BASIC ALGEBRA - EXERCISE 2

1. Let G be a group. Construct an isomorphism of the group ring $\mathbb{C}G$ with its opposite.
2. Construct an isomorphism of the Weyl algebra A as defined in Exercise 1 with its opposite.
3. Recall that the Weyl algebra $A = A_k$ is defined as the subring of $\mathrm{End}_k(k[x])$ of k-linear endomorphisms of $k[x]$ generated by X (multiplication by x) and D (derivative). Naturally, $k[x]$ becomes a left A_k-module.
   a. Prove that, if $\mathrm{char}(k) = 0$, $k[x]$ is a simple A_k-module.
   b. Prove that, if $\mathrm{char}(k) = 0$, A_k has no finite dimensional (over k) modules. (Hint: use the properties of trace of a matrix).
4. Describe commutative semisimple rings.
5. Prove that any module over a semisimple ring is both projective and injective.
6. Prove that the ring $R = k[x]/(x^n)$ is injective as an R-module (k is a field).