LIE GROUPS, HOME ASSIGNMENT 4

1. Describe the Lie algebra of the group $Sp(2n, \mathbb{R})$. What is its dimension?
2. Let $f : X \to B$ be a locally trivial fibration with fiber F. Prove that if B and X are path-connected, X is also path-connected.
3. Prove that $GL(n, \mathbb{R})$ has two connected components.
4. Calculate the kernels of the following homomorphisms
   a. $\pi : SU(2) \to SO(3)$.
   b. $SL(2, \mathbb{C}) \to SO^+(1, 3)$ described in 5.6.2.
   c. $\rho : SU(2) \times SU(2) \to SO(4)$.