

## BASIC ALGEBRA - EXERCISE 4

1. Calculate  $\mathbb{Q} \otimes_{\mathbb{Z}} (\mathbb{Q}/\mathbb{Z})$ .
2. Prove that the homomorphism  $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q} \rightarrow \mathbb{Q}$  given by multiplication  $a \otimes b \mapsto ab$ , is an isomorphism.
3. Find the kernel of the homomorphism

$$\mathbb{Z}[x] \otimes_{\mathbb{Z}} \mathbb{Z}[x] \rightarrow \mathbb{Z}[x]$$

given by the multiplication.

4. Let  $f : A \rightarrow B$  be a ring homomorphism. Prove that, if  $M$  is a free  $A$ -module,  $B \otimes_A M$  is a free  $B$ -module. The same for projective and flat  $A$ -modules.
5. Let  $A$  be commutative. Prove that if  $A^m$  is isomorphic to  $A^n$  then  $m = n$ . (Hint: find a maximal ideal in  $A$ ).