ERRATA ET ADDENDUM TO "TAMARKIN'S PROOF..."

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1. Proof of 4.3.1, p. 12

The morphism $\tau(1)$ defined by formulas (15) does not preserve the units. Therefore, strictly speaking, the collection $\{\tau(n)\}$ is not a morphism of operads. This can be, however, easily fixed.

Add formally units to the operads $\mathrm{Endop}(B)$ and $\mathrm{Endop}(A)$, so that τ will induce a quasi-isomorphism of the new extended operads. Then the rest of the proof works well for the extended operads.

2. Etingof-Kazhdan theory, 7.2

The general formulation of Etingof-Kazhdan quantization-dequantization is not so easy to grasp. Trying to simplify the exposition, I presented in Theorem 7.2.1 a special case of Etingof-Kazhdan theorem which I believed was sufficient for our needs. This was not completely correct. The problem is with the definition of the category $\mathcal{A}(R)$: if $\mathcal{A}$ is the category of complexes of k-vector spaces then according to the definition of 7.2 $\mathcal{A}(R)$ is NOT equivalent to the category of complexes of free R-modules but to the full subcategory consisting of complexes of the form $R \otimes X$, X being a complex of k-vector spaces.

Let me be more accurate now.

Let $\mathcal{A}$ be an abelian k-linear tensor category with Hom-vector spaces complete with respect to a decreasing filtration

$$\mathrm{Hom}(x, y) = F^0 \supseteq F^1 \supseteq F^2 \supseteq \dots ,$$

and with compositions compatible with these filtrations.

In the application we have in mind $\mathcal{A}$ is the category of complexes of free $k[[h]]$-modules, endowed with a k^*-action compatible with the action on $k[[h]]$ given by the formula

$$\lambda(h) = \lambda^{-1} h.$$

Let $\mathtt{LBA}_0$ be the category of Lie bialgebras in $\mathcal{A}$ whose cobracket belongs to the F^1 of the corresponding Hom.

Let $\mathtt{HA}_0$ denotes the category of Hopf algebras in $\mathcal{A}$ with comultiplication Δ and antipode S such that $\Delta - \Delta^{\mathrm{op}}$ and $S - S^{-1}$ belong to the F^1 of the corresponding Hom.

Then the quantization functor of Etingof-Kazhdan

$$Q : \mathtt{LBA}_0 \to \mathtt{HA}_0$$

is defined. The Hopf algebra $Q(\mathfrak{g})$ is isomorphic to the symmetric algebra $S(\mathfrak{g})$ as an object of $\mathcal{A}$, but has different operations expressible by universal formulas through the bracket and cobracket of $\mathfrak{g}$.

One has to be slightly more careful with the dequantization functor. The existence of dequantization is formulated in [EK] only for the special cases which do not include complexes. However, the most dificult step of dequantization procedure, that of construction of a co-Poisson Hopf algebra, is given by universal formulas, and, therefore, is valid in any category $\mathcal{A}$. The only "$\mathcal{A}$-sensitive" step is the passage from co-Poisson Hopf algebra to Lie bialgebra. This passage is given by taking the primitive part of the Hopf algebra.

We know that this passage is an equivalence of categories if one forgets about the differentials. Since the collection of primitive elements form a subcomplex and since this subcomplex generates the enveloping algebra as an associative algebra, we are done.

3. Addendum

Theorem 5.3.3 proven in the text claims that the Hochschild complex $C^*(A,A)$ admits a structure of $\mathcal{G}_\infty$-algebra structure inducing the usual Gerstenhaber algebra on the cohomology. I have forgotten, however, to check that the induced $\mathtt{LIE}_\infty$ algebra structure on $C^*(A,A)$ comes from the standard Lie bracket.

The action of $\mathcal{G}_\infty$ on $C^*(A,A)$ comes from an action of the operad $\widetilde{\mathcal{B}}$ on $C^*(A,A)$. The operad morphism from $\mathcal{G}_\infty$ to $\widetilde{\mathcal{B}}$ is constructed in 6.3.

Let us calculate the composition

$$\mathtt{LIE}\{1\}_\infty \to \mathcal{G}_\infty \to \widetilde{\mathcal{B}}.$$

$\widetilde{\mathcal{B}}$-algebra structure on X induces, via ℓ_{11}, a Lie algebra on $X[1]$. The functor $\mathbb{F}^*_{\mathtt{COM}}(\_[1])$ applied to the map of Lie algebras $X[1] \to \mathbb{F}^*_{\mathtt{LIE}}(X[1])$ gives a map

$$\mathbb{F}^*_{\mathtt{COM}}(X[2]) \to \mathbb{F}^*_{\mathcal{G}}(X)[2].$$

This shows that $\mathtt{LIE}_\infty$-algebra structure on $C^*(A,A)[1]$ comes from $\ell_{11} \in \widetilde{\mathcal{B}}(2)$. Since $\widetilde{\mathcal{B}}(2) = \mathcal{B}_\infty(2)$, we are done.