LIE GROUPS, HOME ASSIGNMENT 5

1. Describe all two-dimensional Lie algebras.
2. Prove that the complexification of the real Lie algebra $\mathfrak{o}(p, n-p, \mathbb{R})$ is isomorphic to $\mathfrak{o}(n, \mathbb{C})$.

3*. Prove that the complexifications of the Lie algebras $\mathfrak{sl}(2, \mathbb{R})$ and $\mathfrak{o}(3, \mathbb{R})$ are isomorphic.

4. Give an example of a non-semisimple Lie algebra $\mathfrak{g}$ that admits a non-degenerate invariant symmetric bilinear form.
5. Recall that the Grassmannian $Gr(m, n)$ of m-subspaces in a real vector space of dimension n admits a transitive $GL(n, \mathbb{R})$ action. Let P be a stabilizer of this action and let $\mathfrak{p} = \mathtt{Lie}(P)$. Present $\mathfrak{p}$ as a semidirect product of its radical and a semisimple Lie algebra.