*BB21+x*BB22),

```

```

BA00222=(x+x*BA000222+x*BA002222+x*A221x0+x*BB21+x*BB22) ,
BA01111=(x+x*BA001111+x*(A141x0+A131x0+A121x0)+x*BB11) ,
BA000011=(x+x*BA0000111+x*A421x0+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA000022=(x+x*BA0000222+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA000033=(x+x*BA0000333+x*BB42+x*BB43+x*BB44) ,
BA000044=(x+x*BA0000444+x*BB43+x*BB44) ,
BA000111=(x+x*BA0000111+x*BA0001111+x*(A331x0+A321x0)+x*BB31+x*BB32+x*BB33) ,
BA000222=(x+x*BA0000222+x*BA0002222+x*A321x0+x*BB31+x*BB32+x*BB33) ,
BA000333=(x+x*BA0000333+x*BA0003333+x*BB31+x*BB32+x*BB33) ,
BA001111=(x+x*BA0001111+x*(A241x0+A231x0+A221x0)+x*BB21+x*BB22) ,
BA002222=(x+x*BA0002222+x*(A231x0+A221x0)+x*BB21+x*BB22) ,
BA0000111=(x+x*BA00001111+x*(A431x0+A421x0)+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA0000222=(x+x*BA00002222+x*A421x0+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA0000333=(x+x*BA00003333+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA0000444=(x+x*BA00004444+x*BB42+x*BB43+x*BB44) ,
BA0001111=(x+x*BA00001111+x*(A341x0+A331x0+A321x0)+x*BB31+x*BB32+x*BB33) ,
BA0002222=(x+x*BA00002222+x*(A331x0+A321x0)+x*BB31+x*BB32+x*BB33) ,
BA0003333=(x+x*BA00003333+x*A321x0+x*BB31+x*BB32+x*BB33) ,
BA00001111=(x+x*(A441x0+A431x0+A421x0)+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA00002222=(x+x*(A431x0+A421x0)+x*BB41+x*BB42+x*BB43+x*BB44) ,
BA00003333=(x+x*A421x0+x*BB41+x*BB42+x*BB43+x*BB44) } :

```

```

> VAR:={BA0, BA00, BB11, BA000, BB21, BB22, BA011, BA0000, BB31,
BB32, BB33, BA0011, BA0022, BA0111, BB41, BB42, BB43, BB44,
BA00011, BA00022, BA00033, BA00111, BA00222, BA01111, BA000011,
BA000022, BA000033, BA000044, BA000111, BA000222, BA000333,
BA001111, BA002222, BA0000111, BA0000222, BA0000333, BA0000444,
BA0001111, BA0002222, BA0003333, BA00001111, BA00002222,
BA00003333} :

```

```

> ssf:=solve(SYS,VAR) :

```

```

> op(ssf[1])[1]; finA021x00000:=simplify(op(ssf[1])[2]) :

```

BA0

```

> alias(vv=RootOf(x^12+v*x^9-3*v*x^8+v^2*x^6-3*v^2*x^5+v^3*x^3+3*v^2
*x^4+v^4+3*v^3*x-v^3=0,v)) :

```

```

> series(subs(u4=vv,finA021x00000),x,60) ;

```

$$\begin{aligned}
& x + 2x^2 + 6x^3 + 22x^4 + 89x^5 + 377x^6 + 1630x^7 + 7099x^8 + 30911x^9 + 134122x^{10} + 579055x^{11} \\
& + 2486122x^{12} + 10613550x^{13} + 45061160x^{14} + 190314295x^{15} + 799860726x^{16} + 3346439323x^{17} \\
& + 13941999966x^{18} + 57860268300x^{19} + 239264677555x^{20} + 986139459335x^{21} + \\
& 4051986668110x^{22} + 16602214224885x^{23} + 67845590135527x^{24} + 276576825849574x^{25} +
\end{aligned}$$

$$\begin{aligned}
& 1124920771473566 x^{26} + 4565693415516002 x^{27} + 18493999452648271 x^{28} + \\
& 74773795711942922 x^{29} + 301795571949692439 x^{30} + 1216094059524950197 x^{31} + \\
& 4892766241235145695 x^{32} + 19656826517742615155 x^{33} + 78864124509747661270 x^{34} + \\
& 315998774926173592790 x^{35} + 1264623613587434277900 x^{36} + 5055161090667858845315 x^{37} \\
& + 20185165287741374949425 x^{38} + 80515057805155850159600 x^{39} + \\
& 320842269162414713346825 x^{40} + 1277310107879055414071239 x^{41} + \\
& 5080548839456028658413392 x^{42} + 20190745014057430759450524 x^{43} + \\
& 80174863254254118244596305 x^{44} + O(x^{45})
\end{aligned}$$

```

> series(1*x+2*x^2+6*x^3+22*x^4+89*x^5+377*x^6+1630*x^7+7099*x^8+309
11*x^9+134122*x^10+579055*x^11+2486122*x^12+10613550*x^13+45061160
*x^14+190314295*x^15+799860726*x^16+3346439323*x^17+13941999966*x^
18+57860268300*x^19+239264677555*x^20+986139459335*x^21+4051986668
110*x^22+16602214224885*x^23+67845590135527*x^24+276576825849574*x
^25+1124920771473566*x^26+4565693415516002*x^27+18493999452648271*
x^28+74773795711942922*x^29+301795571949692439*x^30+12160940595249
50197*x^31+4892766241235145695*x^32+19656826517742615155*x^33+7886
4124509747661270*x^34+315998774926173592790*x^35+12646236135874342
77900*x^36+5055161090667858845315*x^37+20185165287741374949425*x^3
8+80515057805155850159600*x^39+320842269162414713346825*x^40+12773
10107879055414071239*x^41+5080548839456028658413392*x^42+201907450
14057430759450524*x^43+80174863254254118244596305*x^44+O(x^45), x, 4
5);

```

$$\begin{aligned}
& x + 2 x^2 + 6 x^3 + 22 x^4 + 89 x^5 + 377 x^6 + 1630 x^7 + 7099 x^8 + 30911 x^9 + 134122 x^{10} + 579055 x^{11} \\
& + 2486122 x^{12} + 10613550 x^{13} + 45061160 x^{14} + 190314295 x^{15} + 799860726 x^{16} + 3346439323 \\
& x^{17} + 13941999966 x^{18} + 57860268300 x^{19} + 239264677555 x^{20} + 986139459335 x^{21} + \\
& 4051986668110 x^{22} + 16602214224885 x^{23} + 67845590135527 x^{24} + 276576825849574 x^{25} + \\
& 1124920771473566 x^{26} + 4565693415516002 x^{27} + 18493999452648271 x^{28} + \\
& 74773795711942922 x^{29} + 301795571949692439 x^{30} + 1216094059524950197 x^{31} + \\
& 4892766241235145695 x^{32} + 19656826517742615155 x^{33} + 78864124509747661270 x^{34} + \\
& 315998774926173592790 x^{35} + 1264623613587434277900 x^{36} + 5055161090667858845315 x^{37} \\
& + 20185165287741374949425 x^{38} + 80515057805155850159600 x^{39} + \\
& 320842269162414713346825 x^{40} + 1277310107879055414071239 x^{41} + \\
& 5080548839456028658413392 x^{42} + 20190745014057430759450524 x^{43} + \\
& 80174863254254118244596305 x^{44} + O(x^{45})
\end{aligned}$$

```

> finA021x000:=factor(simplify(subs(u4=vv, finA021x00000))) ;

```

$$(-8203125 x^{21} vv - 468750 x^{20} vv - 5078125 x^{18} vv^2 + 1953125 x^{16} vv^3 + 78125 x^{17} vv^2$$

$$\begin{aligned}
& -390625 x^{15} v v^3 - 2921875 x^{16} v v^2 - 4609375 x^{14} v v^3 + 3546875 x^{13} v v^3 - 3059375 x^{12} v v^3 \\
& + 3203125 x^{19} v v - 22912500 x^{15} v v^2 - 7703750 v v^3 x^{11} - 1009781 x^{10} - 1788936 x^{13} \\
& + 2839572 x^{11} - 2704378 x^{12} - 8505 x^8 + 154080 x^9 - 5000000 x^{21} + 12779875 x^{19} \\
& + 11446875 x^{20} - 14491750 x^{18} - 13143527 x^{15} + 1595415 x^{16} + 9406595 x^{17} + 7338375 x^{14} \\
& + 9546875 x^{22} - 5088705 x^7 v v + 1721699 x^6 v v - 276030 x^5 v v + 17010 v v x^4 + 4420744 x^3 v v^2 \\
& - 1173711 x^2 v v^2 + 157752 x v v^2 - 3515625 x^{24} + 3125000 x^{23} - 25915625 v v x^{18} \\
& + 10040625 v v x^{17} + 4703250 v v x^{16} + 11662500 v v^2 x^{14} - 33813225 v v x^{15} - 5755875 v v^2 x^{13} \\
& + 28404825 v v x^{14} - 26732525 v v^2 x^{12} - 3265507 v v x^{13} + 8943875 v v^3 x^{10} + 37191910 v v^2 x^{11} \\
& - 18069436 v v x^{12} - 3558775 v v^3 x^9 - 13951143 v v^2 x^{10} + 25346008 v v x^{11} - 3133860 v v^3 x^8 \\
& - 10453444 v v^2 x^9 - 10525772 v v x^{10} + 8574039 v v^3 x^7 + 30982743 v v^2 x^8 - 1761959 v v x^9 \\
& - 4400941 v v^3 x^6 - 21158423 v v^2 x^7 + 6890115 v v x^8 + 253023 v v^3 x^5 + 3731245 v v^2 x^6 \\
& + 2270761 v v^3 x^4 + 7215579 v v^2 x^5 - 2081239 v v^3 x^3 - 8638210 v v^2 x^4 + 777000 v v^3 x^2 \\
& - 132237 v v^3 x - 8505 v v^2 + 8505 v v^3) / (x^{10} (125 x^4 + 50 x^3 + 15 x^2 + 94 x - 27)^3 (x^3 + x^2 - 1))
\end{aligned}$$

> finaA021x00000:=(-8203125*x^21*vv-468750*x^20*vv-5078125*x^18*vv^2+1953125*x^16*vv^3+78125*x^17*vv^2-390625*x^15*vv^3-2921875*x^16*vv^2-4609375*x^14*vv^3+3546875*x^13*vv^3-3059375*x^12*vv^3+3203125*x^19*vv-22912500*x^15*vv^2-7703750*vv^3*x^11-1009781*x^10-1788936*x^13+2839572*x^11-2704378*x^12-8505*x^8+154080*x^9-5000000*x^21+12779875*x^19+11446875*x^20-14491750*x^18-13143527*x^15+1595415*x^16+9406595*x^17+7338375*x^14+9546875*x^22-5088705*x^7*vv+1721699*x^6*vv-276030*x^5*vv+17010*vv*x^4+4420744*x^3*vv^2-1173711*x^2*vv^2+157752*x*vv^2-3515625*x^24+3125000*x^23-25915625*vv*x^18+10040625*vv*x^17+4703250*vv*x^16+11662500*vv^2*x^14-33813225*vv*x^15-5755875*vv^2*x^13+28404825*vv*x^14-26732525*vv^2*x^12-3265507*vv*x^13+8943875*vv^3*x^10+37191910*vv^2*x^11-18069436*vv*x^12-3558775*vv^3*x^9-13951143*vv^2*x^10+25346008*vv*x^11-3133860*vv^3*x^8-10453444*vv^2*x^9-10525772*vv*x^10+8574039*vv^3*x^7+30982743*vv^2*x^8-1761959*vv*x^9-4400941*vv^3*x^6-21158423*vv^2*x^7+6890115*vv*x^8+253023*vv^3*x^5+3731245*vv^2*x^6+2270761*vv^3*x^4+7215579*vv^2*x^5-2081239*vv^3*x^3-8638210*vv^2*x^4+777000*vv^3*x^2-132237*vv^3*x-8505*vv^2+8505*vv^3)/x^10/(125*x^4+50*x^3+15*x^2+94*x-27)^3/(x^3+x^2-1);

$$\begin{aligned}
finaA021x00000 & := (-3515625 x^{24} + 3125000 x^{23} - 8203125 v v x^{21} + 9546875 x^{22} - 468750 v v x^{20} \\
& - 5000000 x^{21} - 5078125 v v^2 x^{18} + 3203125 v v x^{19} + 11446875 x^{20} + 1953125 v v^3 x^{16} \\
& + 78125 v v^2 x^{17} - 25915625 v v x^{18} + 12779875 x^{19} - 390625 v v^3 x^{15} - 2921875 v v^2 x^{16} \\
& + 10040625 v v x^{17} - 14491750 x^{18} - 4609375 v v^3 x^{14} - 22912500 v v^2 x^{15} + 4703250 v v x^{16} \\
& + 9406595 x^{17} + 3546875 v v^3 x^{13} + 11662500 v v^2 x^{14} - 33813225 v v x^{15} + 1595415 x^{16} \\
& - 3059375 v v^3 x^{12} - 5755875 v v^2 x^{13} + 28404825 v v x^{14} - 13143527 x^{15} - 7703750 v v^3 x^{11}
\end{aligned}$$

$$\begin{aligned}
& -26732525 \, \nu \nu^2 x^{12} - 3265507 \, \nu \nu x^{13} + 7338375 x^{14} + 8943875 \, \nu \nu^3 x^{10} + 37191910 \, \nu \nu^2 x^{11} \\
& - 18069436 \, \nu \nu x^{12} - 1788936 x^{13} - 3558775 \, \nu \nu^3 x^9 - 13951143 \, \nu \nu^2 x^{10} + 25346008 \, \nu \nu x^{11} \\
& - 2704378 x^{12} - 3133860 \, \nu \nu^3 x^8 - 10453444 \, \nu \nu^2 x^9 - 10525772 \, \nu \nu x^{10} + 2839572 x^{11} \\
& + 8574039 \, \nu \nu^3 x^7 + 30982743 \, \nu \nu^2 x^8 - 1761959 \, \nu \nu x^9 - 1009781 x^{10} - 4400941 \, \nu \nu^3 x^6 \\
& - 21158423 \, \nu \nu^2 x^7 + 6890115 \, \nu \nu x^8 + 154080 x^9 + 253023 \, \nu \nu^3 x^5 + 3731245 \, \nu \nu^2 x^6 \\
& - 5088705 \, \nu \nu x^7 - 8505 x^8 + 2270761 \, \nu \nu^3 x^4 + 7215579 \, \nu \nu^2 x^5 + 1721699 \, \nu \nu x^6 - 2081239 \, \nu \nu^3 x^3 \\
& - 8638210 \, \nu \nu^2 x^4 - 276030 \, \nu \nu x^5 + 777000 \, \nu \nu^3 x^2 + 4420744 \, \nu \nu^2 x^3 + 17010 \, \nu \nu x^4 - 132237 \, \nu \nu^3 x \\
& - 1173711 \, \nu \nu^2 x^2 + 8505 \, \nu \nu^3 + 157752 \, \nu \nu^2 x - 8505 \, \nu \nu^2) / (x^{10} \\
& (125 x^4 + 50 x^3 + 15 x^2 + 94 x - 27)^3 (x^3 + x^2 - 1))
\end{aligned}$$

[>