LIE GROUPS, HOME ASSIGNMENT 6

1. Prove that any two universal coverings of a connected space X are isomorphic.
2. Calculate the fundamental group of a real Grassmannian $Gr(p, n)$ that classifies all p-dimensional subspaces in $\mathbb{R}^n$.
3. Calculate the fundamental groups of the following Lie groups:
   a) $SO(n, \mathbb{R})$.
   b) $SO(n, \mathbb{C})$.
   c) $PSL(n, \mathbb{R})$, $PSL(n, \mathbb{C})$, where $PSL(n, \mathbb{F})$ is defined as the factor of $SL(n, \mathbb{F})$ by the scalar matrices.
4. Prove that S^n is simply connected for $n > 1$, using van Kampen theorem.

5*. Using the description of covering spaces in terms of the fundamental group, prove van Kampen theorem.