```
> #case 021,0000
> restart:
> #checking the rules
> R[0]:=0: R[0,0]:=0: R[0,1]:=0: R[0,0,0]:=0: R[0,0,1]:=0:
  R[0,0,2]:=0: R[0,1,1]:=0: R[0,0,0,1]:=0: R[0,0,0,2]:=0:
  R[0,0,0,3]:=0: R[0,0,1,1]:=0: R[0,0,2,2]:=0: R[0,1,1,1]:=0:
  R[0,1,1,2]:=0: R[0,0,0,1,1]:=0: R[0,0,0,2,2]:=0: R[0,0,0,3,3]:=0:
  R[0,0,1,1,1]:=0: R[0,0,1,1,2]:=0: R[0,0,2,2,2]:=0:
  R[0,1,1,1,2]:=0: R[0,1,1,2,2]:=0: R[0,0,0,1,1,1]:=0:
  R[0,0,0,1,1,2]:=0: R[0,0,0,2,2,2]:=0: R[0,0,1,1,1,2]:=0:
  R[0,0,1,1,2,2]:=0: R[0,1,1,1,2,2]:=0: R[0,1,1,2,2,2]:=0:
  R[0,0,0,1,1,1,2]:=0: R[0,0,0,1,1,2,2]:=0: R[0,0,1,1,1,2,2]:=0:
  R[0,0,1,1,2,2,2]:=0: R[0,1,1,1,2,2,2]:=0: R[0,1,1,1,2,2,3]:=0:
  R[0,0,0,1,1,1,2,2]:=0: R[0,0,0,1,1,2,2,2]:=0:
  R[0,0,1,1,1,2,2,2]:=0: R[0,0,1,1,1,2,2,3]:=0:
  R[0,1,1,1,2,2,2,3]:=0: R[0,1,1,1,2,2,3,3]:=0:
  R[0,0,0,1,1,1,2,2,2]:=0: R[0,0,0,1,1,1,2,2,3]:=0:
  R[0,0,1,1,1,2,2,2,3]:=0: R[0,0,1,1,1,2,2,3,3]:=0:
  R[0,1,1,1,2,2,2,3,3]:=0: R[0,1,1,1,2,2,3,3,3]:=0:
  R[0,0,0,1,1,1,2,2,2,3]:=0: R[0,0,0,1,1,1,2,2,3,3]:=0:
  R[0,0,1,1,1,2,2,2,3,3]:=0: R[0,0,1,1,1,2,2,3,3,3]:=0:
  R[0,1,1,1,2,2,2,3,3,3]:=0: R[0,1,1,1,2,2,2,3,3,4]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3]:=0: R[0,0,0,1,1,1,2,2,3,3,3]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3]:=0: R[0,0,1,1,1,2,2,2,3,3,4]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4]:=0: R[0,1,1,1,2,2,2,3,3,4,4]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,3]:=0: R[0,0,0,1,1,1,2,2,2,3,3,4]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4]:=0: R[0,0,1,1,1,2,2,2,3,3,4,4]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4]:=0: R[0,1,1,1,2,2,2,3,3,4,4,4]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,3,4]:=0: R[0,0,0,1,1,1,2,2,2,3,3,4,4]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4,4]:=0: R[0,0,1,1,1,2,2,2,3,3,4,4,4]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4,4]:=0: R[0,1,1,1,2,2,2,3,3,3,4,4,5]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,3,4,4]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,4,4,4]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4,4,4]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4]:=0:
  R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=0:
  R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]:=0:
  R[0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]:=0:
```

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R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=0:
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=0:
R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]:=0:
R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]:=0:
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5]:=0:
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,6]:=0:
> for i from 1 to 15 do R[0]:=simplify(x+x*(R[0,0]+R[0,1])):
R[0,0]:=simplify(x+x*(R[0,0,0]+R[0,0,1]+R[0,0,2])):
R[0,1]:=simplify(x+x*(R[0,0,1]+R[0,1,1]+R[0,1])):
R[0,0,0]:=simplify(x+x*(R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1]:=simplify(x+x*(R[0,0,0,1]+R[0,0,1,1]+R[0,0,1]+R[0,0,2])):
R[0,0,2]:=simplify(x+x*(R[0,0,0,2]+R[0,0,2,2]+R[0,0,2])):
R[0,1,1]:=simplify(x+x*(R[0,0,1,1]+R[0,1,1,1]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1]:=simplify(x+x*(R[0,0,0,1,1]+R[0,0,0,1]+R[0,0,0,2]+R[0,0
,0,3])):
R[0,0,0,2]:=simplify(x+x*(R[0,0,0,2,2]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,3]:=simplify(x+x*(R[0,0,0,3,3]+R[0,0,0,3])):
R[0,0,1,1]:=simplify(x+x*(R[0,0,0,1,1]+R[0,0,1,1,1]+R[0,0,1,1,2]+R
[0,0,1]+R[0,0,2])):
R[0,0,2,2]:=simplify(x+x*(R[0,0,0,2,2]+R[0,0,2,2,2]+R[0,0,1]+R[0,0
,2])):
R[0,1,1,1]:=simplify(x+x*(R[0,0,1,1,1]+R[0,1,1,1,2]+R[0,1,1,2]+R[0
,1])):
R[0,1,1,2]:=simplify(x+x*(R[0,0,1,1,2]+R[0,1,1,2,2]+R[0,1,1,2]+R[0
,1])):
R[0,0,0,1,1]:=simplify(x+x*(R[0,0,0,1,1,1]+R[0,0,0,1,1,2]+R[0,0,0,
1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,2,2]:=simplify(x+x*(R[0,0,0,2,2,2]+R[0,0,0,1]+R[0,0,0,2]+R
[0,0,0,3])):
R[0,0,0,3,3]:=simplify(x+x*(R[0,0,0]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1]:=simplify(x+x*(R[0,0,0,1,1,1]+R[0,0,1,1,1,2]+R[0,0,1,
1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,2]:=simplify(x+x*(R[0,0,0,1,1,2]+R[0,0,1,1,2,2]+R[0,0,1,
1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,2,2,2]:=simplify(x+x*(R[0,0,0,2,2,2]+R[0,0,1,1,2]+R[0,0,1]+R
[0,0,2])):
R[0,1,1,1,2]:=simplify(x+x*(R[0,0,1,1,1,2]+R[0,1,1,1,2,2]+R[0,1,1,
1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,2,2]:=simplify(x+x*(R[0,0,1,1,2,2]+R[0,1,1,2,2,2]+R[0,1,1,
1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1]:=simplify(x+x*(R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,
0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,1,1,2]:=simplify(x+x*(R[0,0,0,1,1,2,2]+R[0,0,0,1,1,2]+R[0,
```

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0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,2,2,2]:=simplify(x+x*(R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]
+R[0,0,0,3])):
R[0,0,1,1,1,2]:=simplify(x+x*(R[0,0,0,1,1,1,2]+R[0,0,1,1,1,2,2]+R[
0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,2,2]:=simplify(x+x*(R[0,0,0,1,1,2,2]+R[0,0,1,1,2,2,2]+R[
0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2]:=simplify(x+x*(R[0,0,1,1,1,2,2]+R[0,1,1,1,2,2,2]+R[
0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,2,2,2]:=simplify(x+x*(R[0,0,1,1,2,2,2]+R[0,1,1,1,2,2,3]+R[
0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2]+R[0,0,0,1,1,1,2
]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,1,1,2,2]:=simplify(x+x*(R[0,0,0,1,1,2,2,2]+R[0,0,0,1,1,1,2
]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2]+R[0,0,1,1,1,2,2
,2]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,
2])):
R[0,0,1,1,2,2,2]:=simplify(x+x*(R[0,0,0,1,1,2,2,2]+R[0,0,1,1,1,2,2
,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2]:=simplify(x+x*(R[0,0,1,1,1,2,2,2]+R[0,1,1,1,2,2,2
,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,3]+R[0,1,1,1,2,2,3
,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2]+R[0,0,0,1,1
,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[
0,0,0,3])):
R[0,0,0,1,1,2,2,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1
,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2]+R[0,0,1,1,1
,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+
R[0,0,2])):
R[0,0,1,1,1,2,2,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3]+R[0,0,1,1,1
,2,2,3,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+
R[0,0,2])):
R[0,1,1,1,2,2,2,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3]+R[0,1,1,1,2
,2,2,3,3]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1
,2]+R[0,1])):
R[0,1,1,1,2,2,3,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,3,3]+R[0,1,1,1,2
,2,3,3,3]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1
,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0
,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2
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]+R[0,0,0,3])):
R[0,0,0,1,1,1,2,2,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3,3]+R[0,0,0
,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2
]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3]+R[0,0,1
,1,1,2,2,2,3,3]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,
1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,1,2,2,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3,3]+R[0,0,1
,1,1,2,2,3,3,3]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,
1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3]+R[0,1,1
,1,2,2,2,3,3,3]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,
1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,3,3,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,3,3,3]+R[0,1,1
,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R
[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3]+R[0
,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,
1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,1,1,1,2,2,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3,3,3]+R[0
,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,
1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3]+R[0
,0,1,1,1,2,2,2,3,3,3]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3
]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2]
)):
R[0,0,1,1,1,2,2,3,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,3,3,3]+R[0
,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,
0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,3]+R[0
,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[
0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,4]+R[0
,1,1,1,2,2,2,3,3,4,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[
0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,3]
+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2
,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,
0,3])):
R[0,0,0,1,1,1,2,2,3,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,4]
+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,
0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,3]
```

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+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2
,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[
0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,4]
+R[0,0,1,1,1,2,2,2,3,3,4,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2
,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[
0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,3,4]
+R[0,1,1,1,2,2,2,3,3,3,4,4]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2
,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1
,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,4,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,4,4]
+R[0,1,1,1,2,2,2,3,3,4,4,4]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2
,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1
,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3,3]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,
3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1
,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[
0,0,0,3])):
R[0,0,0,1,1,1,2,2,2,3,3,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,
4,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1
,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[
0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,
3,4]+R[0,0,1,1,1,2,2,2,3,3,3,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0
,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,
1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,3,
4,4]+R[0,0,1,1,1,2,2,2,3,3,4,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0
,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,
1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,3,
4,4]+R[0,1,1,1,2,2,2,3,3,3,4,4,4]+R[0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0
,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[
0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,4,4,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,4,
4,4]+R[0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1
,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R
[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,
3,3,4,4]+R[0,0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R
[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,
0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
```

```
R[0,0,0,1,1,1,2,2,2,3,3,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,
3,4,4,4]+R[0,0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R
[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,
0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,
3,3,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4,
4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1
,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1
]+R[0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,4,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,3,
3,4,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4]
+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+
R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4,4,4]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,
3,4,4,4]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,
5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2
,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,3,4,4,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,3,
3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,5,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,
5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2
,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,
3,3,3,4,4,4]+R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,0,1,1,1,2,2,2,
3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0
,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2
]+R[0,0,0,3])):
R[0,0,0,1,1,1,2,2,2,3,3,4,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,
3,3,3,4,4,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,
4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[
0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,
3,3,3,4,4,4]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,0,1,1,1,2,2,2,3,
3,3,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0
,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R
[0,0,1]+R[0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,2,
3,3,3,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5]+R[0,0,1,1,1,2,2,2,3,
3,3,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0
,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R
[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,
3,3,4,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]+R[0,1,1,1,2,2,2,3,3,
3,4,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R
```

```
[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,
1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,3,
3,3,4,4,5,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]+R[0,1,1,1,2,2,2,3,3,
3,4,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R
[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,
1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,
2,3,3,3,4,4,4,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,0,1,1,1,2,
2,2,3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0
,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0
,0,2]+R[0,0,0,3])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,
2,3,3,3,4,4,5,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,0,1,1,1,2,
2,2,3,3,3,4]+R[0,0,0,1,1,1,2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0
,0,0,1,1,1,2,2,3]+R[0,0,0,1,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0
,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,
2,3,3,3,4,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]+R[0,0,1,1,1,2,
2,2,3,3,3,4,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,1,1,1,2,2,
2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1
,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,2,
2,3,3,3,4,4,5,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]+R[0,0,1,1,1,2,
2,2,3,3,3,4,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,1,1,1,2,2,
2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1
,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,
3,3,3,4,4,4,5,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5]+R[0,1,1,1,2,2,
2,3,3,3,4,4,4,5,5,6]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,1,1,1,2,2,
2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0
,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,2,
3,3,3,4,4,5,5,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,6]+R[0,1,1,1,2,2,
2,3,3,3,4,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,
3,4]+R[0,1,1,1,2,2,2,3,3,4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[
0,1,1,1,2]+R[0,1,1,2]+R[0,1])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,
2,2,3,3,3,4,4,4,5,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,0,0,1,
1,1,2,2,2,3,3,3,4,4,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,0,1,1,1,
2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1
,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,5,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,
```

```
2,2,3,3,3,4,4,5,5,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,0,0,1,
1,1,2,2,2,3,3,3,4,4,5]+R[0,0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,0,1,1,1,
2,2,2,3,3,4]+R[0,0,0,1,1,1,2,2,2,3]+R[0,0,0,1,1,1,2,2,3]+R[0,0,0,1
,1,1,2]+R[0,0,0,1,1,2]+R[0,0,0,1]+R[0,0,0,2]+R[0,0,0,3])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,
2,2,3,3,3,4,4,4,5,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5]+R[0,0,1,
1,1,2,2,2,3,3,3,4,4,4,5,5,6]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,
0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4]+R[0,0,1,1,1,
2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0,0,1,1,1,2,2,3]+R[0,0,1,1,1,2
]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,0,1,1,1,2,2,2,3,3,3,4,4,5,5,5]:=simplify(x+x*(R[0,0,0,1,1,1,2,
2,2,3,3,3,4,4,5,5,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,6]+R[0,0,1,
1,1,2,2,2,3,3,3,4,4,4,5]+R[0,0,1,1,1,2,2,2,3,3,3,4,4,5]+R[0,0,1,1,
1,2,2,2,3,3,3,4]+R[0,0,1,1,1,2,2,2,3,3,4]+R[0,0,1,1,1,2,2,2,3]+R[0
,0,1,1,1,2,2,3]+R[0,0,1,1,1,2]+R[0,0,1,1,2]+R[0,0,1]+R[0,0,2])):
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5]:=simplify(x+x*(R[0,0,1,1,1,2,2,
2,3,3,3,4,4,4,5,5,5]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5,6]+R[0,1,1,
1,2,2,2,3,3,3,4,4,4,5,5,6]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,1,1,
1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,
4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0
,1])):
R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,6]:=simplify(x+x*(R[0,0,1,1,1,2,2,
2,3,3,3,4,4,4,5,5,6]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,6,6]+R[0,1,1,
1,2,2,2,3,3,3,4,4,4,5,5,6]+R[0,1,1,1,2,2,2,3,3,3,4,4,4,5]+R[0,1,1,
1,2,2,2,3,3,3,4,4,5]+R[0,1,1,1,2,2,2,3,3,3,4]+R[0,1,1,1,2,2,2,3,3,
4]+R[0,1,1,1,2,2,2,3]+R[0,1,1,1,2,2,3]+R[0,1,1,1,2]+R[0,1,1,2]+R[0
,1])):  od:
```

```
> taylor(R[0],x,15);
```

$$x + 2x^2 + 6x^3 + 21x^4 + 78x^5 + 296x^6 + 1126x^7 + 4285x^8 + 16281x^9 + 61690x^{10} + 233078x^{11} + 878164x^{12} + 3299936x^{13} + 12370320x^{14} + O(x^{15})$$

```
> #to save memory, and after we see that the rules give the right
  numbers, we restart
> #Checking the eqations
> restart:
> BA0:=0: BA00:=0: BA01:=0: BA000:=0: BA001:=0: BA002:=0: BA011:=0:
  BA0001:=0: BA0002:=0: BA0003:=0: BA0011:=0: BA0022:=0: BA0111:=0:
  BA00011:=0: BA00022:=0: BA00033:=0: BA00111:=0: BA00222:=0:
  BA000111:=0: BA000222:=0: BA121:=0: BA122:=0: BA123:=0: BA131:=0:
  BA132:=0: BA133:=0: BA221:=0: BA222:=0: BA223:=0: BA231:=0:
  BA232:=0: BA233:=0: BA321:=0: BA322:=0: BA323:=0: BA331:=0:
  BA332:=0: BA333:=0:
> for i from 1 to 21 do BA0:=simplify(x+x*BA00+x*BA01):
```

```
BA00:=simplify(x+x*BA000+x*BA001+x*BA002):
BA01:=simplify(x+x*BA001+x*BA011+x*BA01):
BA000:=simplify(x+x*BA0001+x*BA0002+x*BA0003):
BA001:=simplify(x+x*BA0001+x*BA0011+x*BA001+x*BA002):
BA002:=simplify(x+x*BA0002+x*BA0022+x*BA002):
BA011:=simplify(x+x*BA0011+x*BA0111+x*subs(v=0,BA121)+x*BA01):
BA0001:=simplify(x+x*BA00011+x*BA0001+x*BA0002+x*BA0003):
BA0002:=simplify(x+x*BA00022+x*BA0002+x*BA0003):
BA0003:=simplify(x+x*BA00033+x*BA0003):
BA0011:=simplify(x+x*BA00011+x*BA00111+x*subs(v=0,BA221)+x*BA001+x
*BA002): BA0022:=simplify(x+x*BA00022+x*BA00222+x*BA001+x*BA002):
BA0111:=simplify(x+x*BA00111+x*subs(v=0,BA131)+x*subs(v=0,BA121)+x
*BA01):
BA00011:=simplify(x+x*BA000111+x*subs(v=0,BA321)+x*BA0001+x*BA0002
+x*BA0003):
BA00022:=simplify(x+x*BA000222+x*BA0001+x*BA0002+x*BA0003):
BA00033:=simplify(x+x*BA000+x*BA0002+x*BA0003):
BA00111:=simplify(x+x*BA000111+x*subs(v=0,BA231)+x*subs(v=0,BA221)
+x*BA001+x*BA002):
BA00222:=simplify(x+x*BA000222+x*subs(v=0,BA221)+x*BA001+x*BA002):
BA000111:=simplify(x+x*subs(v=0,BA331)+x*subs(v=0,BA321)+x*BA0001+
x*BA0002+x*BA0003):
BA000222:=simplify(x+x*subs(v=0,BA321)+x*BA0001+x*BA0002+x*BA0003)
:
BA121:=simplify(x/(1-v)+x*BA221+x*BA122+x/(1-v)*BA121+x*v/(1-v)*BA
131+x/(1-v)*BA01):
BA122:=simplify(x/(1-v)+x*BA222+x*BA123+x/(1-v)*(BA131+BA121)+x/(1
-v)*BA01):
BA123:=simplify(x/(1-v)+x*BA223+x/v*(BA121-subs(v=0,BA121))+x/(1-v
)*(BA121+BA131)+x/(1-v)*BA01):
BA131:=simplify(x/(1-v)+x*BA231+x*BA132+x/(1-v)*(BA131+BA121)+x/(1
-v)*BA01):
BA132:=simplify(x/(1-v)+x*BA232+x*BA133+x/(1-v)*BA121+x/v*(BA121-s
ubs(v=0,BA121))+x/(1-v)*BA131+x/(1-v)*BA01):
BA133:=simplify(x/(1-v)+x*BA233+x/v*(BA131+BA121-subs(v=0,BA131)-s
ubs(v=0,BA121))+x/(1-v)*(BA131+BA121)+x/(1-v)*BA01):
BA221:=simplify(x/(1-v)+x*BA321+x*BA222+x/(1-v)*BA221+x*v/(1-v)*BA
231+x/(1-v)*BA001+x/(1-v)*BA002):
BA222:=simplify(x/(1-v)+x*BA322+x*BA223+x/(1-v)*BA231+x/(1-v)*BA22
1+x/(1-v)*BA001+x/(1-v)*BA002):
BA223:=simplify(x/(1-v)+x*BA323+x/v*(BA221-subs(v=0,BA221))+x/(1-v
)*(BA221+BA231)+x/(1-v)*BA001+x/(1-v)*BA002):
BA231:=simplify(x/(1-v)+x*BA331+x*BA232+x/(1-v)*BA231+x/(1-v)*BA22
```

```
1+x/(1-v)*BA001+x/(1-v)*BA002):
BA232:=simplify(x/(1-v)+x*BA332+x*BA233+x/v*(BA221-subs(v=0,BA221)
)+x/(1-v)*(BA221+BA231)+x/(1-v)*BA001+x/(1-v)*BA002):
BA233:=simplify(x/(1-v)+x*BA333+x/v*(BA231+BA221-subs(v=0,BA231)-s
ubs(v=0,BA221))+x/(1-v)*(BA231+BA221)+x/(1-v)*BA001+x/(1-v)*BA002)
:
BA321:=simplify(x/(1-v)+x*BA322+x/(1-v)*BA321+x*v/(1-v)*BA331+x/(1
-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA322:=simplify(x/(1-v)+x*BA323+x/(1-v)*(BA331+BA321)+x/(1-v)*BA00
01+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA323:=simplify(x/(1-v)+x/v*(BA321-subs(v=0,BA321))+x/(1-v)*(BA321
+BA331)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA331:=simplify(x/(1-v)+x*BA332+x/(1-v)*(BA331+BA321)+x/(1-v)*BA00
01+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA332:=simplify(x/(1-v)+x*BA333+x/v*(BA321-subs(v=0,BA321))+x/(1-v
)*(BA321+BA331)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v)*BA0003):
BA333:=simplify(x/(1-v)+x/v*(BA331+BA321-subs(v=0,BA331)-subs(v=0,
BA321))+x/(1-v)*(BA321+BA331)+x/(1-v)*BA0001+x/(1-v)*BA0002+x/(1-v
)*BA0003):  od:
> taylor(BA0,x,15);
```

$$x + 2x^2 + 6x^3 + 21x^4 + 78x^5 + 296x^6 + 1126x^7 + 4285x^8 + 16281x^9 + 61690x^{10} + 233078x^{11} + 878164x^{12} + 3299936x^{13} + 12370320x^{14} + O(x^{15})$$

```
> ##the system A2+S3 of the eqautions:
> eq121:=-A121(v)+(x/(1-v)+x*A221(v)+x*A122(v)+x/(1-v)*A121(v)+x*v/(
1-v)*A131(v)+x/(1-v)*A01);
eq122:=-A122(v)+(x/(1-v)+x*A222(v)+x*A123(v)+x/(1-v)*(A131(v)+A121
(v))+x/(1-v)*A01);
eq123:=-A123(v)+(x/(1-v)+x*A223(v)+x/v*(A121(v)-A121(0))+x/(1-v)*(
A121(v)+A131(v))+x/(1-v)*A01);
eq131:=-A131(v)+(x/(1-v)+x*A231(v)+x*A132(v)+x/(1-v)*(A131(v)+A121
(v))+x/(1-v)*A01);
eq132:=-A132(v)+(x/(1-v)+x*A232(v)+x*A133(v)+x/(1-v)*A121(v)+x/v*(
A121(v)-A121(0))+x/(1-v)*A131(v)+x/(1-v)*A01);
eq133:=-A133(v)+(x/(1-v)+x*A233(v)+x/v*(A131(v)+A121(v)-A131(0)-A1
21(0))+x/(1-v)*(A131(v)+A121(v))+x/(1-v)*A01);
eq221:=-A221(v)+(x/(1-v)+x*A321(v)+x*A222(v)+x/(1-v)*A221(v)+x*v/(
1-v)*A231(v)+x/(1-v)*A001+x/(1-v)*A002);
eq222:=-A222(v)+(x/(1-v)+x*A322(v)+x*A223(v)+x/(1-v)*A231(v)+x/(1-
v)*A221(v)+x/(1-v)*A001+x/(1-v)*A002);
eq223:=-A223(v)+(x/(1-v)+x*A323(v)+x/v*(A221(v)-A221(0))+x/(1-v)*(
A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002);
eq231:=-A231(v)+(x/(1-v)+x*A331(v)+x*A232(v)+x/(1-v)*A231(v)+x/(1-
```

```
v)*A221(v)+x/(1-v)*A001+x/(1-v)*A002);
eq232:=-A232(v)+(x/(1-v)+x*A332(v)+x*A233(v)+x/v*(A221(v)-A221(0))
+x/(1-v)*(A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002);
eq233:=-A233(v)+(x/(1-v)+x*A333(v)+x/v*(A231(v)+A221(v)-A231(0)-A2
21(0))+x/(1-v)*(A231(v)+A221(v))+x/(1-v)*A001+x/(1-v)*A002);
eq321:=-A321(v)+(x/(1-v)+x*A322(v)+x/(1-v)*A321(v)+x*v/(1-v)*A331(
v)+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
eq322:=-A322(v)+(x/(1-v)+x*A323(v)+x/(1-v)*(A331(v)+A321(v))+x/(1-
v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
eq323:=-A323(v)+(x/(1-v)+x/v*(A321(v)-A321(0))+x/(1-v)*(A321(v)+A3
31(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
eq331:=-A331(v)+(x/(1-v)+x*A332(v)+x/(1-v)*(A331(v)+A321(v))+x/(1-
v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
eq332:=-A332(v)+(x/(1-v)+x*A333(v)+x/v*(A321(v)-A321(0))+x/(1-v)*(
A321(v)+A331(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
eq333:=-A333(v)+(x/(1-v)+x/v*(A331(v)+A321(v)-A331(0)-A321(0))+x/(
1-v)*(A321(v)+A331(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003);
```

$$eq121 := -\mathrm{A121}(v) + \frac{x}{1-v} + x\,\mathrm{A221}(v) + x\,\mathrm{A122}(v) + \frac{x\,\mathrm{A121}(v)}{1-v} + \frac{x\,v\,\mathrm{A131}(v)}{1-v} + \frac{x\,A01}{1-v}$$

$$eq122 := -\mathrm{A122}(v) + \frac{x}{1-v} + x\,\mathrm{A222}(v) + x\,\mathrm{A123}(v) + \frac{x\,(\mathrm{A131}(v) + \mathrm{A121}(v))}{1-v} + \frac{x\,A01}{1-v}$$

$$eq123 := -\mathrm{A123}(v) + \frac{x}{1-v} + x\,\mathrm{A223}(v) + \frac{x\,(\mathrm{A121}(v) - \mathrm{A121}(0))}{v} + \frac{x\,(\mathrm{A131}(v) + \mathrm{A121}(v))}{1-v} + \frac{x\,A01}{1-v}$$

$$eq131 := -\mathrm{A131}(v) + \frac{x}{1-v} + x\,\mathrm{A231}(v) + x\,\mathrm{A132}(v) + \frac{x\,(\mathrm{A131}(v) + \mathrm{A121}(v))}{1-v} + \frac{x\,A01}{1-v}$$

$$eq132 := -\mathrm{A132}(v) + \frac{x}{1-v} + x\,\mathrm{A232}(v) + x\,\mathrm{A133}(v) + \frac{x\,\mathrm{A121}(v)}{1-v} + \frac{x\,(\mathrm{A121}(v) - \mathrm{A121}(0))}{v} + \frac{x\,\mathrm{A131}(v)}{1-v} + \frac{x\,A01}{1-v}$$

$$eq133 := -\mathrm{A133}(v) + \frac{x}{1-v} + x\,\mathrm{A233}(v) + \frac{x\,(\mathrm{A131}(v) + \mathrm{A121}(v) - \mathrm{A131}(0) - \mathrm{A121}(0))}{v} + \frac{x\,(\mathrm{A131}(v) + \mathrm{A121}(v))}{1-v} + \frac{x\,A01}{1-v}$$

$$eq221 := -\mathrm{A221}(v) + \frac{x}{1-v} + x\,\mathrm{A321}(v) + x\,\mathrm{A222}(v) + \frac{x\,\mathrm{A221}(v)}{1-v} + \frac{x\,v\,\mathrm{A231}(v)}{1-v} + \frac{x\,A001}{1-v} + \frac{x\,A002}{1-v}$$

$eq222 :=$

$$-\mathrm{A222}(v)+\frac{x}{1-v}+x\,\mathrm{A322}(v)+x\,\mathrm{A223}(v)+\frac{x\,\mathrm{A231}(v)}{1-v}+\frac{x\,\mathrm{A221}(v)}{1-v}+\frac{x\,A001}{1-v}+\frac{x\,A002}{1-v}$$

$$eq223 := -\mathrm{A223}(v)+\frac{x}{1-v}+x\,\mathrm{A323}(v)+\frac{x\,(\mathrm{A221}(v)-\mathrm{A221}(0))}{v}+\frac{x\,(\mathrm{A221}(v)+\mathrm{A231}(v))}{1-v}+\frac{x\,A001}{1-v}+\frac{x\,A002}{1-v}$$

$eq231 :=$

$$-\mathrm{A231}(v)+\frac{x}{1-v}+x\,\mathrm{A331}(v)+x\,\mathrm{A232}(v)+\frac{x\,\mathrm{A231}(v)}{1-v}+\frac{x\,\mathrm{A221}(v)}{1-v}+\frac{x\,A001}{1-v}+\frac{x\,A002}{1-v}$$

$$eq232 := -\mathrm{A232}(v)+\frac{x}{1-v}+x\,\mathrm{A332}(v)+x\,\mathrm{A233}(v)+\frac{x\,(\mathrm{A221}(v)-\mathrm{A221}(0))}{v}+\frac{x\,(\mathrm{A221}(v)+\mathrm{A231}(v))}{1-v}+\frac{x\,A001}{1-v}+\frac{x\,A002}{1-v}$$

$$eq233 := -\mathrm{A233}(v)+\frac{x}{1-v}+x\,\mathrm{A333}(v)+\frac{x\,(\mathrm{A231}(v)+\mathrm{A221}(v)-\mathrm{A231}(0)-\mathrm{A221}(0))}{v}+\frac{x\,(\mathrm{A221}(v)+\mathrm{A231}(v))}{1-v}+\frac{x\,A001}{1-v}+\frac{x\,A002}{1-v}$$

$eq321 :=$

$$-\mathrm{A321}(v)+\frac{x}{1-v}+x\,\mathrm{A322}(v)+\frac{x\,\mathrm{A321}(v)}{1-v}+\frac{x\,v\,\mathrm{A331}(v)}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

$eq322 :=$

$$-\mathrm{A322}(v)+\frac{x}{1-v}+x\,\mathrm{A323}(v)+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v))}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

$$eq323 := -\mathrm{A323}(v)+\frac{x}{1-v}+\frac{x\,(\mathrm{A321}(v)-\mathrm{A321}(0))}{v}+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v))}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

$eq331 :=$

$$-\mathrm{A331}(v)+\frac{x}{1-v}+x\,\mathrm{A332}(v)+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v))}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

$$eq332 := -\mathrm{A332}(v)+\frac{x}{1-v}+x\,\mathrm{A333}(v)+\frac{x\,(\mathrm{A321}(v)-\mathrm{A321}(0))}{v}+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v))}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

$$eq333 := -\mathrm{A333}(v)+\frac{x}{1-v}+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v)-\mathrm{A331}(0)-\mathrm{A321}(0))}{v}$$

$$+\frac{x\,(\mathrm{A331}(v)+\mathrm{A321}(v))}{1-v}+\frac{x\,A0001}{1-v}+\frac{x\,A0002}{1-v}+\frac{x\,A0003}{1-v}$$

```
>
> ##The E1 expressions:
> A121:=(v)->-x*(-A221(v)*v-A122(v)*v+v*A131(v)+A01+A221(v)+A122(v)+
  1)/(-1+v+x); #by solving eq121
```

$$A121 := v \to -\frac{x\,(-\mathrm{A221}(v)\,v-\mathrm{A122}(v)\,v+v\,\mathrm{A131}(v)+A01+\mathrm{A221}(v)+\mathrm{A122}(v)+1)}{-1+v+x}$$

```
> A122:=(v)->-(x*A221(v)-A131(v)*x-v*A222(v)-x*A222(v)-v*A123(v)-x*A
  123(v)+A01+A131(v)+A222(v)+A123(v)+1)*x/(x^2+v+x-1); #then by
  solving eq122
```

$$A122 := v \to -(x\,\mathrm{A221}(v)-\mathrm{A131}(v)\,x-v\,\mathrm{A222}(v)-x\,\mathrm{A222}(v)-v\,\mathrm{A123}(v)-x\,\mathrm{A123}(v)+A01+\mathrm{A131}(v)+\mathrm{A222}(v)+\mathrm{A123}(v)+1)\,x/(x^2+v+x-1)$$

```
> A131:=v->-x*(-A231(v)*x^2-A132(v)*x^2+A222(v)*x^2+A123(v)*x^2-v*A2
  31(v)-x*A231(v)-v*A132(v)-x*A132(v)+x*A221(v)+A01+A231(v)+A132(v)+
  1)/(v+2*x-1); #then by solve eq132
```

$$A131 := v \to -x\,(-\mathrm{A231}(v)\,x^2-\mathrm{A132}(v)\,x^2+\mathrm{A222}(v)\,x^2+\mathrm{A123}(v)\,x^2-v\,\mathrm{A231}(v)-x\,\mathrm{A231}(v)-v\,\mathrm{A132}(v)-x\,\mathrm{A132}(v)+x\,\mathrm{A221}(v)+A01+\mathrm{A231}(v)+\mathrm{A132}(v)+1)/(v+2\,x-1)$$

```
> A133:=v->(v+x*A221(v)+x*A231(v)+x*A132(v)-x*A221(0)-3*A231(v)*x^2-
  3*A132(v)*x^2+A222(v)*x^2+A123(v)*x^2+2*A233(v)*v^2*x+4*A233(v)*v*
  x^2-4*A233(v)*v*x+A231(0)*v*x+A132(0)*v*x-A231(0)*v*x^2-A132(0)*v*
  x^2+2*A132(v)*x^3-A233(v)*v^2+A233(v)*v+A222(0)*v*x^2+A123(0)*v*x^
  2+A221(0)*v*x+2*A231(v)*x^3-2*A222(v)*x^3-2*A123(v)*x^3-2*A221(v)*
  x^2-2*A231(0)*x^3-2*A132(0)*x^3+2*A222(0)*x^3+2*A123(0)*x^3+2*x^2*
  A221(0)+A01*v+3*A231(0)*x^2+3*A132(0)*x^2-A222(0)*x^2-A123(0)*x^2-
  x*A231(0)-x*A132(0))*x/(2*x-1)/v/(v+2*x-1); #then by solving eq133
```

$$A133 := v \to (v+x\,\mathrm{A221}(v)-3\,\mathrm{A231}(v)\,x^2-3\,\mathrm{A132}(v)\,x^2+\mathrm{A222}(v)\,x^2+\mathrm{A123}(v)\,x^2+x\,\mathrm{A231}(v)+x\,\mathrm{A132}(v)-x\,\mathrm{A221}(0)-\mathrm{A222}(0)\,x^2-\mathrm{A123}(0)\,x^2-x\,\mathrm{A231}(0)-x\,\mathrm{A132}(0)-\mathrm{A233}(v)\,v^2+\mathrm{A233}(v)\,v+A01\,v+2\,\mathrm{A132}(v)\,x^3+2\,\mathrm{A231}(v)\,x^3-2\,\mathrm{A222}(v)\,x^3-2\,\mathrm{A123}(v)\,x^3-2\,\mathrm{A221}(v)\,x^2-2\,\mathrm{A231}(0)\,x^3-2\,\mathrm{A132}(0)\,x^3+2\,\mathrm{A222}(0)\,x^3+2\,\mathrm{A123}(0)\,x^3+2\,x^2\,\mathrm{A221}(0)+3\,\mathrm{A231}(0)\,x^2+3\,\mathrm{A132}(0)\,x^2+2\,\mathrm{A233}(v)\,v^2\,x+\mathrm{A123}(0)\,v\,x^2+\mathrm{A221}(0)\,v\,x-4\,\mathrm{A233}(v)\,v\,x+4\,\mathrm{A233}(v)\,v\,x^2+\mathrm{A222}(0)\,v\,x^2-\mathrm{A132}(0)\,v\,x^2-\mathrm{A231}(0)\,v\,x^2+\mathrm{A231}(0)\,v\,x+\mathrm{A132}(0)\,v\,x)\,x/((2\,x-1)\,v\,(v+2\,x-1))$$

```
> A221:=v->-x*(v*A231(v)-v*A222(v)-A321(v)*v+A222(v)+A321(v)+A001+A0
  02+1)/(-1+v+x); #then by solving eq221;
```

$A221 :=$

$$v \to -\frac{x\,(v\,\mathrm{A231}(v) - v\,\mathrm{A222}(v) - \mathrm{A321}(v)\,v + \mathrm{A222}(v) + \mathrm{A321}(v) + A001 + A002 + 1)}{-1 + v + x}$$

```
> A222:=v->(x*A231(v)+v*A322(v)+x*A322(v)+v*A223(v)+x*A223(v)-x*A321
  (v)-A231(v)-A322(v)-A223(v)-A001-A002-1)*x/(x^2+v+x-1); #then by
  solving eq222
```

$$A222 := v \to (x\,\mathrm{A231}(v) + v\,\mathrm{A322}(v) + x\,\mathrm{A322}(v) + v\,\mathrm{A223}(v) + x\,\mathrm{A223}(v) - x\,\mathrm{A321}(v) - \mathrm{A231}(v) - \mathrm{A322}(v) - \mathrm{A223}(v) - A001 - A002 - 1)\,x\,/\,(x^2 + v + x - 1)$$

```
> A231:=v->x*(A331(v)*x^2-A322(v)*x^2-A223(v)*x^2+A232(v)*x^2+v*A331
  (v)+x*A331(v)-x*A321(v)+A232(v)*v+x*A232(v)-A331(v)-A232(v)-A001-A
  002-1)/(v+2*x-1); #then by solving eq231
```

$$A231 := v \to x\,(\mathrm{A331}(v)\,x^2 - \mathrm{A322}(v)\,x^2 - \mathrm{A223}(v)\,x^2 + \mathrm{A232}(v)\,x^2 + v\,\mathrm{A331}(v) + x\,\mathrm{A331}(v) - x\,\mathrm{A321}(v) + \mathrm{A232}(v)\,v + x\,\mathrm{A232}(v) - \mathrm{A331}(v) - \mathrm{A232}(v) - A001 - A002 - 1)\,/\,(v + 2\,x - 1)$$

```
> A233:=v->(v+x*A232(v)+x*A321(v)+x*A331(v)-A333(v)*v^2+A333(v)*v+2*
  A232(v)*x^3+2*A333(v)*v^2*x+4*A333(v)*v*x^2-4*A333(v)*v*x+A331(0)*
  v*x+A232(0)*v*x-A331(0)*v*x^2-A232(0)*v*x^2-2*A331(0)*x^3-2*A232(0
  )*x^3+2*A331(v)*x^3-2*A223(v)*x^3+3*A331(0)*x^2+3*A232(0)*x^2-x*A3
  31(0)-x*A232(0)+A322(0)*v*x^2+A223(0)*v*x^2+A321(0)*v*x-3*A331(v)*
  x^2+A223(v)*x^2-3*A232(v)*x^2-2*A322(v)*x^3+A322(v)*x^2-2*A321(v)*
  x^2+2*A322(0)*x^3+2*A223(0)*x^3-A322(0)*x^2-A223(0)*x^2+2*A321(0)*
  x^2-x*A321(0)+A002*v+A001*v)*x/(2*x-1)/v/(v+2*x-1); #then by
  solving eq233
```

$$A233 := v \to (v - 3\,\mathrm{A331}(v)\,x^2 + \mathrm{A322}(v)\,x^2 + \mathrm{A223}(v)\,x^2 - 3\,\mathrm{A232}(v)\,x^2 + x\,\mathrm{A331}(v) + x\,\mathrm{A232}(v) - x\,\mathrm{A331}(0) - x\,\mathrm{A232}(0) - \mathrm{A322}(0)\,x^2 - \mathrm{A223}(0)\,x^2 - x\,\mathrm{A321}(0) - \mathrm{A333}(v)\,v^2 + \mathrm{A333}(v)\,v + A002\,v + A001\,v + 2\,\mathrm{A232}(v)\,x^3 - 2\,\mathrm{A331}(0)\,x^3 - 2\,\mathrm{A232}(0)\,x^3 + 2\,\mathrm{A331}(v)\,x^3 - 2\,\mathrm{A223}(v)\,x^3 + 3\,\mathrm{A331}(0)\,x^2 + 3\,\mathrm{A232}(0)\,x^2 - 2\,\mathrm{A322}(v)\,x^3 - 2\,\mathrm{A321}(v)\,x^2 + 2\,\mathrm{A322}(0)\,x^3 + 2\,\mathrm{A223}(0)\,x^3 + 2\,\mathrm{A321}(0)\,x^2 + x\,\mathrm{A321}(v) + \mathrm{A322}(0)\,v\,x^2 - \mathrm{A232}(0)\,v\,x^2 + \mathrm{A232}(0)\,v\,x - \mathrm{A331}(0)\,v\,x^2 + \mathrm{A331}(0)\,v\,x + 2\,\mathrm{A333}(v)\,v^2\,x + \mathrm{A223}(0)\,v\,x^2 + \mathrm{A321}(0)\,v\,x - 4\,\mathrm{A333}(v)\,v\,x + 4\,\mathrm{A333}(v)\,v\,x^2)\,x\,/\,((2\,x - 1)\,v\,(v + 2\,x - 1))$$

```
> A321:=v->-x*(v*A331(v)-v*A322(v)+A322(v)+A0001+A0002+A0003+1)/(-1+
  v+x); #then by solving eq321
```

$$A321 := v \to -\frac{x\,(v\,\mathrm{A331}(v) - v\,\mathrm{A322}(v) + \mathrm{A322}(v) + A0001 + A0002 + A0003 + 1)}{-1 + v + x}$$

```
> A322:=v->(x*A331(v)+A323(v)*v+x*A323(v)-A331(v)-A323(v)-A0001-A000
  2-A0003-1)*x/(x^2+v+x-1);  #then by solving eq322
```

$$A322 := v \to \frac{(x\,\mathrm{A331}(v) + \mathrm{A323}(v)\,v + x\,\mathrm{A323}(v) - \mathrm{A331}(v) - \mathrm{A323}(v) - A0001 - A0002 - A0003 - 1)\,x}{x^2 + v + x - 1}$$

```
> A331:=v->x*(A332(v)*x^2-A323(v)*x^2+A332(v)*v+x*A332(v)-A332(v)-A0
```

```
001-A0002-A0003-1)/(v+2*x-1); #then by solving eq331
```

$$A331 := v \to x \left(\mathrm{A332}(v)\, x^2 - \mathrm{A323}(v)\, x^2 + \mathrm{A332}(v)\, v + x\, \mathrm{A332}(v) - \mathrm{A332}(v) - A0001 - A0002 - A0003 - 1\right) / (v + 2\,x - 1)$$

```
> A333:=v->(2*A332(v)*x^3-2*A323(v)*x^3-A332(0)*v*x^2-2*A332(0)*x^3+
A323(0)*v*x^2+2*A323(0)*x^3-3*A332(v)*x^2+A323(v)*x^2+A332(0)*v*x+
3*A332(0)*x^2-A323(0)*x^2+x*A332(v)-x*A332(0)+A0001*v+A0002*v+A000
3*v+v)*x/(2*x-1)/(v+2*x-1)/v; #then by solving eq333
```

$$A333 := v \to \left(2\,\mathrm{A332}(v)\, x^3 - 2\,\mathrm{A323}(v)\, x^3 - \mathrm{A332}(0)\, v\, x^2 - 2\,\mathrm{A332}(0)\, x^3 + \mathrm{A323}(0)\, v\, x^2 + 2\,\mathrm{A323}(0)\, x^3 - 3\,\mathrm{A332}(v)\, x^2 + \mathrm{A323}(v)\, x^2 + \mathrm{A332}(0)\, v\, x + 3\,\mathrm{A332}(0)\, x^2 - \mathrm{A323}(0)\, x^2 + x\,\mathrm{A332}(v) - x\,\mathrm{A332}(0) + A0001\, v + A0002\, v + A0003\, v + v\right) x / \left((2\,x - 1)\,(v + 2\,x - 1)\, v\right)$$

```
> ##System S3
> eq123:=-A123(v)+(x/(1-v)+x*A223(v)+x/v*(A121(v)-A121(0))+x/(1-v)*(
A121(v)+A131(v))+x/(1-v)*A01):
eq132:=-A132(v)+(x/(1-v)+x*A232(v)+x*A133(v)+x/(1-v)*A121(v)+x/v*(
A121(v)-A121(0))+x/(1-v)*A131(v)+x/(1-v)*A01):
eq223:=-A223(v)+(x/(1-v)+x*A323(v)+x/v*(A221(v)-A221(0))+x/(1-v)*(
A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002):
eq232:=-A232(v)+(x/(1-v)+x*A332(v)+x*A233(v)+x/v*(A221(v)-A221(0))
+x/(1-v)*(A221(v)+A231(v))+x/(1-v)*A001+x/(1-v)*A002):
eq323:=-A323(v)+(x/(1-v)+x/v*(A321(v)-A321(0))+x/(1-v)*(A321(v)+A3
31(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003):
eq332:=-A332(v)+(x/(1-v)+x*A333(v)+x/v*(A321(v)-A321(0))+x/(1-v)*(
A321(v)+A331(v))+x/(1-v)*A0001+x/(1-v)*A0002+x/(1-v)*A0003):
> #---matrix form of these 6 equations:
> with(linalg): A:=matrix(6,13,0):
> d:=1: eq:=eq123: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
```

```
> d:=2: eq:=eq132: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
> d:=3: eq:=eq223: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
> d:=4: eq:=eq232: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
```

```
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
> d:=5: eq:=eq323: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
> d:=6: eq:=eq332: A[d,1]:=factor(coeff(eq,A123(v))):
A[d,2]:=factor(coeff(eq,A132(v))):
A[d,3]:=factor(coeff(eq,A223(v))):
A[d,4]:=factor(coeff(eq,A232(v))):
A[d,5]:=factor(coeff(eq,A323(v))):
A[d,6]:=factor(coeff(eq,A332(v))):
A[d,7]:=factor(coeff(eq,A123(0))):
A[d,8]:=factor(coeff(eq,A132(0))):
A[d,9]:=factor(coeff(eq,A223(0))):
A[d,10]:=factor(coeff(eq,A232(0))):
A[d,11]:=factor(coeff(eq,A323(0))):
A[d,12]:=factor(coeff(eq,A332(0))):
A[d,13]:=factor(subs(A123(v)=0,A132(v)=0,A223(v)=0,A232(v)=0,A323(
v)=0,A332(v)=0,A123(0)=0,A132(0)=0,A223(0)=0,A232(0)=0,A323(0)=0,A
332(0)=0,eq)):
> A:=rref(A):
B:=evalm((-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v^2*x-v^2)*A):
B:=map(factor,B):
>
> #Solving to find A_r(0) and A_r(v) for all
r=123,132,223,232,323,332
> KK:=(-x^6-v*x^4+v^2*x^2+2*v*x^3+v^3+2*v^2*x-v^2); #the kernel:
```

$$KK := -x^6 - v\,x^4 + v^2\,x^2 + 2\,v\,x^3 + v^3 + 2\,v^2\,x - v^2$$

```
> i:=6:
eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A232(v)
```

```
+B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+B[i,9
]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A332(0)+
B[i,13];
```

$$eq_6 := (-x^6 - v\,x^4 + v^2\,x^2 + 2\,v\,x^3 + v^3 + 2\,v^2\,x - v^2)\,\mathrm{A332}(v) - \frac{(v + 2\,x - 1)\,v\,x^3\,\mathrm{A323}(0)}{2\,x - 1} - \frac{(-2\,x^4 + x^3 + v^2 + 2\,v\,x - v)\,x^3\,\mathrm{A332}(0)}{2\,x - 1} - \frac{(1 + A0001 + A0002 + A0003)\,x\,v^2}{2\,x - 1}$$

```
> i:=4:
eq[i]:=factor(KK*B[i,1])*A123(v)+factor(KK*B[i,2])*A132(v)+factor(
KK*B[i,3])*A223(v)+DD(factor(KK*B[i,4])*A232(v))+factor(KK*B[i,5])
*A323(v)+factor(diff(KK*B[i,6],v))*A332(v)+factor(diff(KK*B[i,7],v
))*A123(0)+factor(diff(KK*B[i,8],v))*A132(0)+factor(diff(KK*B[i,9]
,v))*A223(0)+factor(diff(KK*B[i,10],v))*A232(0)+factor(diff(KK*B[i
,11],v))*A323(0)+factor(diff(KK*B[i,12],v))*A332(0)+factor(diff(KK
*B[i,13],v));
```

$$eq_4 := \mathrm{DD}((-x^6 - v\,x^4 + v^2\,x^2 + 2\,v\,x^3 + v^3 + 2\,v^2\,x - v^2)^2\,\mathrm{A232}(v)) - x^3\,(-2\,v\,x^6 - 2\,x^7 - 3\,v^2\,x^4 - 4\,v\,x^5 + x^6 + 4\,v^3\,x^2 + 12\,v^2\,x^3 + 10\,v\,x^4 + 5\,v^4 + 16\,v^3\,x + 9\,v^2\,x^2 - 4\,v\,x^3 - 8\,v^3 - 12\,v^2\,x + 3\,v^2)\,\mathrm{A223}(0)\,/\,(2\,x - 1) - x^3\,(2\,x^8 - 6\,v\,x^6 - 7\,x^7 - 9\,v^2\,x^4 - 10\,v\,x^5 + 3\,x^6 + 4\,v^3\,x^2 + 15\,v^2\,x^3 + 18\,v\,x^4 + 5\,v^4 + 16\,v^3\,x + 9\,v^2\,x^2 - 6\,v\,x^3 - 8\,v^3 - 12\,v^2\,x + 3\,v^2)\,\mathrm{A232}(0)\,/\,(2\,x - 1) - x^4\,(16\,v\,x^7 + 12\,x^8 + 12\,v^2\,x^5 + 2\,v\,x^6 - 12\,x^7 - 3\,v^2\,x^4 - 8\,v\,x^5 + 3\,x^6 + 20\,v^4\,x + 60\,v^3\,x^2 + 36\,v^2\,x^3 + 2\,v\,x^4 - 15\,v^4 - 80\,v^3\,x - 81\,v^2\,x^2 + 24\,v^3 + 48\,v^2\,x - 9\,v^2)\,\mathrm{A323}(0)\,/\,(2\,x - 1)^2 - x^4\,(8\,x^9 - 16\,v\,x^7 - 20\,x^8 - 60\,v^2\,x^5 - 46\,v\,x^6 + 14\,x^7 + 69\,v^2\,x^4 + 112\,v\,x^5 - 3\,x^6 + 20\,v^4\,x + 60\,v^3\,x^2 + 18\,v^2\,x^3 - 66\,v\,x^4 - 15\,v^4 - 80\,v^3\,x - 81\,v^2\,x^2 + 12\,v\,x^3 + 24\,v^3 + 48\,v^2\,x - 9\,v^2)\,\mathrm{A332}(0)\,/\,(2\,x - 1)^2 + x\,v\,(-24\,x^8\,A0001 - 24\,x^8\,A0002 - 24\,x^8\,A0003 - 24\,v\,x^6\,A0001 - 24\,v\,x^6\,A0002 - 24\,v\,x^6\,A0003 - 24\,x^8 + 10\,x^7\,A0001 + 10\,x^7\,A0002 + 10\,x^7\,A0003 + 4\,x^7\,A001 + 4\,x^7\,A002 + 16\,v^2\,x^4\,A0001 + 16\,v^2\,x^4\,A0002 + 16\,v^2\,x^4\,A0003 - 24\,v\,x^6 + 45\,v\,x^5\,A0001 + 45\,v\,x^5\,A0002 + 45\,v\,x^5\,A0003 + 6\,v\,x^5\,A001 + 6\,v\,x^5\,A002 + 14\,x^7 - 2\,x^6\,A001 - 2\,x^6\,A002 + 16\,v^2\,x^4 + 12\,v^2\,x^3\,A0001 + 12\,v^2\,x^3\,A0002 + 12\,v^2\,x^3\,A0003 - 8\,v^2\,x^3\,A001 - 8\,v^2\,x^3\,A002 + 51\,v\,x^5 - 12\,v\,x^4\,A0001 - 12\,v\,x^4\,A0002 - 12\,v\,x^4\,A0003 - 15\,v\,x^4\,A001 - 15\,v\,x^4\,A002 - 2\,x^6 + 5\,v^3\,x\,A0001 + 5\,v^3\,x\,A0002 + 5\,v^3\,x\,A0003 - 10\,v^3\,x\,A001 - 10\,v^3\,x\,A002 + 4\,v^2\,x^3 - 12\,v^2\,x^2\,A001 - 12\,v^2\,x^2\,A002 - 27\,v\,x^4 + 6\,v\,x^3\,A001 + 6\,v\,x^3\,A002 - 5\,v^3\,x + 5\,v^3\,A001 + 5\,v^3\,A002 - 12\,v^2\,x^2 - 4\,v^2\,x\,A0001 - 4\,v^2\,x\,A0002 - 4\,v^2\,x\,A0003 + 16\,v^2\,x\,A001 + 16\,v^2\,x\,A002 + 6\,v\,x^3 + 5\,v^3 + 12\,v^2\,x - 4\,v^2\,A001 - 4\,v^2\,A002 - 4\,v^2)\,/\,(2\,x - 1)^2$$

```
> i:=2:
eq[i]:=factor(KK^2*B[i,1])*A123(v)+DD2(factor(KK^2*B[i,2])*A132(v)
```

```
)+factor(KK^2*B[i,3])*A223(v)+factor(KK^2*B[i,4])*A232(v)+factor(K
K^2*B[i,5])*A323(v)+factor(diff(KK^2*B[i,6],v,v))*A332(v)+factor(d
iff(KK^2*B[i,7],v,v))*A123(0)+factor(diff(KK^2*B[i,8],v,v))*A132(0
)+factor(diff(KK^2*B[i,9],v,v))*A223(0)+factor(diff(KK^2*B[i,10],v
,v))*A232(0)+factor(diff(KK^2*B[i,11],v,v))*A323(0)+factor(diff(KK
^2*B[i,12],v,v))*A332(0)+factor(diff(KK^2*B[i,13],v,v));
```

$$
\begin{aligned}
eq_2 := {} & \mathrm{DD2}((-x^6 - v\,x^4 + v^2\,x^2 + 2\,v\,x^3 + v^3 + 2\,v^2\,x - v^2)^3\,\mathrm{A132}(v)) - 2\,x^3\,(x^{12} + 6\,v\,x^{10} + 4\,x^{11} \\
& - 6\,v^2\,x^8 - 18\,v\,x^9 - 10\,x^{10} - 40\,v^3\,x^6 - 96\,v^2\,x^7 - 45\,v\,x^8 + 4\,x^9 - 15\,v^4\,x^4 - 20\,v^3\,x^5 + 60\,v^2\,x^6 \\
& + 60\,v\,x^7 + 42\,v^5\,x^2 + 180\,v^4\,x^3 + 270\,v^3\,x^4 + 120\,v^2\,x^5 - 18\,v\,x^6 + 28\,v^6 + 126\,v^5\,x + 120\,v^4\,x^2 \\
& - 80\,v^3\,x^3 - 108\,v^2\,x^4 - 63\,v^5 - 180\,v^4\,x - 100\,v^3\,x^2 + 24\,v^2\,x^3 + 45\,v^4 + 60\,v^3\,x - 10\,v^3)\,\mathrm{A123}(0) \\
& / (2\,x - 1) - 2\,x^3\,(3\,x^{12} + 30\,v\,x^{10} + 19\,x^{11} + 6\,v^2\,x^8 - 30\,v\,x^9 - 30\,x^{10} - 80\,v^3\,x^6 - 198\,v^2\,x^7 \\
& - 105\,v\,x^8 + 10\,x^9 - 45\,v^4\,x^4 - 80\,v^3\,x^5 + 84\,v^2\,x^6 + 114\,v\,x^7 + 42\,v^5\,x^2 + 195\,v^4\,x^3 + 350\,v^3\,x^4 \\
& + 180\,v^2\,x^5 - 30\,v\,x^6 + 28\,v^6 + 126\,v^5\,x + 120\,v^4\,x^2 - 100\,v^3\,x^3 - 144\,v^2\,x^4 - 63\,v^5 - 180\,v^4\,x \\
& - 100\,v^3\,x^2 + 30\,v^2\,x^3 + 45\,v^4 + 60\,v^3\,x - 10\,v^3)\,\mathrm{A132}(0) / (2\,x - 1) - 2\,x^4\,(-8\,x^{13} - 36\,v\,x^{11} \\
& - 13\,x^{12} + 24\,v^2\,x^9 + 84\,v\,x^{10} + 40\,x^{11} + 80\,v^3\,x^7 + 138\,v^2\,x^8 + 18\,v\,x^9 - 28\,x^{10} - 40\,v^3\,x^6 - 96\,v^2\,x^7 \\
& - 45\,v\,x^8 + 6\,x^9 + 84\,v^5\,x^3 + 375\,v^4\,x^4 + 520\,v^3\,x^5 + 240\,v^2\,x^6 + 12\,v\,x^7 + 112\,v^6\,x + 420\,v^5\,x^2 \\
& + 240\,v^4\,x^3 - 470\,v^3\,x^4 - 384\,v^2\,x^5 - 84\,v^6 - 630\,v^5\,x - 1140\,v^4\,x^2 - 380\,v^3\,x^3 + 204\,v^2\,x^4 + 189\,v^5 \\
& + 720\,v^4\,x + 560\,v^3\,x^2 - 36\,v^2\,x^3 - 135\,v^4 - 220\,v^3\,x + 30\,v^3)\,\mathrm{A223}(0) \big/ (2\,x - 1)^2 - 2\,x^4\,(\\
& 108\,v\,x^{11} + 59\,x^{12} + 120\,v^2\,x^9 - 60\,v\,x^{10} - 110\,x^{11} - 320\,v^3\,x^7 - 822\,v^2\,x^8 - 426\,v\,x^9 + 64\,x^{10} \\
& - 360\,v^4\,x^5 - 520\,v^3\,x^6 + 552\,v^2\,x^7 + 651\,v\,x^8 - 12\,x^9 + 84\,v^5\,x^3 + 735\,v^4\,x^4 + 1780\,v^3\,x^5 \\
& + 792\,v^2\,x^6 - 324\,v\,x^7 + 112\,v^6\,x + 420\,v^5\,x^2 + 150\,v^4\,x^3 - 1170\,v^3\,x^4 - 1116\,v^2\,x^5 + 54\,v\,x^6 \\
& - 84\,v^6 - 630\,v^5\,x - 1140\,v^4\,x^2 - 260\,v^3\,x^3 + 480\,v^2\,x^4 + 189\,v^5 + 720\,v^4\,x + 560\,v^3\,x^2 - 72\,v^2\,x^3 \\
& - 135\,v^4 - 220\,v^3\,x + 30\,v^3)\,\mathrm{A232}(0) \big/ (2\,x - 1)^2 - 2\,x^5\,(24\,x^{14} + 36\,v\,x^{12} - 24\,x^{13} + 264\,v^2\,x^{10} \\
& + 360\,v\,x^{11} + 111\,x^{12} + 1200\,v^3\,x^8 + 2016\,v^2\,x^9 + 393\,v\,x^{10} - 150\,x^{11} + 720\,v^4\,x^6 + 80\,v^3\,x^7 \\
& - 3168\,v^2\,x^8 - 1836\,v\,x^9 + 73\,x^{10} - 84\,v^5\,x^4 - 1080\,v^4\,x^5 - 2460\,v^3\,x^6 + 504\,v^2\,x^7 + 1656\,v\,x^8 \\
& - 12\,x^9 + 336\,v^6\,x^2 + 1512\,v^5\,x^3 + 2445\,v^4\,x^4 + 2940\,v^3\,x^5 + 1386\,v^2\,x^6 - 606\,v\,x^7 - 448\,v^6\,x \\
& - 2709\,v^5\,x^2 - 4770\,v^4\,x^3 - 3050\,v^3\,x^4 - 1008\,v^2\,x^5 + 81\,v\,x^6 + 168\,v^6 + 1764\,v^5\,x + 4410\,v^4\,x^2 \\
& + 3000\,v^3\,x^3 + 300\,v^2\,x^4 - 378\,v^5 - 1800\,v^4\,x - 1770\,v^3\,x^2 - 36\,v^2\,x^3 + 270\,v^4 + 520\,v^3\,x - 60\,v^3) \\
& \mathrm{A323}(0) \big/ (2\,x - 1)^3 - 2\,x^5\,(-48\,x^{14} - 108\,v\,x^{12} + 52\,x^{13} + 168\,v^2\,x^{10} + 288\,v\,x^{11} + 33\,x^{12} \\
& - 960\,v^3\,x^8 - 1872\,v^2\,x^9 - 1107\,v\,x^{10} - 57\,x^{11} - 1800\,v^4\,x^6 - 2280\,v^3\,x^7 + 1224\,v^2\,x^8 + 1566\,v\,x^9 \\
& + 23\,x^{10} - 84\,v^5\,x^4 + 2700\,v^4\,x^5 + 7680\,v^3\,x^6 + 3228\,v^2\,x^7 - 972\,v\,x^8 - 3\,x^9 + 336\,v^6\,x^2 \\
& + 1512\,v^5\,x^3 + 555\,v^4\,x^4 - 6030\,v^3\,x^5 - 5346\,v^2\,x^6 + 270\,v\,x^7 - 448\,v^6\,x - 2709\,v^5\,x^2 - 4455\,v^4\,x^3 \\
& + 170\,v^3\,x^4 + 3222\,v^2\,x^5 - 27\,v\,x^6 + 168\,v^6 + 1764\,v^5\,x + 4410\,v^4\,x^2 + 2580\,v^3\,x^3 - 876\,v^2\,x^4
\end{aligned}
$$

$$- 378\, v^5 - 1800\, v^4\, x - 1770\, v^3\, x^2 + 90\, v^2\, x^3 + 270\, v^4 + 520\, v^3\, x - 60\, v^3)\ \mathrm{A332}(0) \Big/ (2\, x - 1)^3 -$$
$$2\, x\, (675\, v^4\, x^5 - 390\, v^4\, x^4 - 680\, v^3\, x^5 - 240\, v^4\, x^3 + 280\, v^3\, x^4 + 285\, v^4\, x^2 - 105\, v^4\, x + 570\, v^4\, x^6$$
$$+ 273\, v^5\, x^4 + 360\, v^3\, x^6 + 168\, v^5\, x^3 + 36\, v^2\, x^6 - 378\, v^5\, x^2 + 210\, v^5\, x - 40\, v^3\, x^3 + 15\, v^4 - 42\, v^5$$
$$+ 4\, A01\, x^{14} - 4\, A01\, x^{13} + A01\, x^{12} + 24\, A01\, v\, x^{12} - 24\, A01\, v^2\, x^{10} - 72\, A01\, v\, x^{11} - 160\, A01\, v^3\, x^8$$
$$- 168\, A01\, v^2\, x^9 + 54\, A01\, v\, x^{10} - 60\, A01\, v^4\, x^6 + 160\, A01\, v^3\, x^7 + 330\, A01\, v^2\, x^8 - 12\, A01\, v\, x^9$$
$$+ 168\, A01\, v^5\, x^4 + 540\, A01\, v^4\, x^5 + 360\, A01\, v^3\, x^6 - 192\, A01\, v^2\, x^7 + 112\, A01\, v^6\, x^2 + 168\, A01\, v^5\, x^3$$
$$- 375\, A01\, v^4\, x^4 - 560\, A01\, v^3\, x^5 + 36\, A01\, v^2\, x^6 - 112\, A01\, v^6\, x - 462\, A01\, v^5\, x^2 - 240\, A01\, v^4\, x^3$$
$$+ 260\, A01\, v^3\, x^4 + 252\, A01\, v^5\, x + 330\, A01\, v^4\, x^2 - 40\, A01\, v^3\, x^3 - 120\, A01\, v^4\, x + 204\, v\, x^{14}\, A0001$$
$$+ 204\, v\, x^{14}\, A0002 + 204\, v\, x^{14}\, A0003 + 144\, v^2\, x^{12}\, A0001 + 144\, v^2\, x^{12}\, A0002 + 144\, v^2\, x^{12}\, A0003$$
$$- 360\, v\, x^{13}\, A0001 - 360\, v\, x^{13}\, A0002 - 360\, v\, x^{13}\, A0003 - 120\, v\, x^{13}\, A001 - 120\, v\, x^{13}\, A002$$
$$+ 720\, v^3\, x^{10}\, A0001 + 720\, v^3\, x^{10}\, A0002 + 720\, v^3\, x^{10}\, A0003 - 96\, v^2\, x^{11}\, A0001 - 96\, v^2\, x^{11}\, A0002$$
$$- 96\, v^2\, x^{11}\, A0003 + 96\, v^2\, x^{11}\, A001 + 96\, v^2\, x^{11}\, A002 + 135\, v\, x^{12}\, A0001 + 135\, v\, x^{12}\, A0002$$
$$+ 135\, v\, x^{12}\, A0003 + 324\, v\, x^{12}\, A001 + 324\, v\, x^{12}\, A002 + 660\, v^4\, x^8\, A0001 + 660\, v^4\, x^8\, A0002$$
$$+ 660\, v^4\, x^8\, A0003 - 480\, v^3\, x^9\, A0001 - 480\, v^3\, x^9\, A0002 - 480\, v^3\, x^9\, A0003 + 480\, v^3\, x^9\, A001$$
$$+ 480\, v^3\, x^9\, A002 - 594\, v^2\, x^{10}\, A0001 - 594\, v^2\, x^{10}\, A0002 - 594\, v^2\, x^{10}\, A0003 + 588\, v^2\, x^{10}\, A001$$
$$+ 588\, v^2\, x^{10}\, A002 - 12\, v\, x^{11}\, A0001 - 12\, v\, x^{11}\, A0002 - 12\, v\, x^{11}\, A0003 - 216\, v\, x^{11}\, A001$$
$$- 216\, v\, x^{11}\, A002 - 1440\, v^4\, x^7\, A0001 - 1440\, v^4\, x^7\, A0002 - 1440\, v^4\, x^7\, A0003 + 120\, v^4\, x^7\, A001$$
$$+ 120\, v^4\, x^7\, A002 - 780\, v^3\, x^8\, A0001 - 780\, v^3\, x^8\, A0002 - 780\, v^3\, x^8\, A0003 - 400\, v^3\, x^8\, A001$$
$$- 400\, v^3\, x^8\, A002 + 576\, v^2\, x^9\, A0001 + 576\, v^2\, x^9\, A0002 + 576\, v^2\, x^9\, A0003 - 990\, v^2\, x^9\, A001$$
$$- 990\, v^2\, x^9\, A002 + 42\, v\, x^{10}\, A001 + 42\, v\, x^{10}\, A002 + 168\, v^5\, x^5\, A0001 + 168\, v^5\, x^5\, A0002$$
$$+ 168\, v^5\, x^5\, A0003 - 168\, v^5\, x^5\, A001 - 168\, v^5\, x^5\, A002 + 1440\, v^4\, x^6\, A0001 + 1440\, v^4\, x^6\, A0002$$
$$+ 1440\, v^4\, x^6\, A0003 - 810\, v^4\, x^6\, A001 - 810\, v^4\, x^6\, A002 + 1560\, v^3\, x^7\, A0001 + 1560\, v^3\, x^7\, A0002$$
$$+ 1560\, v^3\, x^7\, A0003 - 720\, v^3\, x^7\, A001 - 720\, v^3\, x^7\, A002 - 126\, v^2\, x^8\, A0001 - 126\, v^2\, x^8\, A0002$$
$$- 126\, v^2\, x^8\, A0003 + 480\, v^2\, x^8\, A001 + 480\, v^2\, x^8\, A002 + 189\, v^5\, x^4\, A0001 + 189\, v^5\, x^4\, A0002$$
$$+ 189\, v^5\, x^4\, A0003 - 84\, v^5\, x^4\, A001 - 84\, v^5\, x^4\, A002 - 240\, v^4\, x^5\, A0001 - 240\, v^4\, x^5\, A0002$$
$$- 240\, v^4\, x^5\, A0003 + 375\, v^4\, x^5\, A001 + 375\, v^4\, x^5\, A002 - 840\, v^3\, x^6\, A0001 - 840\, v^3\, x^6\, A0002$$
$$- 840\, v^3\, x^6\, A0003 + 840\, v^3\, x^6\, A001 + 840\, v^3\, x^6\, A002 - 72\, v^2\, x^7\, A001 - 72\, v^2\, x^7\, A002$$
$$- 135\, v^4\, x^4\, A0001 - 135\, v^4\, x^4\, A0002 - 135\, v^4\, x^4\, A0003 + 120\, v^4\, x^4\, A001 + 120\, v^4\, x^4\, A002$$
$$+ 140\, v^3\, x^5\, A0001 + 140\, v^3\, x^5\, A0002 + 140\, v^3\, x^5\, A0003 - 260\, v^3\, x^5\, A001 - 260\, v^3\, x^5\, A002$$
$$- 42\, v^5\, x^2\, A0001 - 42\, v^5\, x^2\, A0002 - 42\, v^5\, x^2\, A0003 + 126\, v^5\, x^2\, A001 + 126\, v^5\, x^2\, A002$$
$$+ 20\, v^3\, x^4\, A001 + 20\, v^3\, x^4\, A002 - 42\, v^5\, x\, A001 - 42\, v^5\, x\, A002 + 15\, v^4\, x^2\, A0001 + 15\, v^4\, x^2\, A0002$$
$$+ 15\, v^4\, x^2\, A0003 - 60\, v^4\, x^2\, A001 - 60\, v^4\, x^2\, A002 + 15\, v^4\, x\, A001 + 15\, v^4\, x\, A002$$
$$- 252\, x^6\, v^5\, A0001 - 252\, x^6\, v^5\, A0002 - 252\, x^6\, v^5\, A0003 + 28\, x^2\, v^6\, A0001 + 28\, x^2\, v^6\, A0002$$

$$+ 28\, x^2\, v^6\, A0003 - 56\, x^2\, v^6\, A001 - 56\, x^2\, v^6\, A002 + 28\, x\, v^6\, A001 + 28\, x\, v^6\, A002 + 60\, x^{16} - 72\, x^{15}$$
$$+ 204\, v\, x^{14} + 36\, x^{14} - 9\, x^{13} + x^{12} - 480\, v\, x^{13} + 144\, v^2\, x^{12} + 483\, v\, x^{12} - 300\, v\, x^{11} + 720\, v^3\, x^{10}$$
$$- 30\, v^2\, x^{10} + 96\, v\, x^{10} + 28\, v^6 - 582\, v^2\, x^9 - 12\, v\, x^9 + 660\, v^4\, x^8 - 1340\, v^3\, x^8 + 684\, v^2\, x^8 - 252\, v^5\, x^6$$
$$- 1320\, v^4\, x^7 + 1000\, v^3\, x^7 - 264\, v^2\, x^7 + 84\, v^6\, x^2 - 84\, v^6\, x + 28\, A01\, v^6 - 42\, A01\, v^5 + 15\, A01\, v^4$$
$$+ 60\, x^{16}\, A0001 + 60\, x^{16}\, A0002 + 60\, x^{16}\, A0003 - 24\, x^{15}\, A001 - 24\, x^{15}\, A002 + 10\, x^{14}\, A0001$$
$$+ 10\, x^{14}\, A0002 + 10\, x^{14}\, A0003 - 48\, x^{15}\, A0001 - 48\, x^{15}\, A0002 - 48\, x^{15}\, A0003 + 22\, x^{14}\, A001$$
$$+ 22\, x^{14}\, A002 - 5\, x^{13}\, A001 - 5\, x^{13}\, A002) \Big/ (2\, x - 1)^3$$

```
> #the roots of the kernel
> alias(kv1=RootOf(KK=0,v)): KAV:=allvalues(kv1):
> simplify(series(KAV[1],x,10)) assuming x>0 and x<0.1; u0:=KAV[1]:
  u1:=KAV[2]: u2:=KAV[3]:
```

$$1 - 2\, x - x^2 - 2\, x^3 - 3\, x^4 - 8\, x^5 - 22\, x^6 - 62\, x^7 - 182\, x^8 - 548\, x^9 + O(x^{10})$$

```
> ##finding A323(0) and A332(0)
> S1:=solve({subs(v=uu1,subs(KK=0,eq[6])),subs(v=uu2,subs(KK=0,eq[6]
  ))},{A323(0),A332(0)}): S1:=map(factor,S1); op(op(S1)[1])[1];
  A323x0:=op(op(S1)[1])[2]; op(op(S1)[2])[1];
  A332x0:=op(op(S1)[2])[2];
```

$$S1 := \{\mathrm{A323}(0) = \frac{(1 + A0001 + A0002 + A0003)\,(-uu1\, x^3 - uu2\, x^3 + uu1\, uu2)}{x^5\,(uu1 + uu2 + 2\, x - 1)},$$
$$\mathrm{A332}(0) = -\frac{uu1\, uu2\,(1 + A0001 + A0002 + A0003)}{x^5\,(uu1 + uu2 + 2\, x - 1)}\}$$

$$\mathrm{A323}(0)$$

$$A323x0 := \frac{(1 + A0001 + A0002 + A0003)\,(-uu1\, x^3 - uu2\, x^3 + uu1\, uu2)}{x^5\,(uu1 + uu2 + 2\, x - 1)}$$

$$\mathrm{A332}(0)$$

$$A332x0 := -\frac{uu1\, uu2\,(1 + A0001 + A0002 + A0003)}{x^5\,(uu1 + uu2 + 2\, x - 1)}$$

```
> #finding A223(0) and A232(0)
> ee:=subs(A323(0)=A323x0,A332(0)=A332x0,subs(DD(0)=0,subs(KK=0,eq[4
  ]))):
> S2:=solve({subs(v=uu1,ee)=0,subs(v=uu2,ee)},{A223(0),A232(0)}):
  S2:=map(factor,S2):
> op(op(S2)[1])[1]; A223x0:=op(op(S2)[1])[2]: op(op(S2)[2])[1];
  A232x0:=op(op(S2)[2])[2]:
```

$$\mathrm{A223}(0)$$

$$\mathrm{A232}(0)$$

```
> #finding A123(0) and A132(0)
```

```
> ee:=factor(subs(A223(0)=A223x0,A232(0)=A232x0,A323(0)=A323x0,A332(
  0)=A332x0,subs(DD2(0)=0,subs(KK=0,eq[2])))):
> S3:=solve({subs(v=uu1,ee)=0,subs(v=uu2,ee)},{A123(0),A132(0)}):
  S3:=map(factor,S3):
> op(op(S3)[1])[1]; A123x0:=op(op(S3)[1])[2]: op(op(S3)[2])[1];
  A132x0:=op(op(S3)[2])[2]:
```

$$A123(0)$$

$$A132(0)$$

```
> ##Now we simplify the expreesions of all gf A_r(0)
  (r=123,132,223,232,323,332) with using the facts that u1*u2=x^6/u0
  and u1+u2=1-2*x-x^2-u0:
> solve({x1+x^6/uu0/x1=1-2*x-x^2-uu0},{x1}):
  xx1:=1/2/uu0*(-uu0*x^2-uu0^2-2*uu0*x+uu0+(-4*uu0*x^6+uu0^2*x^4+2*u
  u0^3*x^2+4*uu0^2*x^3+uu0^4+4*uu0^3*x+2*uu0^2*x^2-2*uu0^3-4*uu0^2*x
  +uu0^2)^(1/2));
  xx2:=-1/2*(uu0*x^2+uu0^2+2*uu0*x+(-4*uu0*x^6+uu0^2*x^4+2*uu0^3*x^2
  +4*uu0^2*x^3+uu0^4+4*uu0^3*x+2*uu0^2*x^2-2*uu0^3-4*uu0^2*x+uu0^2)^
  (1/2)-uu0)/uu0;
```

$$xx1 := (-uu0\,x^2 - uu0^2 - 2\,uu0\,x + uu0 + (-4\,uu0\,x^6 + uu0^2\,x^4 + 2\,uu0^3\,x^2 + 4\,uu0^2\,x^3 + uu0^4 + 4\,uu0^3\,x + 2\,uu0^2\,x^2 - 2\,uu0^3 - 4\,uu0^2\,x + uu0^2)^{(1/2)})\,/\,(2\,uu0)$$

$$xx2 := -(uu0\,x^2 + uu0^2 + 2\,uu0\,x + (-4\,uu0\,x^6 + uu0^2\,x^4 + 2\,uu0^3\,x^2 + 4\,uu0^2\,x^3 + uu0^4 + 4\,uu0^3\,x + 2\,uu0^2\,x^2 - 2\,uu0^3 - 4\,uu0^2\,x + uu0^2)^{(1/2)} - uu0)\,/\,(2\,uu0)$$

```
> A332x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A332x0))+1)-1));
```

$$A332x0 := \frac{x\,(1 + A0001 + A0002 + A0003)}{uu0\,(x^2 + uu0)}$$

```
> A323x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A323x0))+1)-1));
```

$$A323x0 := -\frac{(1 + A0001 + A0002 + A0003)\,(uu0\,x^2 + x^3 + uu0^2 + 2\,uu0\,x - uu0)}{uu0\,x^2\,(x^2 + uu0)}$$

```
> A232x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A232x0))+1)-1)):
> A223x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A223x0))+1)-1)):
> A132x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A132x0))+1)-1)):
> A123x0:=factor(simplify(simplify(rationalize(subs(uu1=xx1,uu2=xx2,
  A123x0))+1)-1)):
>
```

```
> #---first terms of these functions and finding A_r(v) for each of
  these six g.f.
> taylor(subs(v=0,BA332),x,10);taylor(subs(v=0,BA323),x,10);
```

$$x + 7x^2 + 34x^3 + 145x^4 + 577x^5 + 2202x^6 + 8184x^7 + 29885x^8 + 107809x^9 + O(x^{10})$$

$$x + 6x^2 + 27x^3 + 110x^4 + 423x^5 + 1572x^6 + 5720x^7 + 20526x^8 + 72969x^9 + O(x^{10})$$

```
> A332x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A332x0)))):
  simplify(series(subs(A0003=BA0003,A0002=BA0002,A0001=BA0001,subs(u
  u0=u0,A332x0)),x,14)) assuming x>0 and x<0.1;
  A323x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A323x0)))):
  simplify(series(subs(A0003=BA0003,A0002=BA0002,A0001=BA0001,subs(u
  u0=u0,A323x0)),x,14)) assuming x>0 and x<0.1;
```

$$O(x^7) + 2202x^6 + 577x^5 + 145x^4 + 34x^3 + 7x^2 + x$$

$$1572x^6 + 423x^5 + O(x^7) + 110x^4 + 27x^3 + 6x^2 + x$$

```
> #--finding A323(v) and A332(v)
> i:=5:
  eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A232(v)
  +B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+B[i,9
  ]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A332(0)+
  B[i,13]:
> T1:=solve({eq[6]=0,eq[5]=0},{A323(v),A332(v)}):
  T1:=map(factor,T1):
  op(op(T1)[1])[1];A323v:=factor(subs(A323(0)=A323x0,A332(0)=A332x0,
  op(op(T1)[1])[2])):
  op(op(T1)[2])[1];A332v:=factor(subs(A323(0)=A323x0,A332(0)=A332x0,
  op(op(T1)[2])[2])):
```

$$A323(v)$$

$$A332(v)$$

```
>
> taylor(subs(v=0,BA232),x,10);taylor(subs(v=0,BA223),x,10);
```

$$x + 7x^2 + 39x^3 + 194x^4 + 895x^5 + 3911x^6 + 16424x^7 + 66968x^8 + 267017x^9 + O(x^{10})$$

$$x + 6x^2 + 31x^3 + 147x^4 + 655x^5 + 2784x^6 + 11432x^7 + 45760x^8 + 179641x^9 + O(x^{10})$$

```
> A232x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A232x0)))):
  simplify(series(subs(A002=BA002,A001=BA001,A0003=BA0003,A0002=BA00
  02,A0001=BA0001,simplify(subs(uu0=u0,A232x0))),x,13)) assuming x>0
  and x<0.1;
  A223x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A223x0)))):simpl
  ify(series(subs(A002=BA002,A001=BA001,A0003=BA0003,A0002=BA0002,A0
  001=BA0001,simplify(subs(uu0=u0,A223x0))),x,13)) assuming x>0 and
  x<0.1;
```

$$O(x^9) + 3911x^6 + 194x^4 + 39x^3 + 895x^5 + 7x^2 + 66968x^8 + 16424x^7 + x$$

$$O(x^9) + x + 2784\,x^6 + 147\,x^4 + 31\,x^3 + 655\,x^5 + 6\,x^2 + 45760\,x^8 + 11432\,x^7$$

```
> ##factor(coeff(coeff(subs(A001=r-A002,A0001=t-A0002-A0003,A232x0),
  t,1),r,0));
> #--find A223(v) and A232(v)
> i:=3:
  eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A232(v)
  +B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+B[i,9
  ]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A332(0)+
  B[i,13]: i:=4:
  eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A232(v)
  +B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+B[i,9
  ]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A332(0)+
  B[i,13]:
> T2:=solve({eq[4]=0,eq[3]=0},{A223(v),A232(v)}):
  T2:=map(factor,T2):
  op(op(T2)[1])[1];A223v:=subs(kv1=uu0,simplify(subs(uu0=kv1,factor(
  subs(A323(0)=A323x0,A332(0)=A332x0,A223(0)=A223x0,A232(0)=A232x0,o
  p(op(T2)[1])[2])))):
  op(op(T2)[2])[1];A232v:=subs(kv1=uu0,simplify(subs(uu0=kv1,factor(
  subs(A323(0)=A323x0,A332(0)=A332x0,A223(0)=A223x0,A232(0)=A232x0,o
  p(op(T2)[2])[2])))):
```

$$A223(v)$$

$$A232(v)$$

```
>
> taylor(subs(v=0,BA132),x,10);taylor(subs(v=0,BA123),x,10);
```

$$x + 6\,x^2 + 31\,x^3 + 155\,x^4 + 754\,x^5 + 3560\,x^6 + 16311\,x^7 + 72682\,x^8 + 316052\,x^9 + O(x^{10})$$

$$x + 5\,x^2 + 24\,x^3 + 115\,x^4 + 542\,x^5 + 2495\,x^6 + 11190\,x^7 + 48968\,x^8 + 209660\,x^9 + O(x^{10})$$

```
> A132x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A132x0)))):
  simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
  0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,A132x0))),x,10))
  assuming x>0 and x<0.1;
  A123x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,A123x0)))):simpl
  ify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A0002=
  BA0002,A0001=BA0001,simplify(subs(uu0=u0,A123x0))),x,10)) assuming
  x>0 and x<0.1;
```

$$155\,x^4 + 31\,x^3 + 754\,x^5 + 6\,x^2 + O(x^6) + x$$

$$x + 5\,x^2 + 24\,x^3 + 115\,x^4 + 542\,x^5 + O(x^6)$$

```
> #--find A123(v) and A132(v)
> i:=1:eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A2
  32(v)+B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+
  B[i,9]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A33
```

```
2(0)+B[i,13]:
i:=2:eq[i]:=B[i,1]*A123(v)+B[i,2]*A132(v)+B[i,3]*A223(v)+B[i,4]*A2
32(v)+B[i,5]*A323(v)+B[i,6]*A332(v)+B[i,7]*A123(0)+B[i,8]*A132(0)+
B[i,9]*A223(0)+B[i,10]*A232(0)+factor(B[i,11])*A323(0)+B[i,12]*A33
2(0)+B[i,13]:
```

```
> T3:=solve({eq[1]=0,eq[2]=0},{A123(v),A132(v)}):
T3:=map(factor,T3):
op(op(T3)[1])[1];A123v:=subs(kv1=uu0,simplify(subs(uu0=kv1,factor(
subs(A323(0)=A323x0,A332(0)=A332x0,A223(0)=A223x0,A232(0)=A232x0,A
123(0)=A123x0,A132(0)=A132x0,op(op(T3)[1])[2])))):
op(op(T3)[2])[1];A132v:=subs(kv1=uu0,simplify(subs(uu0=kv1,factor(
subs(A323(0)=A323x0,A332(0)=A332x0,A223(0)=A223x0,A232(0)=A232x0,A
123(0)=A123x0,A132(0)=A132x0,op(op(T3)[2])[2])))):
```

$$A123(v)$$

$$A132(v)$$

```
> #finding all A_r(v) for all
r=333,331,322,321,233,231,222,221,133,131,122,121 by using E1
```

```
> A333v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A333(v))): A333x0:=subs(v=0,A333v):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A333x0-subs(v=0,BA333))))),x,14)) assuming x>0 and x<0.1;
```

$$O(x^9)$$

```
> A331v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A331(v))): A331x0:=subs(v=0,A331v):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A331x0-subs(v=0,BA331))))),x,14)) assuming x>0 and x<0.1;
```

$$O(x^{11})$$

```
> A322v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A322(v))): A322x0:=subs(v=0,A322v):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A322x0-subs(v=0,BA322))))),x,14)) assuming x>0 and x<0.1;
```

$$O(x^{11})$$

```
> A321v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A321(v))): A321x0:=subs(v=0,A321v):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
```

```
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A321x0-subs(v=0,BA321))))),x,14)) assuming x>0 and x<0.1;
```

$$O(x^{12})$$

```
> A222v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A222(v))):
A222x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A222v))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A222x0-subs(v=0,BA222))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^{7})$$

```
> A221v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A221(v))):
A221x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A221v))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A221x0-subs(v=0,BA221))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^{8})$$

```
> A233v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A233(v))):
A233x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A233v))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A233x0-subs(v=0,BA233))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^{5})$$

```
> A231v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A231(v))):
A231x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A231v))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
```

```
x0,A231x0-subs(v=0,BA231)))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^7)$$

```
> A133v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A132(v)=A132v,A123(v)=A123v,A132(0)=A132x0,A123(0)=A123x0,A133(
v))):
A133x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A133v))))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A133x0-subs(v=0,BA133)))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^5)$$

```
> A131v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A132(v)=A132v,A123(v)=A123v,A132(0)=A132x0,A123(0)=A123x0,A131(
v))):
A131x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A131v))))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A131x0-subs(v=0,BA131)))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^7)$$

```
> A122v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A132(v)=A132v,A123(v)=A123v,A132(0)=A132x0,A123(0)=A123x0,A122(
v))):
A122x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A122v))))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A122x0-subs(v=0,BA122)))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^7)$$

```
> A121v:=factor(subs(A332(v)=A332v,A332(0)=A332x0,A323(v)=A323v,A323
(0)=A323x0,A223(v)=A223v,A232(v)=A232v,A223(0)=A223x0,A232(0)=A232
x0,A132(v)=A132v,A123(v)=A123v,A132(0)=A132x0,A123(0)=A123x0,A121(
v))):
A121x0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,factor(subs(v=0,
A121v))))))):
simplify(series(subs(A01=BA01,A002=BA002,A001=BA001,A0003=BA0003,A
```

```
0002=BA0002,A0001=BA0001,simplify(subs(uu0=u0,simplify(subs(A123(0
)=A123x0,A132(0)=A132x0,A223(0)=A223x0,A323(0)=A323x0,A332(0)=A332
x0,A121x0-subs(v=0,BA121))))),x,10)) assuming x>0 and x<0.1;
```

$$O(x^8)$$

```
>
> #----------Main system for finding A0-(System S1 with using
expressions A_r(0) that we found)-------------
> meq1:=A0=x+x*A00+x*A01: meq2:=A00=x+x*A000+x*A001+x*A002:
meq3:=A01=x+x*A001+x*A011+x*A01:
meq4:=A000=x+x*A0001+x*A0002+x*A0003:
meq5:=A001=x+x*A0001+x*A0011+x*A001+x*A002:
meq6:=A002=x+x*A0002+x*A0022+x*A002:
meq7:=A011=x+x*A0011+x*A0111+x*A121x0+x*A01:
meq8:=A0001=x+x*A00011+x*A0001+x*A0002+x*A0003:
meq9:=A0002=x+x*A00022+x*A0002+x*A0003:
meq10:=A0003=x+x*A00033+x*A0003:
meq11:=A0011=x+x*A00011+x*A00111+x*A221x0+x*A001+x*A002:
meq12:=A0022=x+x*A00022+x*A00222+x*A001+x*A002:
meq13:=A0111=x+x*A00111+x*(A131x0+A121x0)+x*A01:
meq14:=A00011=x+x*A000111+x*A321x0+x*A0001+x*A0002+x*A0003:
meq15:=A00022=x+x*A000222+x*A0001+x*A0002+x*A0003:
meq16:=A00033=x+x*A000+x*A0002+x*A0003:
meq17:=A00111=x+x*A000111+x*(A231x0+A221x0)+x*A001+x*A002:
meq18:=A00222=x+x*A000222+x*A221x0+x*A001+x*A002:
meq19:=A000111=x+x*(A331x0+A321x0)+x*A0001+x*A0002+x*A0003:
meq20:=A000222=x+x*A321x0+x*A0001+x*A0002+x*A0003:
>
> MainSys:=solve({meq1,meq2,meq3,meq4,meq5,meq6,meq7,meq8,meq9,meq10
,meq11,meq12,meq13,meq14,meq15,meq16,meq17,meq18,meq19,meq20},{A0,
A00,A01,A000,A001,A002,A011,A0001,A0002,A0003,A0011,A0022,A0111,A0
0011,A00022,A00033,A00111,A00222,A000111,A000222}):
> FinA0:=factor(op(op(MainSys)[1])[2]):
>
> #first termsof A0
> simplify(series(subs(uu0=u0,FinA0),x,16)) assuming x>0 and x<0.1;
```

$$O(x^{14}) + x + 296\,x^6 + 21\,x^4 + 6\,x^3 + 78\,x^5 + 2\,x^2 + 3299936\,x^{13} + 878164\,x^{12} + 233078\,x^{11} + 61690\,x^{10} + 16281\,x^9 + 4285\,x^8 + 1126\,x^7$$

```
>
> ##expression of A0
> FinA0:=subs(kv1=uu0,factor(simplify(subs(uu0=kv1,FinA0))));
```

$$FinA0 := (-256\,x^{12} + 256\,uu0^2\,x^8 + 256\,uu0\,x^9 - 560\,x^{10} - 576\,uu0\,x^8 - 132\,x^9 + 112\,uu0^2\,x^6$$

$$- 440\, uu0\, x^7 - 792\, x^8 - 452\, uu0^2\, x^5 - 1306\, uu0\, x^6 - 168\, x^7 - 198\, uu0^2\, x^4 - 104\, uu0\, x^5 - 235\, x^6 - 191\, uu0^2\, x^3 - 85\, uu0\, x^4 + 115\, x^5 + 56\, uu0^2\, x^2 + 335\, uu0\, x^3 + 31\, x^4 + 42\, uu0^2\, x + 18\, uu0\, x^2 - 10\, x^3 - 10\, uu0^2 - 62\, uu0\, x + 10\, uu0) \Big/ (x^5 (16\, x^3 + 8\, x^2 + 11\, x - 4)^2)$$

```
> FinA0:=factor(coeff(FinA0,uu0,0))+factor(coeff(FinA0,uu0,1))*uu0+f
  actor(coeff(FinA0,uu0,2))*uu0^2;
```

$$FinA0 := -\frac{256\, x^9 + 560\, x^7 + 132\, x^6 + 792\, x^5 + 168\, x^4 + 235\, x^3 - 115\, x^2 - 31\, x + 10}{x^2 (16\, x^3 + 8\, x^2 + 11\, x - 4)^2} + \frac{(256\, x^9 - 576\, x^8 - 440\, x^7 - 1306\, x^6 - 104\, x^5 - 85\, x^4 + 335\, x^3 + 18\, x^2 - 62\, x + 10)\, uu0}{x^5 (16\, x^3 + 8\, x^2 + 11\, x - 4)^2} + \frac{(256\, x^8 + 112\, x^6 - 452\, x^5 - 198\, x^4 - 191\, x^3 + 56\, x^2 + 42\, x - 10)\, uu0^2}{x^5 (16\, x^3 + 8\, x^2 + 11\, x - 4)^2}$$

```
>
> #checking last form
> simplify(series(subs(uu0=u0,FinA0)-BA0,x,19)) assuming x>0 and
  x<0.1;
```

$$O(x^{14})$$

```
>
>
```