Finite basis properties of wreath products

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ABSTRACT

A pattern class of permutations which is closed under pattern involvement may be described in terms of its basis. The wreath product construction $X \wr Y$ of two closed pattern classes X and Y is also closed, and we aim to investigate classes with the wreath finite basis property, i.e. those classes Y such that $X \wr Y$ is finitely based for all finitely based classes X.

The profile of a permutation has previously been used to reduce permutations by contracting consecutive increasing symbols into a single symbol. Here we extend this notion to the Y-profile, where we reduce permutations by contracting patterns from a closed class Y. The relation between the Y-profile and the wreath product is demonstrated, which reduces the wreath finite basis property to a simpler bounding problem, relying on considering pairs of symbols from basis elements of the class X involved within a given Y-profile.

Pairs of symbols, within a permutation, partition the permutation into nine regions. They also describe their own unique minimal block (the smallest interval containing both symbols), and we see these concepts are fundamental to the success or failure of the bounding problem. The method is applied to several classes, including the class of stack-sortable permutations $\mathcal{A}(231)$. A direct corollary is a bound on the size of basis elements of their wreath products, and hence the wreath finite basis property for each of the classes

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considered. Limitations of this method are also given, with the class $\mathcal{A}(123)$ being an example where the bounding problem fails.