

## LIE GROUPS

- Introduction. Examples. Topological groups.  $GL(n, \mathbb{R})$  as a topological group.
- Smooth manifolds. Smooth maps. Direct product. Definition of a Lie group.
- Tangent space. Immersions and embeddings. Lie subgroup.
- Elementary properties of Lie groups. Action of a Lie group on a manifold. Representation of a Lie group. Factor group.
- Closed linear groups. Lie algebra of a closed linear group.
- Classical groups.
- Closed Lie subgroups. Factor modulo a closed Lie subgroup. Homomorphism  $SU(2) \rightarrow SO(3)$ .
- Covering spaces. Fundamental group. Universal covering spaces. Simply connected Lie groups.
- Vector fields on a manifold. Lie bracket on vector fields. Lie algebra of a Lie group. Closed subgroups of a Lie group (Cartan theorem).
- Fundamental Lie theorems.
- Lie groups versus Lie algebras. Commutative groups, center, normal subgroups. Direct product, semidirect product.
- Compact Lie groups. Representations. Peter-Weyl theorem.

## REFERENCES

- [1] Lecture notes, an updated version will appear at the site of the course.
- [2] P. Etingof, Lie groups and Lie algebras.
- [3] A. Kirillov Jr. An introduction to Lie groups and Lie algebras.
- [4] A. Knap, Lie groups beyond an introduction.
- [5] A. Onishchik, E. Vinberg, Lie groups and algebraic groups.