LIE GROUPS, HOME ASSIGNMENT 2

1. Prove Proposition 3.4.4 from the lecture notes: if $f = g \circ p$ where f and p are smooth and p is a quotient map, then g is smooth.
2. Prove the following properties of locally trivial fibrations (ltf).
   - Any ltf is a quotient map.
   - For a closed Lie subgroup H of a Lie group G, the canonical map $G \to G/H$ is a ltf.
3. Present S^n as the quotient of $O(n+1, \mathbb{R})$ by a subgroup isomorphic to $O(n, \mathbb{R})$.
4. Present S^{2n+1} as the quotient of $U(n+1)$ by a subgroup isomorphic to $U(n)$.
5. Define a smooth structure on the grassmannian $Gr(m, n)$, defining a transitive action of $GL(n, \mathbb{R})$ on it and verifying that the stabilizers are Lie subgroups.
6. Define a structure of a smooth manifold on the set of lattices in $\mathbb{R}^2$, using a transitive action of the group $GL(2, \mathbb{R})$ (a lattice in $\mathbb{R}^2$ is a subgroup $\mathrm{Span}_{\mathbb{Z}}(x, y)$ generated by two linearly independent vectors x, y in $\mathbb{R}^2$).