We saw a great construction of the spaces in the spectrum MU: $MU_n = \mathrm{Thom}(\xi_n \downarrow BU(n))$
$= \mathrm{Thom}(\xi_n \downarrow BGL_n\mathbb{C})$.

The structure maps arise from $\xi_{n+1}|_{BU_n} = \xi_n \oplus 1 \Rightarrow \mathrm{Thom}(\xi_n \oplus 1 \downarrow BU_n) \to \mathrm{Thom}(\xi_{n+1})$.

Now observe that everything is Galois equivariant! $S^{\mathbb{C}} \wedge MU_n \longrightarrow MU_{n+1}$

(point in BU_n = $\mathbb{C}$ n-plane in $\mathbb{C}^\infty$. $\bar{V}$ is another). $\quad S^{\rho} \wedge MU(n\rho) \longrightarrow MU((n+1)\rho)$

One of the ways to describe an equivariant spectrum is to give a space for all finite dim'l reps of G & structure maps $\Sigma^V X(W) \to X(V \oplus W)$. We haven't quite given all of 'em, but since $\dim V < \infty \Rightarrow V \subseteq k\rho$ for some k, this is enough.

Def $MU_{\mathbb{R}}$ is $MU(n\rho) = \mathrm{Thom}(\xi_n \downarrow BU(n))$.

This has geometric meaning: ← slight lie here. a Real bundle on X (a C_2-space) is a $\mathbb{C}$ bundle $\begin{smallmatrix} V \\ \downarrow \\ X \end{smallmatrix}$ s.t.

$$\begin{tikzcd} V \arrow[r, "\sigma"] \arrow[d] & V \arrow[d] \\ X \arrow[r, "\sigma"'] & X \end{tikzcd}$$ is $\overline{(\cdot)}$ linear: $\sigma(\lambda \vec{v}) = \bar{\lambda} \cdot \sigma(\vec{v})$.

All the generators we know and love for MU_* are actually equivariant!

This isn't quite good enough. We need a nice C_8 action

So let's think about rings or G-modules. We talked about $\mathbb{Z}[G] \otimes_{\mathbb{Z}[H]} M$, $\mathrm{Ind}_H^G M$. What were we really doing? We take the direct sum of G/H objects, and we give this the permutation action (let's assume H is normal; otherwise we have to twist). & we saw this had a universal property: $G\text{-Mod}(\mathrm{Ind}_H^G M, N) \cong H\text{-Mod}(M, N)$.

G-Mod has another sym. monoidal product: $\otimes$ (& $\otimes$ distributes over $\oplus$). We can play the same game! $N_H^G M = M^{\otimes G/H}$ with the permutation action. Very weird! But commutes with $\otimes$.

Ex: $M = \mathbb{Z}[X]$, X an H-set. Then $N_H^G M = \mathbb{Z}[X]^{\otimes G/H} = \mathbb{Z}[\coprod_{G/H} X] = \mathbb{Z}[G \times_H X]$.

Ex: $M = \mathbb{Z}[x]$, $G = C_2$. $N_H^G(M) = \mathbb{Z}[x,y]$ $\quad x \rightleftarrows y$. What are the fixed points?

① $x + y = tr(x)$

② $\underset{N(x)}{\underset{\shortparallel}{x \cdot y}} \notin \mathrm{Im}(tr)$. $\Big\}$ $N: M \to (N_H^G M)/tr$ is a ring map

This also has a universal property, but for com. rings: $G\text{-Comm}(N_H^G R, T) = H\text{-Comm}(R, T)$.

It's also not terrible to work out the "distributive" law. (but it's a pain!)

So what made the universal property go?

① Everything is a comm mon. for $\oplus$: $M \oplus M \to M$. $(m,n) \longmapsto m+n$

② Every comm ring is a com. monoid for $\otimes$. $\Rightarrow$ $\otimes$ is the coproduct.

Let's do this in spectra: $\oplus \rightsquigarrow \vee$, $\otimes \rightsquigarrow \wedge$ } Aside, we have a (bi)sym. monoidal cat with these.

This gives us a symmetric monoidal functor $N_H^G : \mathcal{S}p^H \to \mathcal{S}p^G$.

One way to think about this: $(N_H^G X)(\mathrm{Ind}_H^G V) \simeq \left(\bigwedge_{G/H} X(V)\right)$. This is a little thorny, theoretically.

Basic Properties:

① $N_H^G(S^V) = S^{\mathrm{Ind}_H^G(V)}$.

② $N_H^G : \mathrm{Comm}_H \rightleftarrows \mathrm{Comm}_G : L_H^*$. <u>Huge</u> benefit.

③ "Fixed points mod transfers are understandable"

④ $\pi_*^u(N_H^G M) = N_H^G(\pi_*^u(M))$

Now look at $H = C_2$, $G = C_8$; $MU_{\mathbb{R}}$: $MU^{(G)} = N_{C_2}^G MU_{\mathbb{R}} \simeq MU_{\mathbb{R}} \wedge_{\mathbb{R}} \cdots \wedge MU_{\mathbb{R}}$.

What is $\pi_*^u MU^{(G)}$? $N_{C_2}^G(\mathbb{Z}[x_i]) = \mathbb{Z}[\mathrm{Ind}_{C_2}^G x_i]$, as G-modules.

$\sigma x_i = (-1)^i x_i$

Now $L_{C_2}^* MU^{(G)} \simeq MU_{\mathbb{R}}^{\wedge(|G|/2)} \simeq MU_{\mathbb{R}}[\ldots]$, $\bar{x}_i : S^{i\rho_2} \to MU_{\mathbb{R}}$. We therefore get generators $\bar{r}_i : S^{i\rho_2} \to L_{C_2}^* MU^{(G)}$.