

University of Haifa

The Third International Workshop

on

**“Geometric Structures and Interdisciplinary Applications”**

Session B on

**Homotopy Theory**

Wednesday, May 9, 2018

Science and Education Building, room 614, University of Haifa

10:40-11:30	Drew Heard	Invertible objects in stable homotopy theory
11:40-12:30	Emmanuel Farjoun	Transfinite Milnor invariant of knots
12:30-13:30	Lunch	
13:30-14:20	Ishai Dan-Cohen	Rational motivic points & rational motivic path spaces
14:30-15:20	Assaf Horev	Genuine equivariant factorization homology
15:20-16:00	Coffee break	
16:00-16:50	Karol Szumilo	Higher calculus of fractions
17:00-17:50	Shachar Carmeli	A topological Galois first cohomology of algebraic groups

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## “Geometric Structures and Interdisciplinary Applications”

Program of Session B on **Homotopy Theory****Drew Heard:** Invertible objects in stable homotopy theory

The Picard group of the stable homotopy category is known to contain only suspensions of the sphere spectrum. After certain localizations, however, the Picard group becomes much more interesting. We will show how to use Galois descent methods to compute some new Picard groups arising in chromatic homotopy theory. This is joint work with Mathew and Stojanoska.

**Emmanuel Farjoun:** Transfinite Milnor invariant of knots

In his early work on links in three space, J. Milnor defined invariants  $\bar{\mu}$  that distinguish a link group  $G = \pi(S^3 - L)$  from the free group  $F$ . However it is possible for all these invariant to vanish without  $G$  being free. Later on, Orr defined further invariants lying in until now unknown third homotopy group associated to the Bousfield Kan completion of  $BF$ . We show that his target group of invariants is very large, by analyzing the completion tower and using an old exact sequence due to Whitehead.

**Ishai Dan-Cohen:** Rational motivic points & rational motivic path spaces

A central ingredient in Kim’s work on integral points of hyperbolic curves is the “unipotent Kummer map”, which goes from integral points to certain torsors for the prounipotent completion of the fundamental group, and which, roughly speaking, sends an integral point to the torsor of homotopy classes of paths connecting it to a fixed base-point. In joint work with Tomer Schlank, we introduce a space  $\Omega$  of rational motivic loops, and we construct a double factorization of the unipotent Kummer map which may be summarized schematically as

$$\text{points} \Rightarrow \text{rational motivic points} \Rightarrow \Omega\text{-torsors} \Rightarrow \pi_1\text{-torsors.}$$

Our “connectedness theorem” says that any two motivic points are connected by a non-empty torsor. Our “concentration theorem” says that for an affine curve,  $\Omega$  is actually equal to  $\pi_1$ .

**Assaf Horev:** Genuine equivariant factorization homology

Factorization homology is a method for constructing quantum field theories from  $\mathbb{E}_n$ -algebras. We describe a genuine  $G$ -equivariant version of factorization homology for a finite group  $G$ . A  $G$ -factorization homology theory assigns to each smooth manifold with an action of a subgroup  $H < G$  a genuine  $H$ -spectrum. Following Ayala and Francis we give an axiomatic characterization of such theories as satisfying a monoidal version of excision and intertwining topological induction of manifolds with multiplicative transfer of spectra. As a future application we present real THH as genuine  $\mathbb{Z}/2$  factorization homology.

**Karol Szumilo:** Higher calculus of fractions

It is a classical result that a category of fibrant objects in the sense of Brown admits a calculus of fractions which provides a fairly explicit description of its homotopy category. I will discuss the “quasicategory of frames” – a generalization of calculus of fractions which can be used to describe the associated  $(\infty, 1)$ -category in a similar manner.

**Shachar Carmeli:** A topological Galois first cohomology of algebraic groups

Let  $F$  be a field, and  $G$  an algebraic group defined over  $F$ . Even if  $G$  is non-commutative, the Galois cohomology  $H^1(F, G)$  is defined as a pointed set. In the case where  $F$  is of characteristic 0 and  $G$  is a connected linear algebraic group, Michail Borovoi constructs an abelian group  $H_{\text{ab}}^1(F, G)$  with functorial abelianization map  $\text{ab}^1 : H^1(F, G) \rightarrow H_{\text{ab}}^1(F, G)$ .

I will present a general construction of an inverse system of “topological” first cohomology sets  $H_{\text{top},n}^1(F, G)$  together with “topologization” maps  $\text{top}_n^1 : H^1(F, G) \rightarrow H_{\text{top},n}^1(F, G)$ . For  $\text{char}(F) = 0$  and  $G$  a linear algebraic group, we expect to recover Borovoi’s construction for  $n = 2$  and  $n = 3$ . The construction of the topological cohomology uses the notion of relative étale topological type and can be applied to arbitrary sheaf of groups on a site.

This is a work in progress, joint with Ariel Davis.