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M-equivalences and homotopy colimits

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ABSTRACT. Given a fixed "model space" $\boldsymbol{M}$, we call a map $f : \boldsymbol{X} \to \boldsymbol{Y}$ an $\boldsymbol{M}$-equivalence if it induces a weak equivalence $f_* : \boldsymbol{X}^M \to \boldsymbol{Y}^M$ on mapping spaces. We discuss the following question: under what conditions do homotopy colimits preserve $\boldsymbol{M}$-equivalences? For certain $\boldsymbol{M}$'s of interest, this is shown to depend precisely on the connectivities of the spaces.

1. Introduction

Let $\boldsymbol{M}$ be a fixed "model space"; we say that a map $f : \boldsymbol{X} \to \boldsymbol{Y}$ is an $\boldsymbol{M}$-*equivalence* if it induces a weak equivalence of mapping spaces $f_* : \boldsymbol{X}^M \to \boldsymbol{Y}^M$. Our ultimate object is to understand how much of the theory of CWcomplexes still holds when we replace the concept of "weak equivalence" by $\boldsymbol{M}$-equivalence. In particular, we wish to address the following:

Question: What classes of spaces $\mathcal{C}$ have the property that homotopy colimits preserve $\boldsymbol{M}$-equivalences among objects in $\mathcal{C}$?

That is, if $\{\boldsymbol{X}_\alpha\}_{\alpha\in A}$ and $\{\boldsymbol{Y}_\alpha\}_{\alpha\in A}$ are two diagrams of spaces in $\mathcal{C}$, and $\{f_\alpha : \boldsymbol{X}_\alpha \to \boldsymbol{Y}_\alpha\}_{\alpha\in A}$ is a map of diagrams with each f_α an $\boldsymbol{M}$-equivalence, is $\underrightarrow{holim}(f_\alpha) : \underrightarrow{holim}(\boldsymbol{X}_\alpha) \to \underrightarrow{holim}(\boldsymbol{Y}_\alpha)$ an $\boldsymbol{M}$-equivalence, too? Thus we would like to know, for example, whether the fact that a space $\boldsymbol{X}$ is $\boldsymbol{M}$-equivalent to another space $\boldsymbol{Y}$ implies that $\Sigma\boldsymbol{X}$ is $\boldsymbol{M}$-equivalent to $\Sigma\boldsymbol{Y}$.

This is of course one of the defining properties of homotopy colimits with respect to ordinary weak equivalences ($\boldsymbol{S}^0$-equivalences, in our terminology) – see [**BK**, XII, 4.2]. Clearly it fails to hold in complete generality: for example,

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if we take $M = S^1$, then S^0 is M-equivalent to a point, but its suspension is not. More generally, if M is c-connected and $\pi_t X = 0$ for $t > c$ then X is M-*local* (i.e., M-equivalent to a point), but ΣX need not be.

On the other hand, if $\mathcal{C}$ is the class of M-CW complexes (or M-cellular spaces – i.e., conic spaces obtained by a process of "attaching M-cells", analogously to the usual definition [**W**, II, §1], with spheres replaced by suspensions of M – cf. [**BT**], [**DF**]), then any M-equivalence of spaces in $\mathcal{C}$ is a homotopy equivalence, so is preserved by any homotopy colimit. However, M-CW complexes are difficult to identify, in general; we would like a class of spaces $\mathcal{C}$ with a more familiar description – for instance, in terms of connectivities of the spaces.

Our main result in this direction (Theorem 2.5 below) states that for every connected r-dimensional torsion space M there is an integer $e = e(M) < r$, which we can compute in certain cases (see Proposition 2.9), such that:

(i) M-equivalences between e-connected CW complexes are actually mod p equivalences (i.e., inducing an isomorphism in $\tilde{H}_*(-;\mathbb{Z}/p)$) for certain primes p, and so are preserved by all homotopy colimits;
(ii) homotopy colimits in general do *not* preserve M-equivalences between $(e-1)$-connected spaces.

Remark 1.1. Dwyer and Kan have recently shown that there is a concept of *homotopy (co)limits* in any closed model category ([**DK**]; see also [**DS**]); the basic property of such homotopy (co)limits is that they preserve the given weak equivalences. On the other hand, Alex Nofech has described, for each choice of M, a model category structure on the category of (pointed) topological spaces in which the weak equivalences are precisely the M-equivalences (cf. [**N**]); we thus have a concept of M-homotopy (co)limits: namely, the appropriate Dwyer-Kan homotopy colimits in the Nofech M-model category.

Thus our basic question can be reformulated in these terms: for which classes of topological spaces do the usual homotopy colimits agree with the M-homotopy colimits?

1.2. Conventions and notation. Let $\mathcal{T}_\star$ denote the category of connected pointed CW complexes, with base-point preserving maps. All spaces will be assumed to lie in $\mathcal{T}_\star$, unless otherwise stated.

For any r-dimensional co-H-space $M \in \mathcal{T}_\star$, the homotopy groups *with coefficients in* M of $X \in \mathcal{T}_\star$ are defined to be $\pi_t(X;M) \stackrel{Def}{=} [\Sigma^{t-r}M, X]$ (where this makes sense). Thus $f: X \to Y$ is an M-*equivalence* if $\pi_k(f;M)$ is an isomorphism for $t \geq r$. A space X is called M-*local* if it is M-equivalent to a point – i.e., if $\pi_*(X;M) = 0$.

If $\mathbb{P}_0$ is a set of primes, a map $f : X \to Y$ is called a $\mathbb{P}_0$-*equivalence* if $H_*(f;\mathbb{Z}/p) : H_*(X;\mathbb{Z}/p) \to H_*(Y;\mathbb{Z}/p)$ is an isomorphism for all $p \in \mathbb{P}_0$.

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2. $\boldsymbol{M}$-equivalences and $\mathbb{P}_0$-equivalences

In this section we show, essentially, that there are *no* $\boldsymbol{M}$-equivalences between sufficiently connected spaces except for the "trivial" ones:

THEOREM 2.1. *Let $\boldsymbol{M}$ be a pointed connected r-dimensional CW-complex $(r \geq 2)$ and p a prime such that $\tilde{H}_*(\boldsymbol{M};\mathbb{Z})$ is all p-torsion (but $\tilde{H}_s(\boldsymbol{M};\mathbb{Z}) \neq 0$ for some s), and let $\boldsymbol{X}$ be an $(r-1)$-connected $\boldsymbol{M}$-local p-complete space; then $\pi_\star \boldsymbol{X} = 0$.*

PROOF. Let $0 < s < r$ be maximal such that $\tilde{H}_s(\boldsymbol{M};\mathbb{Z}) \neq 0$ – so that $\tilde{H}^{s+1}(\boldsymbol{M};G) \cong Ext(H_s(\boldsymbol{M};\mathbb{Z}),G)$ and $\tilde{H}^i(\boldsymbol{M};G) = 0$ for all $i > s+1$ and all abelian groups G by [**S**, V, §5, Thm. 3]. We shall show by induction on $k \geq r$ that $\pi_k \boldsymbol{X} = 0$:

By assumption $\pi_i \boldsymbol{X} = 0$ for $0 \leq i < k$, so by obstruction theory:

$$(2.2) \qquad [\Sigma^{k-s-1}\boldsymbol{M}, \boldsymbol{X}] \cong \tilde{H}^k(\Sigma^{k-s-1}\boldsymbol{M};\pi_k\boldsymbol{X}) \cong Ext(H_s(\boldsymbol{M};\mathbb{Z}),\pi_k\boldsymbol{X})$$

(cf. [**W**, V, 6.18]), and the left-hand side vanishes since $\boldsymbol{X}$ was assumed to be $\boldsymbol{M}$-local.

However, since $A = \tilde{H}_s(\boldsymbol{M};\mathbb{Z}) \neq 0$ and all the homology of $\boldsymbol{M}$ is p-torsion, there is a short exact sequence $0 \to \mathbb{Z}/p \to A \to A/(\mathbb{Z}/p) \to 0$ and thus (2.2) implies that $Ext(\mathbb{Z}/p, \pi_k\boldsymbol{X}) = 0$ by the corresponding long exact sequence (cf. [**McL**, III, 3.2 & 3.7]). Since $Ext(\mathbb{Z}/p, G) \cong G/pG$ for any abelian group G (cf. [**Ro**, 7.17]), we see that $G = \pi_k\boldsymbol{X}$ is p-divisible. But $\boldsymbol{X}$ was p-complete (and 1-connected) by hypothesis, so the group $G = \pi_k\boldsymbol{X}$ is also Ext-p-complete (see [**BK**, VI, 3.1 & 5.4], and we thus find that $0 = Ext(\mathbb{Z}/p^\infty, G) \cong G = \pi_k\boldsymbol{X}$ by [**BK**, VI, 3.6], which completes the induction step. □

COROLLARY 2.3. *Let $\boldsymbol{M}$ be a pointed connected r-dimensional CW-complex $(r \geq 2)$ such that $H_*(\boldsymbol{M};\mathbb{Z}/p) \neq 0 \Leftrightarrow p \in \mathbb{P}_0$ (for some set of primes $\mathbb{P}_0$), and $\tilde{H}_*(\boldsymbol{M};\mathbb{Z})$ is all torsion. Let $f : \boldsymbol{X} \to \boldsymbol{Y}$ be a map of $(r-1)$-connected spaces. The f is an $\boldsymbol{M}$-equivalence if and only if f is a $\mathbb{P}_0$-equivalence.*

PROOF. By a theorem of Bousfield (cf. [**Mi**, Thm. 1.5]) the map $f : \boldsymbol{X} \to \boldsymbol{Y}$ is an $\boldsymbol{M}$-equivalence if and only if the maps $\hat{f}_p : \hat{\boldsymbol{X}}_p \to \hat{\boldsymbol{Y}}_p$ are $\boldsymbol{M}$-equivalences for each $p \in \mathbb{P}_0$ (where $\hat{(-)}_p$, also written $(\mathbb{F}_p)_\infty(-)$, denotes the p-completion of [**BK**, I, 4.2]).

Now if f is a p-equivalence for each $p \in \mathbb{P}_0$, then $\hat{f}_p : \hat{\boldsymbol{X}}_p \to \hat{\boldsymbol{Y}}_p$ is a homotopy equivalence (cf. [**BK**, I, 5.5]), so

$$(\prod_{p\in\mathbb{P}_0} \hat{f}_p)^{\boldsymbol{M}} : (\prod_{p\in\mathbb{P}_0} \hat{\boldsymbol{X}}_p)^{\boldsymbol{M}} \to (\prod_{p\in\mathbb{P}_0} \hat{\boldsymbol{Y}}_p)^{\boldsymbol{M}}$$

is a weak equivalence, and so by an arithemtic square argument (cf. [**BK**, VI, §8]) $f^{\boldsymbol{M}}$ is a weak equivalence (since $(\boldsymbol{W}_{\mathbb{Q}})^{\boldsymbol{M}} \simeq *$ for any 1-connected $\boldsymbol{W}$ by [**W**, V, 6.18] again).

Conversely, if f is an $\boldsymbol{M}$-equivalence, then each $\hat{f}_p : \hat{\boldsymbol{X}}_p \to \hat{\boldsymbol{Y}}_p$ is an $\boldsymbol{M}$-equivalence, so $\hat{F}_p$, the homotopy fiber of $\hat{f}_p$, is $\boldsymbol{M}$-local. But $\hat{F}_p$ is the p-completion of the homotopy fiber of $f : \boldsymbol{X} \to \boldsymbol{Y}$ by [**BK**, II, 4.8], so Theorem 2.1 applies to it and thus $\pi_*(\hat{F}_p) = 0$, which implies that $\hat{f}_p$ is a homotopy equivalence and thus that f is a $\mathbb{P}_0$-equivalence by [**BK**, I, 5.5] again. □

On the other hand, we have the following

LEMMA 2.4. *If $\boldsymbol{M}$ is any non-trivial c-connected finite-dimensional p-torsion complex ($c \geq 1$) then homotopy colimits do not in general preserve $\boldsymbol{M}$-equivalences of c-connected p-local spaces.*

PROOF. Let $\boldsymbol{X} = K(\mathbb{Z}_{(p)}, c+1)$, so $\boldsymbol{X}$ is $\boldsymbol{M}$-local, since $H^{c+1}(\boldsymbol{M}; \mathbb{Z}_{(p)}) = 0$. Assume s is maximal such that $\tilde{H}_s(\boldsymbol{M}; \mathbb{Z}) \neq 0$, so $\tilde{H}^{s+1}(\boldsymbol{M}; \mathbb{Z}_{(p)}) \neq 0$ (by [**S**, V, 5, Thm. 3], since $\boldsymbol{M}$ is p-torsion) but $\tilde{H}_t(\boldsymbol{M}; G) = 0$ for all $t > s+1$ and any group G. Since $\Sigma^{s-c}\boldsymbol{X}$ is s-connected and $\pi_{s+1}\Sigma^{s-c}\boldsymbol{X} \cong \mathbb{Z}_{(p)}$ by [VII,7.13]GWhE we see $[\boldsymbol{M}, \Sigma^{s-c}\boldsymbol{X}] \cong H^{s+1}(\boldsymbol{M}; \mathbb{Z}_{(p)}) \neq 0$ by [**W**, V, 6.18].

Similarly for some wedge $\boldsymbol{X} \vee \boldsymbol{X} \vee \ldots \vee \boldsymbol{X}$, using the (iterates of) the fibration sequence $\Sigma(\Omega\boldsymbol{X}) \wedge (\Omega\boldsymbol{Y}) \to \boldsymbol{X} \vee \boldsymbol{Y} \to \boldsymbol{X} \times \boldsymbol{Y}$. □

THEOREM 2.5. *Let $\boldsymbol{M}$ be a pointed c-connected r-dimensional CW complex ($1 \leq c < r-1$), with torsion homology; then there is an integer $e = e(\boldsymbol{M})$ ($c < e < r$) such that*

(i) *$\boldsymbol{M}$-equivalences between e-connected CW complexes are actually $\mathbb{P}_0$-equivalences for $\mathbb{P}_0$ as in Corollary 2.3 (and so are preserved by all homotopy colimits);*

(ii) *homotopy colimits in general do* not *preserve $\boldsymbol{M}$-equivalences between $(e-1)$-connected spaces (so these are not generally $\mathbb{P}_0$-equivalences).*

PROOF. Let e be the least integer such that $\boldsymbol{M}$-equivalences between e-connected CW complexes are actually $\mathbb{P}_0$-equivalences: then $c+1 \leq e \leq r-1$ by Corollary 2.3 and Lemma 2.4. Thus there exists a non-trivial $\boldsymbol{M}$-equivalence $f : \boldsymbol{X} \to \boldsymbol{Y}$ between $(e-1)$-connected spaces. Let $\boldsymbol{C}$ denote the homotopy cofiber of f; then $\boldsymbol{C}$ is $(e-1)$-connected, too, but $\boldsymbol{C}$ is not $\mathbb{P}_0$-equivalent to a point.

Assume that all homotopy colimits preserve $\boldsymbol{M}$-equivalences between $(e-1)$-connected spaces. Then in particular the solid vertical maps in

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \longrightarrow & C \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle id} & & \vdots \\ Y & \xrightarrow{id} & Y & \longrightarrow & * \end{array}$$

are $\boldsymbol{M}$-equivalences, so by assumption the dotted vertical map on the cofibers: $C \to *$, is, too – i.e., C is $\boldsymbol{M}$-local. Therefore, by the assumption again ΣC is $\boldsymbol{M}$-local – but ΣC is e-connected, so this implies ΣC is $\mathbb{P}_0$-equivalent to a point, by the definition of e, and thus C is, too (since it is 1-connected) – which is a contradiction. □

EXAMPLE 2.6. If $V_0 = S^r \cup_k e^{r+1}$ is the $(r+1)$-dimensional mod k Moore space $(r \geq 2)$, then necessarily $e(V_0) = r$.

More generally, we can identify $e(\boldsymbol{M})$ (in some cases) for the following class of spaces:

DEFINITION 2.7. Fix a prime p. A space $V \in \mathcal{T}_\star$ will be called *periodically resolvable* of type $(p^\ell, v_1^{k_1}, v_2^{k_2}, \dots, v_n^{k_n})$ if if there is a sequence of spaces $\{V_m\}_{m=-1}^n$ $(n \geq 0)$ with $V = V_n$ such that for each $m \geq 0$, V_m has a v_m-self map $v_m : \Sigma^{d_m} V_{m-1} \to V_{m-1}$ – see [**R2**, §1.5] – with cofiber V_m. We always start with a sphere $V_{-1} = S^j$, and $v_0 : V_{-1} \to V_{-1}$ is the degree p^ℓ map, so V_0 is the $(j+1)$-dimensional mod p^ℓ Moore space. For simplicity we assume that each v_m is a suspension (and all spaces are simply connected).

Our notation implies that (for $m \geq 0$) $d_m = 2k_m(p^m - 1)$ (with $k_m \geq 1$). The dimension of V_m will be denoted by r_m, so $r_{-1} = j$, $r_0 = j + 1$, and in general $r_m = r_{m-1} + d_m + 1$.

Remark 2.8. Such spaces exist for all $n \geq 0$, (though not necessarily for every choice of $(p^\ell, v_1^{k_1}, v_2^{k_2}, \dots, v_n^{k_n}$ – cf. [**T**] and [**R1**, §1.3]). They play a central role in the definition of v_n-periodicity (cf. [**B**]). The concept of V_n-equivalence is in some sense complementary to that of a map inducing an isomorphism in the periodic homotopy groups $v_m^{-1}\pi_*(-; V_{m-1})$ for $0 \leq m \leq n$. Bousfield has answered the question corresponding to ours by showing that such maps are preserved by homotopy colimits if the spaces in question are sufficiently connected (cf. [**B**, Thm. 13.3] and [**BT**, Cor. 7.9]).

PROPOSITION 2.9. *Let* $V = V(n)$ *be an* r*-dimensional periodically resolvable space of type* $(p^\ell, v_1 \dots, v_n)$*; then* $e(V) = r - 1$.

PROOF. Let $\boldsymbol{BP}\langle n \rangle$ denote the spectrum with

$$\pi_\star \boldsymbol{BP}\langle n \rangle \cong \mathbb{Z}_{(p)}[v_1, v_2, \dots, v_n],$$

where in this case $d_m = |v_m| = 2(p^m - 1)$ (cf. [**R1**, §4.2]), and let $\boldsymbol{X}$ denote the infinite loop space corresponding to $\Sigma^{r-1}\boldsymbol{BP}\langle n \rangle$, so that as a graded $\mathbb{Z}/p^\ell$-module (though no longer as an algebra)

$$\pi_*(\boldsymbol{X}; V_0) \cong \Sigma^{r-1}\mathbb{Z}/p^\ell[v_1, v_2, \dots, v_n]. \tag{2.10}$$

We wish to show that $\pi_i(\boldsymbol{X};\boldsymbol{V}) = 0$ for $i \geq r$ – i.e., that $\boldsymbol{X}$ is a non-trivial $(r-2)$-connected $\boldsymbol{V}$-local space, which proves the Proposition. In order to do so, we shall show by induction on $m \geq 1$ that

$$\pi_*(\boldsymbol{X};\boldsymbol{V}_m) \cong \Sigma^{r-1}(\mathbb{Z}/p^\ell[v_{m+1}, v_{m+2}, \dots, v_n]/(\mathbb{Z}/p^\ell)), \tag{2.11}$$

(that is, $\pi_*(\boldsymbol{X};\boldsymbol{V}_m)$ is isomorphic as a graded module to the (shifted) augmentation ideal of the algebra on $v_{m+1}, \dots, v_n$, so in particular $\pi_*(\boldsymbol{X};\boldsymbol{V}_n) = 0$).

Now for each $m > 0$ we have a cofibration sequence

$$\Sigma^{d_m}\boldsymbol{V}_{m-1} \xrightarrow{v_m} \boldsymbol{V}_{m-1} \to \boldsymbol{V}_m,$$

and so a long exact sequence (for $t \geq r_{m-1}$):

$$\dots \longrightarrow \pi_t(\boldsymbol{X};\boldsymbol{V}_{m-1}) \xrightarrow{v_m^\#} \pi_{t+d_m}(\boldsymbol{X};\boldsymbol{V}_{m-1}) \longrightarrow \pi_{t+d_m}(\boldsymbol{X};\boldsymbol{V}_m) \longrightarrow$$
$$\longrightarrow \pi_{t-1}(\boldsymbol{X};\boldsymbol{V}_{m-1}) \xrightarrow{v_m^\#} \pi_{t+d_m-1}(\boldsymbol{X};\boldsymbol{V}_{m-1}) \longrightarrow \dots$$

Since $\pi_t(\boldsymbol{X};\boldsymbol{V}_0) = 0$ for $t \leq r-2$, we see that $\pi_s(\boldsymbol{X};\boldsymbol{V}_0) \cong \pi_s(\boldsymbol{X};\boldsymbol{V}_1)$ for $r \leq s \leq r-2+d_1$, so that in fact $\pi_s(\boldsymbol{X};\boldsymbol{V}_1) = 0$ for $r \leq s \leq r-2+d_1$ (using (2.10). But $v_1 : \Sigma^{d_1}\boldsymbol{V}_0 \to \boldsymbol{V}_0$ induces (formal) "multiplication by v_1" (under the isomorphism of (2.10)), and since this is monic (in a polynomial algebra), we find $\pi_s(\boldsymbol{X};\boldsymbol{V}_0)/(\operatorname{im} v_1) \cong \pi_s(\boldsymbol{X};\boldsymbol{V}_1)$ for $s \geq r-1+d_1$ – i.e., (2.11) holds for $m = 1$.

In general, if (2.11) holds for some $m < n$, then $\pi_t(\boldsymbol{X};\boldsymbol{V}_m) = 0$ for $r \leq t \leq r-1+d_{m+1}$, so the same argument shows (2.11) holds for $m+1$. □

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