ALGEBRAIC GEOMETRY - EXERCISE 8

1. Let $f : X \to Z$ and $g : Y \to Z$ be morphisms of algebraic varieties and let $f' : X \times_Z Y \to Y$ be the base change of f with respect to g. Prove that if f is a closed (resp., open) embedding, then f' is a closed (resp., open) embedding.
2. Prove that a quasi-affine variety (that is, an open subvariety of an affine) is complete iff it is a finite set.
3. Prove that X and Y are complete iff $X \times Y$ is complete.
4. Let $f : X \to Y$ with X complete and Y separated. Prove that the image $f(X) \subset Y$ is complete.