LIE GROUPS, HOME ASSIGNMENT 7

1. Prove that any derivation of the commutative algebra $A = k[x_1, \dots, d_n]$ vanishing at k, has form
$$\delta = \sum_{i=1}^{n} f_i \frac{\partial}{\partial x_i},$$
where $f_i \in A$.
2. Let $G = GL(n, \mathbb{R})$ and $\mathfrak{g} = \mathfrak{gl}(n, \mathbb{R})$ Prove that the Lie bracket on $\mathfrak{g}$ induced by the Lie bracket on the vector fields on G is the standard bracket on the ring of matrices.
3. Prove that the exponential map $\exp : \mathfrak{g} \to G$ is given, in the case when G is a closed Lie group, by the exponential map on matrices.
4. Let G be a Lie group, $\mathfrak{g}$ be the subalgebra of left-invariant vector fields on G and $\mathfrak{g}^R$ be the algebra of right-invariant vector fields on G. Prove that both $\mathfrak{g}$ and $\mathfrak{g}^R$ identify with $T_1(G)$. Prove that the identity map is an anti-isomorphism between $\mathfrak{g}$ and $\mathfrak{g}^R$.
5. Define the direct product of Lie group and of Lie algebras. Prove that $\mathtt{Lie}(G \times H) = \mathtt{Lie}(G) \times \mathtt{Lie}(H)$.