ALGEBRAIC GEOMETRY - EXERCISE 11

1. Prove that the sheafification of the constant presheaf with the value A assigns to U the set of locally constant functions $U \to A$.
2. Let $f : M \to N$ be a homomorphism of sheaves. Prove that the presheaf $U \mapsto \mathrm{Ker}(f_U : M(U) \to N(U))$ is a sheaf.
3. Let $X = S^1$ be the circle, $\mathcal{O}$ be the sheaf of continuous $\mathbb{R}$-valued functions on X and $\mathbb{Z}$ be the constant sheaf with value $\mathbb{Z}$. Verify that the presheaf $U \mapsto \mathcal{O}(U)/\mathbb{Z}(U)$ is not a sheaf.
4. Given a map $f : Y \to X$, define a presheaf S_f on X by the formula
$$S_f(U) = \{s : U \to Y | f \circ s = \mathrm{id}_U\}.$$
Is S_f a sheaf?
5. Complete the proof of the claim $\texttt{Pic}(\mathbb{P}^n) = \mathbb{Z}$. Using the example $X = \mathbb{P}^n$, compare global sections of $L \otimes L'$ with the tensor product of global sections to see that the presheaf $U \mapsto L(U) \otimes_{\mathcal{O}(U)} L'(U)$ is not a sheaf in general.