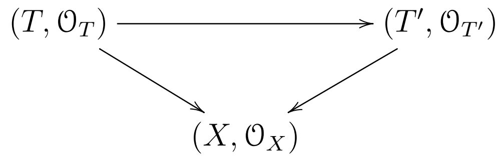
ALGEBRAIC GEOMETRY - EXERCISE 7

1. Let $(X, \mathcal{O}_X)$ be a space with functions and let $Z \subset X$ be a subset. Look at the category whose objects are the morphisms $f : (T, \mathcal{O}_T) \to (X, \mathcal{O}_X)$ such that $f(X) \subset Z$, and the arrows are the commutative diagrams



   Construct the terminal object in this category (it gives a space with functions structure on Z).
2. Construct the fiber product of spaces with functions, realizing it as a subspace of the product.
3. Prove that the fiber product $X \times_Z Y$ is an affine variety, provided X, Y, Z are affine.
4. Deduce that the fiber product of varieties $X \times_Z Y$ is a variety. *Hint: start with a choice of an affine covering of* Z.
5. Let $A = \oplus A_n$ be a graded ring with $A_0 = k$ and let B be a k-algebra. Prove that $\mathtt{proj}_k(A \otimes B) = \mathtt{proj}_k(A) \times \mathtt{spec}_k(B)$. Here $A \otimes B$ is the graded algebra whose components are $(A \otimes B)_n = A_n \otimes B$. Use this to present the blowing-up of $\mathbb{A}^n$ at the origin as a closed subvariety of $\mathbb{A}^n \times \mathbb{P}^{n-1}$.