LIE GROUPS, HOME ASSIGNMENT 9

1. Let $X, Y \in \mathtt{Vect}(M)$ and $f, g \in C^\infty(M)$. Prove that
$$[fX, gY] = fg[X, Y] + fX(g)Y - gY(f)X.$$
2. Prove that the center Z of a Lie group is always closed (even if G is not connected).
3. Let H be a closed Lie subgroup of G and let $\mathfrak{h} = \mathtt{Lie}(H)$, $\mathfrak{g} = \mathtt{Lie}(G)$. Find examples when $\mathfrak{h}$ is an ideal of $\mathfrak{g}$ but H is not a normal Lie subgroup of G:
   - When G is connected but H is not.
   - When H is connected but G is not.
4. Describe irreducible representations of
   - The one-dimensional Lie algebra.
   - The Lie group $(\mathbb{R}, +)$.
   - The Lie group $SO(2, \mathbb{R})$.