ALGEBRAIC GEOMETRY - EXERCISE 12

1. Let A be a commutative ring and let $X = \mathtt{Spec}(A)$ be the set of prime ideals of A endowed with the Zariski topology, see Notes, 2.6.4. Using the recipe of 10.2.1 (definition of the associated sheaf), define on $\mathtt{Spec}(A)$ a sheaf of commutative rings $\mathcal{O}_X$ whose stalks at $\mathfrak{p} \in X$ are $A_{\mathfrak{p}}$ and the sections at $D(f) = \{\mathfrak{p} \not\ni f\}$ are A_f.
2. Let $X = \mathtt{spec}(A)$. Let M be an A-module and F a sheaf of $\mathcal{O}_X$-modules. Prove that any A-module homomorphism $f : M \to F(X)$ gives rise to a morphism of $\mathcal{O}_X$-modules $\widetilde{M} \to F$.
3. Let F be a coherent sheaf and $G \subset F$ a quasicoherent subsheaf. Prove that G is coherent.
4. Intersection of two quasicoherent subsheaves of a quasicoherent sheaf is also quasicoherent.
5. Let $f : X \to Y$ be an affine morphism. Verify that if F is a quasicoherent sheaf on X then $f_*(F)$ is also quasicoherent. *Note: the claim holds for any morphism of algebraic varieties.*