LIE GROUPS, HOME ASSIGNMENT 8

1. The group $G = GL(n, \mathbb{R})$ acts on $\mathbb{R}^n$ in a standard way. Describe the vector fields on $\mathbb{R}^n$ defined by the matrix units $E_{ij} \in M(n, \mathbb{R})$.
2. Let G be a Lie group and $\phi : G \to G$ be an automorphism of G. Prove that $\mathtt{Lie}(\phi) = T_1(\phi)$ is an automorphism of $\mathfrak{g} = \mathtt{Lie}(G)$. Prove that $G^\phi = \{g \in G | \phi(g) = g\}$ is a Lie subgroup of G whose Lie algebra is $\mathfrak{g}^{\mathtt{Lie}(\phi)}$.
3. Let G be a connected Lie group and $\mathfrak{g} = \mathtt{Lie}(G)$. Prove that G is commutative iff $\mathfrak{g}$ is commutative. Is connectedness important for the claim?
4. Let G be a Lie group, $\phi : G \times M \to M$ its action on a manifold M, $x \in M$, $H = \mathtt{Stab}_G(x)$. Describe the kernel of the map $T_1\phi_{g(x)} : \mathfrak{g} \to T_{g(x)}(M)$.