The Third International Workshop
“Geometric Structures and Interdisciplinary Applications”

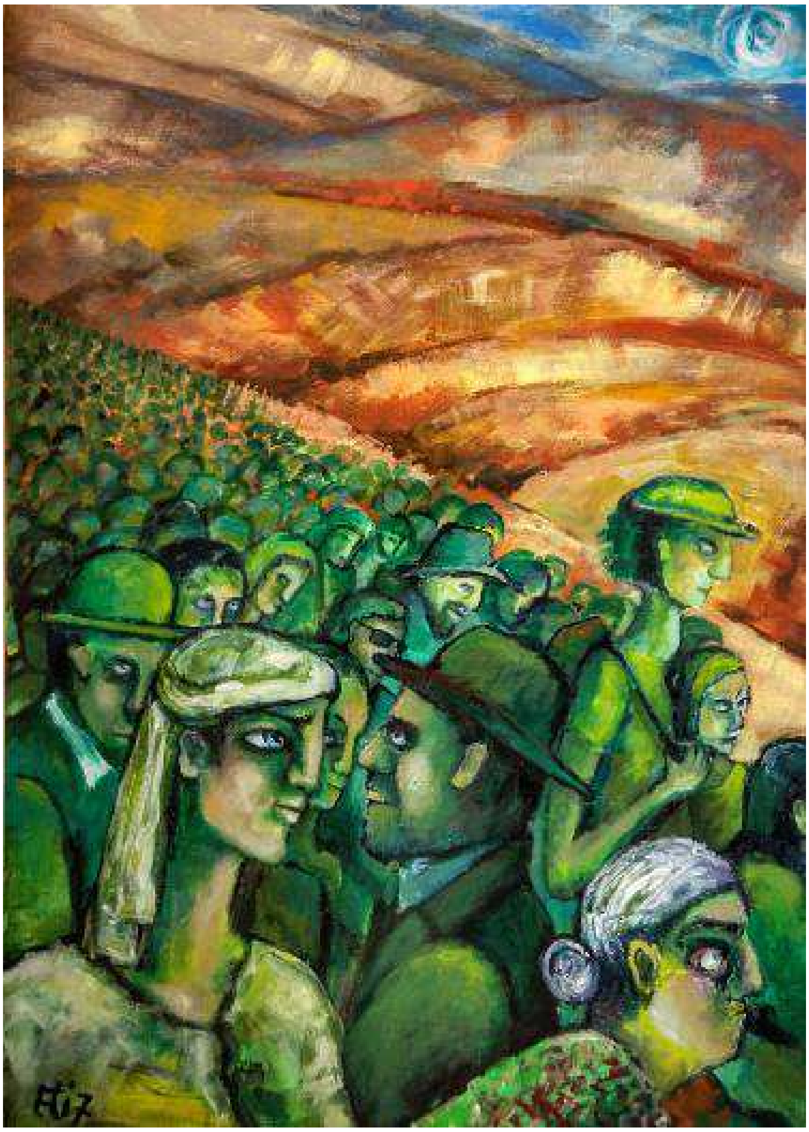
University of Haifa, ISRAEL

Program

and

Book of Abstracts

9 – 12 May, 2018



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Prof. David Blanc: Section B

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The Caesarea Edmond Benjamin de Rothschild
Foundation Institute for Interdisciplinary
Applications of Computer Science
(CRI) at the University of Haifa

CRI Caesarea Rothschild Institute
for Interdisciplinary Applications
of Computer Science

The Department of Mathematics
at the University of Haifa.

אוניברסיטת חיפה
לבלוט בנוף האקדמי

Schedule (Sections A, C)

8 May	9 May	10 May	11 May	12 May
	08:40 – 9:20, r. 624 Registration	room 146 ↓	room 570 ↓	
Excursion	09:30 – 09:45, r. 146 ↓ Opening: I. Albinsky	09:00 – 09:30 R. Langevin	09:00 – 09:30 H. Nozawa	Excursion
	09:50 – 10:20 P. Walczak	09:40 – 10:10 M. Djoric	09:40 – 10:10 C. Bejan	
	10:30 – 11:00 A. Naveira	10:20 – 10:50 J. Mikeš	10:20 – 10:50 A. Tralle	
	11:10 – 11:40 Coffee Break	11:00 – 11:30 Coffee Break	11:00 – 11:30 Coffee Break	
	11:40 – 12:10 K. Udrişte	11:30 – 12:00 V. Rovenski	11:30 – 12:00 Q.B. Tran	
	12:20 – 12:50 I. Vaisman	12.10 – 12.40 M. Soret	12:10 – 12:40 M. Munteanu	
	13:00 – 14:00 Lunch (IBM)	12.50 – 14.10 Lunch (IBM)	12.50 – 13.15 P. Horak	
	Campus tour 14:00 – 14:30	room 570 ↓		
	14:30 – 15:00 r. 146 ↓ V. Balan	14.10 – 14.40 V. Golubyatnikov	13.20 – 13.50 Lunch (r. 570)	
	15:10 – 15:40 Y. Nikonorov	14.50 – 15.20 Y. Yomdin	13.50 – 14.00 Closing	
	15:45 – 16:05 Z. Walczak	15:30 – 15:50 I. Gaissinski		
	16:10 – 16:30 Coffee Break	16:00 – 16:20 Coffee Break		
	16:30 – 17:00 Z. Rakić	16.20 – 16.40 M. Ville		
	17:10 – 17:30 L. Velimirović	16.50 – 17.10 T. Zawadzki	16.00 bus 24 to Dinner area	
Registration 17.00–18.00 r. 624	17.40 – 18.00 A. Velimirović	17.10 – 18.00 Poster session		
	19.00 – optional Touring Haifa	19.00 – optional Touring Haifa	17.30 – 21.30 W/s Dinner	

Registration will be held in room 624 (V.R. office) of the Science and Education Building.
All talks on May 9 will be held in room 146 - Library area, University of Haifa.
All talks on May 10 will be held in room 146 before 13:00, and in room 570 after 14:00.
All talks on May 11 will be held in room 570 of the Science and Education Building.

Section B on
Homotopy Theory

Wednesday, May 9, 2018

Science and Education Building, room 614, University of Haifa

10:40-11:30	Drew Heard	Invertible objects in stable homotopy theory
11:40-12:30	Emmanuel Farjoun	Transfinite Milnor invariant of knots
12:30-13:30	Lunch	
13:30-14:20	Ishai Dan-Cohen	Rational motivic points & rational motivic path spaces
14:30-15:20	Assaf Horev	Genuine equivariant factorization homology
15:20-16:00	Coffee break	
16:00-16:50	Karol Szumilo	Higher calculus of fractions
17:00-17:50	Shachar Carmeli	A topological Galois first cohomology of algebraic groups

For further information contact:

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Preface

Dear Participants: Distinguished Professors and Students, Ladies and Gentlemen!

On behalf of the Organizing Committee of the Third International Workshop "*Geometric Structures and Interdisciplinary Applications*", I extend a warm welcome to all of you.

The workshop will be held from May 9, 2018 until May 12, 2018, at Haifa University. The lectures on May 9 (all day) and May 10 (until 13:00) are held in Room 146, Library Building. The lectures on May 10 (from 13:00) and May 11 (all day) are held in Room 570, Education Building. Pre-workshop Informal Sessions on May 8, and Tours on May 12.

This Workshop is dedicated to Geometric structures (Differential and Computational geometries, geometry–topology–dynamics of Foliations) and Interdisciplinary Applications (e.g. in Mathematical Physics and Biology), as well as other related themes are welcome.

The aims of the workshop: to create a forum for workers in pure and applied mathematics, including students, to promote discussion of state-of-art in modern Geometry; presentation of new results and projects.

The first International Workshop in this series, named "*Reconstruction of Geometrical Objects Using Symbolic Computations*", was on September 2008, University of Haifa.

The second International Workshop in this series, named "*Geometry and Symbolic Computations*", was on May 2013, University of Haifa.

This workshop is supported by The Caesarea Edmond Benjamin de Rothschild Foundation Institute for Interdisciplinary Applications of Computer Science (CRI), and the Department of Mathematics at the University of Haifa.

This booklet presents abstracts, a list of all the activities of the workshop: lectures and posters, presentations and other activities for workshop's days. Booklet is illustrated by paintings of my and Vladimir's friend, painter Elisheva Nesis, and also contains some useful practical information, which I hope will help to make your stay in Haifa, Israel comfortable.

The organizers wish you a very pleasant and a successful workshop.

With warm regards, Irina Albinsky, Ph.D. Computer Sciences and Modeling Specialist
Secretary of Organizing Committee Haifa, Israel, May 2018.

CONTENTS

Section A

Abstracts

Section A

Extensions of the multilinear spectral problem in the Finslerian framework

Vladimir Balan, University Politehnica of Bucharest, Romania)

Key words: Finsler structures, n-way arrays, symmetric covariant tensors, Z-spectra, H-spectra, Cartan tensor, geometric invariance, fundamental tensor field.

Finsler Geometry has provided in the last decade significant models for Special Relativity (e.g., the Pavlov–Chernov, Bogoslovsky m-th root, and Roxbourgh models), for ecology (P.L. Antonelli at al.), and for HARDI biology (Higher Angular Resolution Diffusion Imaging, introduced by L. Astola at al.). As well, Finslerian models were recently obtained, in both the Garner oncologic evolution framework [3] and in the physics of Langmuir–Blodgett monolayers [2].

In the present work, we first present a brief survey of results from the spectral theory of covariant symmetric tensors – regarded as n-way arrays – which mainly rely on the fundamental geometric objects from anisotropic geometric models. These objects are provided by pseudo-norms and play a major role in describing anisotropic metric structures. Their spectral data describe properties of the indicatrices associated to the norms, point out their asymptotic properties, and allow to derive best rank-I approximations – which provide simpler consistent estimates for the original anisotropic structures. We investigate the spectral data of covariant symmetric d-tensor fields, and focus on the metric and Cartan fields of the Finsler structures – including the Euclidean and Riemannian subcases – and further provide and discuss natural alternatives of the spectral equations.

We consider n-dimensional Finsler structures (M, F) with the main axioms relaxed by either dropping the positivity condition, or reducing the domain, and replacing the positive-definiteness of the Finsler metric d-tensor field with the non-degeneracy and constant signature condition. We shall denote by $g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$ and $C_{ijk} = \frac{1}{4}\frac{\partial^3 F^2}{\partial y^i \partial y^j \partial y^k}$ the components of the metric and Cartan d-tensor fields, respectively. One of the important features of the Cartan tensor is that its vanishing makes g quadratic in y, and consequently the Finsler space becomes Riemannian (correspondingly, pseudo-Finsler spaces become, in such case, pseudo-Riemannian).

For a real m-covariant symmetric tensor field T on the flat manifold $V = \mathbb{R}^n$ endowed with the Euclidean metric g, we say that a real λ is a Z-eigenvalue and that a vector y is an Z-eigenvector associated to λ, if they satisfy the system:

$$T.y^{m-1} = \lambda \cdot y, \quad g(y, y) = 1, \quad \text{where} \quad T.y^{m-1} = \sum_{i,i_2,\ldots,i_m \in \overline{1,n}} T_{i,i_2\ldots i_m} y_{i_2} \cdot \ldots \cdot y_{i_m} dx^i,$$

where by lower dot is repeated transvection and the power is tensorial. In the complex case, one calls λ and y as $E-$eigenvalue and $E-$eigenvector, respectively.

L. Qi defined as alternative for spectral objects, the H-eigenvalue λ and its H-eigenvector, a real number λ and a vector y, which satisfy the homogeneous polynomial system of order $m-1$: $(T.y^{m-1})_k = \lambda (y^k)^{m-1}$. In the complex case, λ and y are called N-*eigenvalue* and N-*eigenvector*, respectively.

Regarding the spectra consistency, it is known that in the Euclidean subcase, the Z- and the H-spectra are nonempty for even symmetric tensors, and that a symmetric tensor T is positive defynite/semi-definite if and only if all its $H-$ (or $Z-$) eigenvalues are positive/non-negative. Details on asymptotic rays, recession vectors, degeneracy sets, the best rank-I approximation and the Z- and H-spectral eigendata for various Finsler metrics (Berwald-Moor, Chernov and Bogoslovsky m-th root relativistic models) were studied in [1]. Among the applications of the n-way spectral Finsler approach, starting with 2012, there was studied the relevance of eigendata of the metric and Cartan tensors of the Langmuir–Finsler structure [1, 2, 7] within the physics of monolayers, which studies the interphase boundary of a monomolecular 2D-system [5, 6].

We illustrate, for fixed flagpoles, the Z-eigendata of tensors in Langmuir–Finsler and the Garner–Randers structures. We emphasize that while the Z-eigenproblem may lead to a globally covariant alternative, the H-eigenproblem exhibits a strongly local character. In this respect, we provide and examine the Finslerian geometrically covariant sibling of the Z-eigenproblem:

$$T_{(x,y)} \cdot y^{m-1} = \lambda \cdot g_{(x,y)} \cdot y, \quad g_{(x,y)} \cdot y^2 = a,$$

where g is the Finsler metric tensor field and $a \in \mathbb{R}$. We study, in particular, the eigendata for homogeneous d-tensors T, including the g and C (trivial) subcases.

References

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Sasaki metric on the tangent bundle of a Weyl manifold

Cornelia-Livia Bejan and Ilhan Gul, University Politehnica of Bucharest, Romania)

If $(M;[g])$ is a Weyl manifold of dimension $m > 2$, then by using the Sasaki metric G induced by g, we construct a Weyl structure on TM. We prove that this Weyl structure is never Einstein–Weyl unless $(M;g)$ is flat. The main theorem here extends to the Weyl context a result of Musso and Tricerri.

Main reference. This talk is based on our recent work accepted for publication in Publications de l'Institut Mathématique (Beograd).

Normal curvature of CR submanifolds of maximal CR dimension of complex space forms

Mirjana Djorić (University of Belgrade, Serbia)

Let us suppose that the ambient space is a complex manifold $(\bar{M}^{\frac{n+p}{2}}, J)$, equipped with a Hermitian metric $\bar{g}$ and let M^n be its CR-submanifold of maximal CR-dimension, that is, such that at each point x of M the real dimension of $JT_x(M) \cap T_x(M)$ is $n-1$. Then it follows that M is odd-dimensional and that there exists a unit vector field ξ_x normal to $T_x(M)$ such that $JT_x(M) \subset T_x(M) \oplus \mathrm{span}\{\xi_x\}$, for any $x \in M$. Examples of CR submanifolds of maximal CR dimension are real hypersurfaces of $\bar{M}$, real hypersurfaces of complex submanifolds of $\bar{M}$, odd-dimensional F'-invariant submanifolds of real hypersurfaces of $\bar{M}$, where F' is an almost contact structure of a real hypersurface of $\bar{M}$.

We prove that there do not exist CR submanifolds M^n of maximal CR dimension of a complex projective space $\mathbf{P}^{\frac{n+p}{2}}(\mathbb{C})$ with flat normal connection D of M, when the distinguished normal vector field ξ is parallel with respect to D.

For a submanifold M^n of $\mathbf{P}^{\frac{n+p}{2}}(\mathbb{C})$, it is well-known that $\pi^{-1}(M)$ is a submanifold of $\mathbb{S}^{n+p+1}$, where $\pi^{-1}(M)$ is the circle bundle over M which is compatible with the Hopf map $\pi : \mathbb{S}^{n+p+1} \to \mathbf{P}^{\frac{n+p}{2}}(\mathbb{C})$. If the normal connection of $\pi^{-1}(M)$ in $\mathbb{S}^{n+p+1}$ is flat, we say that the normal connection of M is lift-flat. We prove that if D is lift-flat, then there exists a totally geodesic complex projective subspace $\mathbf{P}^{\frac{n+1}{2}}(\mathbb{C})$ of $\mathbf{P}^{\frac{n+p}{2}}(\mathbb{C})$ such that M is a real hypersurface of $\mathbf{P}^{\frac{n+1}{2}}(\mathbb{C})$.

This is a joint work with Masafumi Okumura.

Some geometric properties of extended Bianchi–Cartan–Vranceanu spaces

Angel Ferrández, Antonio M. Naveira and Ana D. Tarrío (University of Valencia, Spain)

The Bianchi–Cartan–Vranceanu (BCV) spaces are 3-dimensional homogeneous spaces whose differential geometry has been extensively studied in the past two decades. On the occasion of the Differential Geometry Conference held at Bedlewo on June 2017, we had the opportunity to present a 7-dimensional generalization of those spaces, the so-called Extended Bianchi–Cartan–Vranceanu (EBCV) ones. In that lecture we laid the foundation for the construction of these Riemannian manifolds. There we exhibited the values of the Levi–Civita connection, the sectional curvatures, the Ricci tensor, the problem of solving the differential equations that allow us to calculate their geodesics as well as the computations of the Killing fields. In order to deepen into the geometry of those spaces, we now analyze their homogeneous structure as well as the geometric properties of the underlying almost-product structure, which is very close to that of a product manifold. Using Hermann Weyl's theory, we compute the linear invariants and some of the quadratics ones. In particular, it is proved that these spaces have constant scalar curvature. Some of those invariants appear in the power series expansions of the area and volume of a sphere and a geodesic ball. We determine them up to fourth order. Given its interest, due to its special properties, the geometry related to the characteristic connection is considered. Finally, the problem of studying the properties of its sub-Riemannian geometry is addressed.

Modeling in fluid mechanics: instabilities and turbulence

Igor Gaissinski and Vladimir Rovenski (University of Haifa and Technion, Israel)

The talk is devoted to three mathematical models for fluid mechanics developed by the authors, see [1]. We start from the derivation of the Navier–Stokes equation for (non-) relativistic fluid mechanics and reviews the problem of existence and uniqueness of solutions.

We deal with two (among 20 known) hydrodynamic instability types:

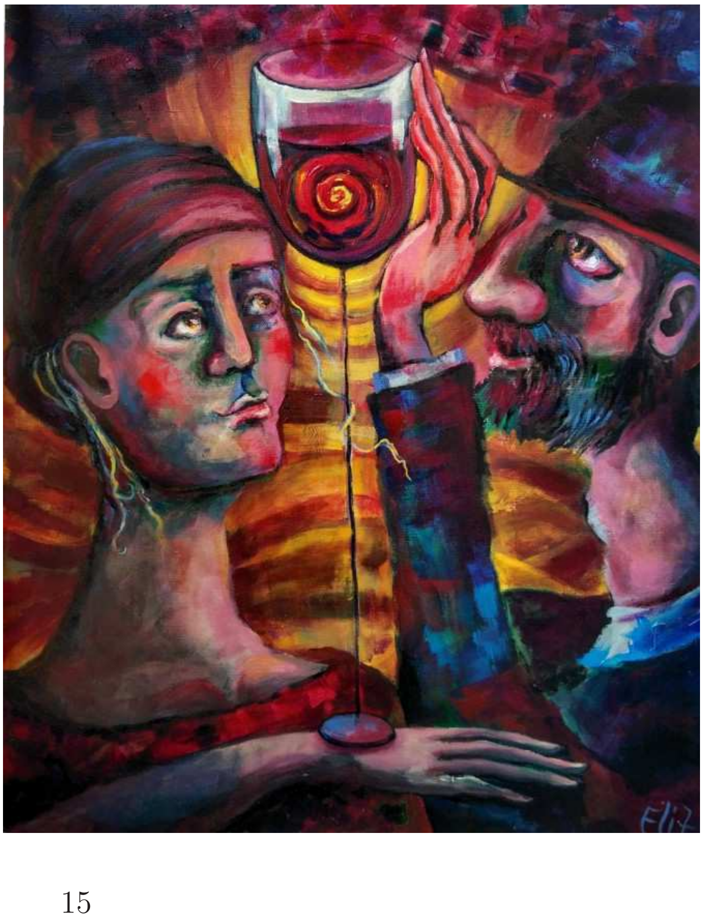
– **Rayleigh-Taylor instability** of a thin liquid film. We deduce the dispersion relation, based on continuity and momentum conservation equations for a viscous compressible jet. A unique nonlinear model is developed for such instability of thin liquid films and solid shells.

– **Kelvin-Helmholtz instability** for a jet of low viscosity. The second example is typical for the behavior of rotated thin liquid films in atomizers, in aircraft engine chambers.

Finally, we discuss the Richardson–Kolmogorov concept of turbulence, which is important for multifractal and hierarchical (shell) models. The main goal is deducing of phenomenological relation based on the main conservation laws (energy, enstrophy and helicity). This relation plays the key role in the choice of shell model for the energy cascade in 2D and 3D fully developed turbulence. The **hierarchical model of turbulence** is based on the assumption that the turbulence is an ensemble of vortices of progressively diminishing scales. The hierarchical base for 2D turbulence describes the ensemble of the vortices, in which any vortex of the given size consists of four vortices of half size and so on. The ensemble of vortices of the same size forms a level. The functions of the hierarchical base are constructed in such a way that Fourier-images of vortices of single level occupy only single octant in the wave-number space and regions of localization of different levels in the Fourier space do not overlap. The Fourier-images space is to be divided by infinite number of shells.

References

[1] Gaissinski I. & Rovenski V. *Modeling in Fluid Mechanics: Instabilities and Turbulence*, CRC, 2018.



Some asymmetric models of circular gene networks

Vladimir Golubyatnikov (Sobolev Institute of Mathematics, Novosibirsk state University, Russia)

We study phase portraits of nonlinear kinetic dynamical systems as models of circular gene networks functioning. Discretization of these phase portraits allows to find conditions of existence of cycles in these portraits.

Key words: Circular gene network; hyperbolic equilibrium points; cycles.

We consider $(4N+2)$-dimensional dynamical systems of the type

$$dm_1/dt = -k_1 m_1 + f_1(p_{2N+1}); \quad dp_1/dt = \mu_1 m_1 - \nu_1 p_1; \ \dots \tag{1}$$

$$dm_j/dt = -k_j m_j + f_j(p_{j-1}); \quad dp_j/dt = \mu_j m_j - \nu_j p_j; \ \dots$$

as a model of a circular gene network functioning. Here $j = 2, 3, \dots N$, all coefficients correspond to kinetic parameters of reactions and are positive, the nonnegative variables m_j, p_j denote concentrations of different mRNA's and proteins, the functions $\{f_j\}$ are smooth, positive and monotonically decreasing, they describe negative feedbacks in synthesis of $\{m_{j+1}\}$, the negative terms describe natural degradations of these species.

Similar 6-dimensional system in its symmetric case $k_1 = k_2 = k_3 = 1$, $\mu_1 = \mu_2 = \mu_3$, $\mu_j = \nu_j$ and $f_j(p) \equiv \alpha(1+p^\gamma)^{-1} + \alpha_0$, was introduced in [1] and later this particular case, symmetric with respect to permutations $(m_1, p_1) \Rightarrow (m_2, p_2) \Rightarrow (m_3, p_3) \Rightarrow (m_1, p_1)$, was studied in [2] and in many other publications. Here we consider the general case. Let $A_j := f_j(0)/k_j$, $B_j := A_j/\nu_j$ and $Q := [0, A_1] \times [0, B_1] \times \dots \times [0, A_{2N+1}] \times [0, B_{2N+1}]$.

One can verify (cf. [3]) that $Q \subset \mathbb{R}_+^{4N+2}$ is an invariant domain of the system (1), and that this system has exactly one equilibrium point S_0 which is contained in the interior of Q. Let $(m_1^0; p_1^0; m_2^0; p_2^0; \dots m_{2N+1}^0; p_{2N+1}^0)$ be coordinates of the point S_0, and $(-q_j) := df_j(p)/dp$ be calculated at $p = p_{j-1}^0$. Here $j - 1 := 2N+1$ for $j = 1$.

Consider partition of the domain Q by the hyperplanes $m_j = m_j^0$, $p_j = p_j^0$. Small 2^{4N+2} parallelepipeds (blocks) of this partition are enumerated by binary indices

$$\mathcal{E} = \{\varepsilon_1 \varepsilon_2 \varepsilon_3 \dots \varepsilon_{4N+1} \varepsilon_{4N+2}\} =$$

$$\left\{ \mathbf{X} \in Q \mid m_1 \gtreqless_{\varepsilon_1} m_1^0; \ p_1 \gtreqless_{\varepsilon_2} p_1^0; \ \dots m_{2N+1} \gtreqless_{\varepsilon_{2N+1}} m_{2N+1}^0; \ p_{2N+2} \gtreqless_{\varepsilon_{2N+2}} p_{2N+2}^0, \right\}, \tag{2}$$

where $\mathbf{X} = (m_1, p_1, m_2, p_2, \dots, m_{2N+1}, p_{2N+1})$, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots \varepsilon_{4N+1}, \varepsilon_{4N+2} \in \{0, 1\}$, the orders are defined as follows: $\gtreqless_0$ means $\leq$, while $\gtreqless_1$ means $\geq$.

For any two incident blocks $\mathcal{E}_1$ and $\mathcal{E}_2$ with common $(4N+1)$-dimensional face $\mathcal{E}_1 \cap \mathcal{E}_2$, trajectories of all points of this face pass to one of these blocks only, either from $\mathcal{E}_1$ to $\mathcal{E}_2$, or vice versa, from $\mathcal{E}_2$ to $\mathcal{E}_1$.

Linearization matrix of the system (1) at S_0 has characteristic polynomial of the form

$$P(\lambda) = \prod_{j=1}^{j=2N+1} (k_j + \lambda)(\nu_j + \lambda) + a^{4N+2},$$

where $a^{4N+2} := \prod_{j=1}^{j=2N+1} q_j \mu_j$. Recall that an equilibrium point S_* of a dynamical system is called *hyperbolic* if linearization matrix of this system at the point S_* has eigenvalues with positive and negative real parts, and does not have imaginary eigenvalues.

Next diagram shows in the simple case $N = 1$ one possible sequence of blocks of the partition (2) of the phase portrait of the system (1) where a cycle of this system can be located. This cycle travels from one block to another according to the arrows of (3). Similar diagrams can be constructed for higher-dimensional systems of the type (1).

$$
\begin{array}{ccccccccc}
\{110010\} & \longrightarrow & \{110011\} & \longrightarrow & \{010011\} & \longrightarrow & \{000011\} & \longrightarrow & \{001011\} \\
\uparrow & & & & & & & & \downarrow \\
\{110000\} & & & & & & & & \{001111\} \\
\uparrow & & & & & & & & \downarrow \\
\{110100\} & \longleftarrow & \{111100\} & \longleftarrow & \{101100\} & \longleftarrow & \{001100\} & \longleftarrow & \{001101\}
\end{array}
\qquad (3)
$$

Let W_N be the unions of the blocks listed in these diagrams, and let U be an appropriate small neighborhood of the point S_0 where the system (1) is linearized according to the Grobman-Hartman theorem. For any N, the union W_N is an invariant domain of (1).

Lemma. *For sufficiently large values of the parameter a, the point S_0 is hyperbolic.*

Theorem. *If the equilibrium point S_0 of the system* (1) *is hyperbolic, then this system has at least one cycle in the domain $W_N \setminus (W_N \cap U)$, which travels from block to block according to the diagram* (3).

Similar considerations are used in biological publications, see for example [4].

Supported by RFBR, grant 18-01-00057.

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Algebraic methods in tilings

Peter Horak and Dong Ryul Kim (University of Washington, and Harvard University, USA)

Tilings and tessellations belong to the oldest structures not only in geometry but in all mathematics. They have attracted attention of best mathematicians. Even one of the Hilbert's problems is on the topic. Tiling problems do not always have a geometric background, sometimes there is even an unexpected relation of a tiling to other parts of mathematics. For example, the roots of the Minkowski conjecture on tiling the n-space by unit cubes can be traced to geometry of numbers and to positive definite quadratic forms; Hao Wang's work on tilings has been inspired by decision problems; there is a well-known relation of Penrose tilings to crystallography, etc. As a part of a brief historic introduction to the area we will describe the Hilbert's 18th problem (1900), and its present status. Our interest in tilings stems from coding theory, especially from the area of error-correcting codes in Lee metric. Therefore in this talk we will focus on tiling the n-space by unit cubes or by a cluster (the union) of unit cubes. Hajos seems to be the first one to use algebraic methods in this type of tilings. In 1939 he reformulated Minkowski conjecture (1904) in terms of abelian groups and thanks to this reformulation he was able to answer this conjecture in the affirmative way. To demonstrate the strength of so-called polynomial method we will provide a short proof of the statement that each tiling of n-space by a cluster of a prime size has to be periodic. This method also enables one to employ fundamental results from algebraic geometry. For example, we will present the first necessary condition for a generic (arbitrary) cluster of cubes to tile the n-space, which has been proved by applying Hilbert's Nullstellensatz. Further, we will also present a sufficient condition for a generic (arbitrary) cluster of cubes to have a property that each tiling by the given tile is periodic; to prove this condition we have used methods of Fourier analysis developed by Lagarias and Wang (1996). Finally, in 2015 we conjectured that each tiling of the n-space by a tile of a prime size is not only periodic but also a lattice tiling. Partial results on this conjecture will be discussed.

The spaces of spheres, circles and the Dupin and Darboux cyclides

Rémi Langevin (Institut de Mathematiques de Bourgogne, Dijon, France)

To cyclides correspond curves in the space of spheres of $\mathbb{S}^3$ or of circles in $\mathbb{S}^3$. Dupin cyclides, a 9-dimensional family of surfaces, are envelope in two ways of one- parameter families of spheres and enjoy nice ways to be glued along curves. Daboux cyclides are spanned by two (or more) families of circles such that two circles, one in each family, are contained in a sphere. They are therefore union of 9-dimensional families, very similar the family of Dupin cyclides.

Going back and forth between $\mathbb{S}^3$ and structured vector spaces containing nice model of the space of spheres or of circles, we deduce the solution of some geometrical problems involving these surfaces. The problems maybe a step towards the computer graphics construction of surfaces using pieces of cyclides.

On geodesic mappings and their generalizations

Josef Mikeš (Department of Algebra and Geometry, Palacky University of Olomouc, Czechia)
Irena Hinterleitner (Department of Mathematics, Brno University of Technology, Czechia)

Diffeomorphisms and automorphisms of geometrically generalized spaces constitute one of the contemporary actual directions in differential geometry. A large number of works is devoted to geodesic, quasigeodesic, holomorphically projective, almost geodesic, F-planar and other mappings, transformations, and deformations.

The issues mentioned above were studied in detail in the monographies and the research works [3, 12, 13, 18, 21, 22], and, recently [8, 14, 15]. This presentation is dedicated to some results concerning the fundamental equations of these mappings, and deformations.

1. First, we consider classic basic equations of geodesic mappings derived by T. Levi-Civita, H. Weyl, L.P. Eisenhart. Then we examine Sinyukov's equation of geodesic mappings of Riemannian spaces. At last we consider fundamental equations of geodesic mappings from spaces with affine connection, Einstein and Berwald spaces onto (pseudo-) Riemannian spaces (J. Mikeš, S. Bácsó, V.E. Berezovsky, S. Formella, and I. Hinterleitner). Continuation of this problem are papers [1, 8, 15, 19, 20].

2. T. Otsuki and Y. Tashiro generalized geodesics on Riemannian spaces by analytic curves and geodesic mappings by holomorphically projective mappings. They derived the fundamental equations for above-mentioned mappings. V. Domashev and J. Mikeš obtained the fundamental equations in closed linear form. Interesting continuation was obtained by I. Hinterleitner [7, 9, 16].

3. F-planar curves and F-planar mappings generalized aforesaid curves and mappings. Fundamental equations were inferred by J. Mikeš and N.S. Sinyukov. We obtained new results of the fundamental equations for these mappings in the paper [11]. Special F-planar mappings are F_2^ε-planar have been studied by Peška, Chudá, Guseva, Hinterleitner, and Mikeš [5, 10].

4. Then we discuss fundamental equations for conformal mappings from Riemannian spaces onto Einstein spaces. Classic equations were deduced by H. Brinkmann, and derived in a new better linearly form by M.L. Gavrilchenko, E. Gladysheva and J. Mikeš. There equations were specified by Evtushik, Guseva, Hinterleitner and Mikeš [2, 4].

5. Finally, we consider new results in the theory of rotary mappings and isoperimetric extremals of rotation. We have obtained new fundamental equations for them, [6, 17].

The above mentioned new equations make it possible to solve regularly the problems whether the given space admits the investigated mappings onto Riemannian spaces.

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Magnetic trajectories on the unit tangent bundle of a Riemannian manifold

Marian Ioan Munteanu (University Al. I. Cuza of Iasi, Romania)

Magnetic curves represent the trajectories of the charged particles moving on a Riemannian manifold under the action of the magnetic fields. They are modeled by a second order differential equation, that is $\nabla_{\gamma'}\gamma' = \phi\gamma'$, usually known as the *Lorentz equation*. Such curves are sometimes called also *magnetic geodesics* since the Lorentz equation generalizes the equation of geodesics under arc-length parametrization, namely, $\nabla_{\gamma'}\gamma' = 0$. In the last years, magnetic curves were studied in Käher manifolds and Sasakian manifolds, respectively, since their fundamental 2-forms provide natural examples of magnetic fields.

Even that some physical terms are involved, when we study contact magnetic curves only we have in mind to obtain perturbations of geodesics obtained by use of the almost contact metric structure on the manifold. Therefore, it is enough for us to consider that these magnetic trajectories are special curves obtained as solutions of the Lorentz equation as we said before.

In this talk we present our recent investigation on magnetic curves on the unit tangent bundle of a Riemannian manifold M. We write the equation of motion for arbitrary M. In the case when M is a space form $M(c)$, we prove that every contact magnetic curve is slant. If $c \neq 1$, a contact normal magnetic curve is slant if and only if it satisfies a conservation law. These results generalize the beautiful paper of Klingenberg and Sasaki published in 1975 about geodesics on the unit tangent bundle of the 2-sphere.

More geometric properties are obtained for magnetic curves on the unit tangent bundle $U\mathbb{S}^2$ of the 2-sphere and the unit tangent bundle $U\mathbb{E}^2$ of the Euclidean plane, respectively.

On mean value points in classical mean value theorems

Nikonorov Yury (Southern Mathematical Institute of Vladikavkaz Scientific Centre, Russia)

We will start with a remarkable property of mean value points in the first integral mean value theorem. Let us consider a continuous function $f : [0,1] \to \mathbb{R}$. For any number $x \in (0,1]$ there is $\tau \in [0,x]$ such that $xf(\tau) = \int_0^x f(t)\,dt$. Such τ is unique when f is increasing or decreasing. In the general case, we put

$$\xi(x) := \max\Big\{\tau \in [0,x] \,|\, xf(\tau) = \int_0^x f(t)\,dt\Big\}.$$

Obviously, $\xi(x)/x \in [0,1]$, and a natural question is: What is the asymptotic of $\frac{\xi(x)}{x}$, when $x \to 0$? In general, one could not expect that there is the limit $\lim\limits_{x\to 0} \frac{\xi(x)}{x}$, therefore, it is natural to study the corresponding upper limit. Professor V.K. Ionin conjectured the following:

In the above notations, for any continuous function $f : [0,1] \to \mathbb{R}$, the inequality

$$\varlimsup_{x\to 0} \frac{\xi(x)}{x} \geq \frac{1}{e},$$

holds, where $e = \lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^n$.

This conjecture was proved in [1]. Now there are several (more or less) different proofs of the above conjecture. It should be noted that Vladimir Kuz'mich Ionin (1935–2014) was the scientific advisor on my first dissertation (Ph.D. thesis) [4].

Note also, that there are continuous functions $f : [0,1] \to \mathbb{R}$ with the property

$$\varliminf_{x\to 0} \frac{\xi(x)}{x} = 0,$$

therefore, there is no problem related to the lower limit.

From that time, sharp asymptotic estimations of the above kind were established for other classical mean-value theorems, i.e., Cauchy's mean value theorem [2, 7], Taylor's theorem [3], the Schwarz theorem for divided differences [5, 8], etc. We will discuss some of them together with corresponding methods of study. On the ground of obtained results and developed tools we will consider also some unsolved questions.

Finally, we consider some more geometrical problems. Let $\gamma : [a,b) \to \mathbb{R}^2$, where $a, b \in \bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$, be a continuous parametric curve in the Euclidean plane such that for every $t \in (a,b)$ there exists a non-zero derivative vector $\gamma'(t)$. Note that this vector defines a direction of the tangent line to the considered curve at the point $\gamma(t)$.

For every $t \in (a,b)$ we denote by $T(t)$ the set of $\tau \in (a,t]$ such that the vector $\gamma'(\tau)$ is collinear to the vector $\overrightarrow{\gamma(a)\gamma(t)}$. It is clear that the set $T(t)$ is non-empty for every $t \in (a,b)$. Let us consider the value

$$DT(t) = \sup\{D(\tau) \,|\, \tau \in T(t)\}.$$

We are interested in the asymptotic of the ratio $DT(t)/D(t)$ when $t \to a$. It should be noted the following result from [6] (that was generalized for support points in [7]):

Theorem. *Let $\gamma : [a,b) \to \mathbb{R}^2$ be an arbitrary continuous parametric curve with non-zero derivative vector $\gamma'(t)$ at every point $t \in (a,b)$. Then the following inequality holds*:

$$\varlimsup_{t\to a} \frac{DT(t)}{D(t)} \geq \frac{1}{e}.$$

The case of curves in multidimensional Euclidean spaces was started to study in [9], where coplanarity points on space differentiable curves and their asymptotic properties are considered. The obtained results lead to some general conjectures on the asymptotic of coplanarity points.

A detailed expositions of discussed above results could be found in the resent book [10].

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Localization of Chern-Simons type invariants of Sasakian manifolds

Hiraku Nozawa (Ritsumeikan University, Japan)

This talk is based on a joint work with Oliver Goertsches & Dirk Tooben [2]. A Sasakian manifold is a Riemannian manifold $(M; g)$ whose Riemannian cone $(\mathbb{R}_{>0} \times M, dr^2 + r^2 g)$ is Kähler. The Reeb flow of a Sasakian manifold is an isometric flow. The volume of Sasakian manifolds is an interesting invariant from both mathematical and phyisical point of view as we mention below. We will discuss how to compute it in terms of the topological data of closed orbits of the Reeb flow. A motivation to study the volume of Sasakian manifolds comes from the AdS/CFT correspondence (the holographic principle) due to Maldacena, which is a duality in string theory. Type IIb string theory on $AdS_5 \times M$ and an $N = 1$ superconformal field theory on the conformal boundary of AdS_5, where AdS_5 is a 5-dimensional anti de Sitter space and M is a Sasaki-Einstein 5-manifold. The volume of M corresponds to the central charge of the superconformal field theory under this duality. Several misterious conjectures on the volume of Sasakian manifolds proposed by physists remain open. Let us describe the main result. Martelli–Sparks–Yau [3] proved, assuming the existence of certain good metrics, a localization formula of the volume of Sasakian manifolds to the fixed point a torus action on a resolution of the singularity of the cone. We explain that the volume of Sasakian manifolds can be localized to the set of closed orbits of the Reeb flow as in the following result.

Theorem ([2]). *Let $(M; g)$ be a $(2n+1)$-dimensional compact Sasakian manifold with only finitely many closed Reeb orbits $L_1, \dots, L_N$. Let T be the closure of the Reeb flow and $\mathfrak{a} = \mathrm{Lie}(T) \setminus \mathbb{R}\xi$, where ξ is the Reeb vector field of the contact form η. Denote the weights of the transverse isotropy $\mathfrak{a}$-representation at L_k by $\{\alpha_j^k\}_{j=1}^n \subset \mathfrak{a}^*$ for $k = 1, \dots, N$. Then, the volume of M is given by*

$$\mathrm{Vol}(M, g) = (-1)^n \frac{\pi^n}{n!} \sum\nolimits_{k=1}^{N} l^k \cdot \frac{\eta|_{L_k}(v^\sharp)^n}{\prod_j \alpha_j^k (v + \mathbb{R}b)},$$

where $l_k = \int_{L_k} \eta$ is the length of the closed Reeb orbit L_k, $v^\sharp$ is the infinitesimal action of $v \in \mathfrak{t}$ and the fractions on the right hand side are considered as rational functions in the variable $v \in \mathfrak{t}$. The total expression is independent of $v \in \mathfrak{t}$.

As a corollary of this formula, we compute the volume of toric Sasakian manifolds and type I deformation of homogeneous Sasakian manifolds. The above localization formula is a special case of an Atiyah–Bott–Berline–Vergne type localization formula for Killing foliations in the context of basic equivariant cohomology [2, Theorem 1]. Roughly, we integrate equivariant basic cohomology class along the orbits of the Reeb flow. We will describe its combinatorial aspect with some examples. This formula can be applied to compute some secondary characteristic classes of Killing foliations. The results of Toben [4] and Casselmann–Fisher [1] are related to these localization formulas.

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Duality principle in Osserman manifolds

Zoran Rakić (Faculty of Mathematics, University of Belgrade, Serbia)

Let (M, g) be a pseudo-Riemannian manifold, with curvature tensor R. The Jacobi operator R_X is the symmetric endomorphism of T_pM defined by $R_X(Y) = R(Y, X)X$. In Riemannian settings, if M is locally a rank-one symmetric space or if M is flat, then the local isometry group acts transitively on the unit sphere bundle SM and hence the eigenvalues of R_X are constant on SM. Osserman in the late eighties, wondered if the converse held; this question is usually known as the *Osserman conjecture*.

In the first part of the lecture we will give an overview of Osserman type problems in the psuedo-Riemannian geometry. The second part is devoted to the equivalence of the Osserman pointwise condition and the duality principle. This part of the lecture consists the new results, which are obtained in collaboration with Yury Nikolayevsky and Vladica Andrejić.

The mixed curvature of foliations and almost-product manifolds

Vladimir Rovenski (University of Haifa, Israel)

There is a definite interest of geometers to problems of existence of metrics on foliations with given (sectional, K; Ricci, Ric; or scalar, S) curvature properties. There are a number of different "weighted" curvatures for a Riemannian manifold (M, g) endowed with a vector field X (e.g., X is a gradient of a density function $f : M \to \mathbb{R}$). In the talk, we deal with the mixed curvature of foliations (and almost-product manifolds), whose components are encoded in Riccati and Jacobi equations on leaf geodesics.

D. Ferus (1970) found a topological obstruction for existence of a totally geodesic foliation $\mathcal{F}^\nu$ of $M^{n+\nu}$: "if $K_{\rm mix} = k = {\rm const} > 0$ along a complete leaf, then $\nu < \rho(n)$". Here $\rho(n) - 1$ is the number of continuous pointwise linear independent vector fields on a sphere S^{n-1}.

Among Toponogov's many contributions to Riemannian geometry is the following conjecture: "the inequality $\nu < \rho(n)$ holds for totally geodesic foliations $\mathcal{F}^\nu$ of a closed manifold $(M^{n+\nu}, g)$ with $K_{\rm mix} > 0$", see survey [2] and the bibliography therein.

We define the "weighted" Jacobi operator $R^X_x : T^\perp\mathcal{F} \to T^\perp\mathcal{F}$ $(x \in T\mathcal{F})$ and weighted modification of the co-nullity tensor, [3]. Then we pose and study the following conjecture:

The inequality $\nu < \rho(n)$ holds for a totally geodesic foliation $\mathcal{F}^\nu$ of a closed $(M^{n+\nu}, g, X)$ under assumption $R^X_x > \|X^\top\|^2\,{\rm id}^\perp$ for all unit vectors $x \in T\mathcal{F}$.

Finally, we discuss some recent results about the mixed scalar curvature $S_{\rm mix}$: integral formulas and splitting of foliations, [4]; the Einstein-Hilbert type action and "mixed gravitational field equations", [1, 5]; the Yamabe type problem, [6].

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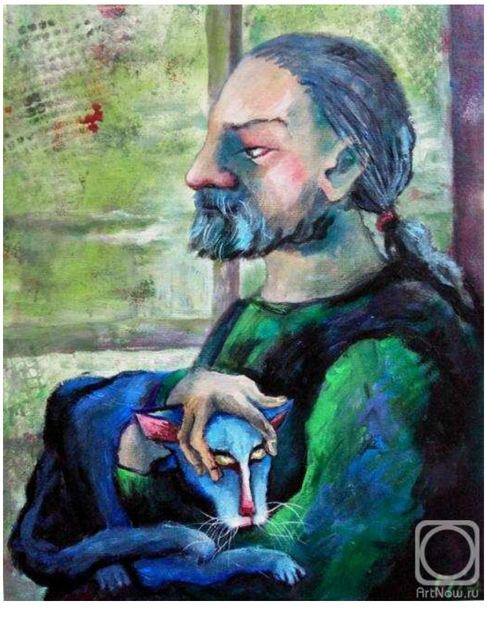


Weierstrass-type representations for minimal surfaces in the Euclidian space $\mathbb{R}^4$ and for marginally trapped surfaces in the Lorentzian space $\mathbb{R}^{1,4}$

Soret Marc (University of Tours, France)

The Weierstrass representation gives an “algebraic” solution to the minimal surface equation in $\mathbb{R}^3$ and, as Calabi noticed, also yields the solution to maximal surfaces in the Lorentzian space $\mathbb{R}^{3,1}$. Maximal surfaces are a particular case of marginally trapped surfaces (MOTS) introduced by Penrose and describing horizons of black holes.

Due to a renewed interest in MOTS, H. Liu showed that some solutions to the MOTS equations in Lorentzian space $\mathbb{R}^{4,1}$ can be described by a Weierstrass-type representation.

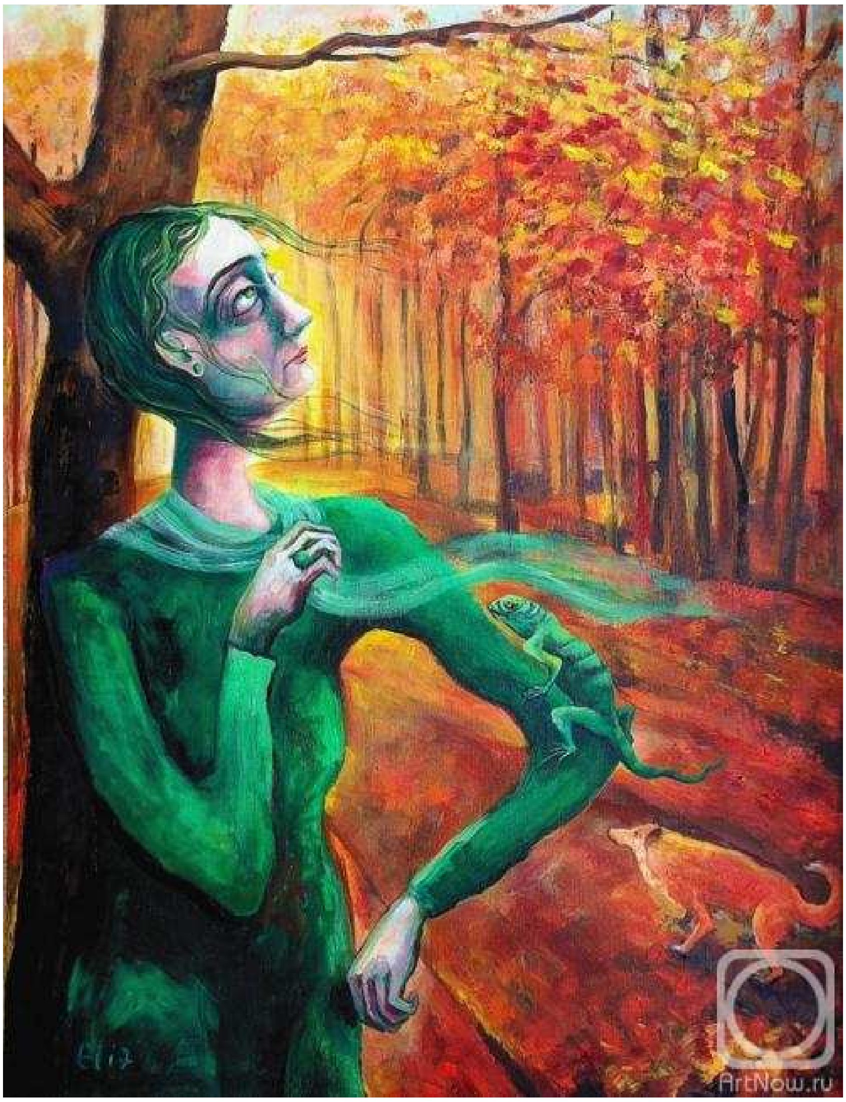
We show that all solutions can be explicitely described this way and we construct explicit examples. In the same spirit, we give a Weierstrass-type representation for complete minimal surfaces in $\mathbb{R}^4$ with applications to their isolated singularities and their ends.

The mean curvature flow by parallel hypersurfaces

Keti Tenenblat (University of Brasilia, Brazil)

It is shown that a hypersurface of a space form is the initial data for a solution to the mean curvature flow by parallel hypersurfaces if, and only if, it is isoparametric. By solving an ordinary differential equation, explicit solutions are given for all isoparametric hypersurfaces of space forms. In particular, for such hypersurfaces of the sphere, the exact collapsing time into a focal submanifold is given in terms of its dimension, the principal curvatures and their multiplicities.

Joint work with Hiuri F. Reis.



On relativistic space form problems

Aleksy Tralle (University of Warmia and Mazury, Poland)

I will describe my recent results on the existence of Clifford–Klein forms of pseudo-Riemannian homogeneous spaces.

Some remarks on a Minkowski space $(\mathbb{R}^n, F)$

Quoc Binh Tran (University of Debrecen, Institute of Mathematics, Hungary)

Key words: Finsler manifold; unit tangent sphere bundle; Sasaki metric; locally symmetric

We consider a complete, totally umbilical hypersurface M of Riemannian space $(\hat{\mathbb{R}}^n, \hat{g})$ induced by a Minkowski space $(\mathbb{R}^n, F)$. Under certain conditions we prove that M is isometric to a "round" hypersphere of the $(n+1)$-dimensional Euclidean space. We also prove that the Minkowski norm F must be arised from an inner product if there exist a non-zero vector field, which is parallel according to Levi–Civita connection of the metric tensor $\hat{g}$.

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Optimal control problems in differential geometry

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In this lecture we want to clarify what are the optimal control problems in differential geometry and how to solve them. The three problems we have raised show no doubt what this research direction must mean.

Key words: optimal control problems in diff. geometry, curvature energy, Ricci energy.

1. Curvature energy functional with affine connection evolution.

Let the triplet $(M; g; \Gamma^i_{jk})$, where M is an n-dimensional manifold, g is a Riemannian metric, and Γ^i_{jk} is a symmetric connection. In differential geometry, the connection Γ^i_{jk} generates the curvature $(1;3)$-tensor field $R = (R^l_{ijk})$, defined by

$$R^l_{ijk} = \frac{\partial \Gamma^l_{kj}}{\partial x^i} - \frac{\partial \Gamma^l_{ki}}{\partial x^j} + \Gamma^s_{kj}\Gamma^l_{si} - \Gamma^s_{ki}\Gamma^l_{sj}, \quad i, j, k, l = 1, \dots, n.$$

Problem 1. Taking $R = (R^l_{ijk})$ like a control, we can imagine the following optimal control problem (a least squares problem):

$$\min_R \frac{1}{2} \int_{\Omega_{0,K}} g_{lp} g^{iq} g^{jr} g^{ks} R^l_{ijk} R^p_{qrs} dx^1 \cdots dx^n$$

subject to the controlled evolution PDE

$$(\delta^m_k \delta^n_j \delta^p_i - \delta^m_k \delta^n_i \delta^p_j)\left(\frac{\partial \Gamma^l_{mn}}{\partial x^p} + \Gamma^s_{mn}\Gamma^l_{sp}\right) = R^l_{ijk}.$$

2. Ricci energy functional with affne connection evolution.

Let the triplet $(M; g; \Gamma^i_{jk})$, where M is an n-dimensional manifold, g is a Riemannian metric, and Γ^i_{jk} is a symmetric connection. A trace of the curvature tensor field, namely $R_{jl} = R^i_{jil}$, is called the Ricci tensor field. Explicitly,

$$\frac{1}{2} R_{ij} = \partial_r \Gamma^r_{ji} - \partial_j \Gamma^l_{ji} + \Gamma^r_{lr}\Gamma^l_{ji} - \Gamma^r_{jl}\Gamma^l_{ri}.$$

Problem 2. Taking R_{ij} like a control, we can imagine the following optimal control problem (a least squares problem):

$$\min_{R_{ij}} \frac{1}{2} \int_{\Omega_{0,K}} g^{ik} g^{jl} R_{ij} R_{kl} dx^1 \cdots dx^n$$

subject to the controlled evolution PDE

$$(\delta^p_l m \delta^m_j \delta^n_i - \delta^p_j \delta^m_l \delta^n_i)\left(\frac{\partial \Gamma^l_{mn}}{\partial x^p} + \Gamma^s_{mn}\Gamma^l_{sp}\right) = \frac{1}{2} R_{ij}.$$

3. Optimal control problem involving the almost contact structure.

Let us consider the 4-tuple $(M; g; \Gamma^i_{jk}; (\phi, \xi, \eta))$, where M is a manifold, g is a Riemannian metric, Γ^i_{jk} is a symmetric affine connection and (ϕ, ξ, η) is an almost contact structure. The divergence of ϕ with respect to Γ^i_{jk}, namely,

$$(\mathrm{div}_\Gamma \phi)_j = \partial_i \phi^i_j + \Gamma^i_{il}\phi^l_j - \Gamma^h_{lj}\phi^l_h = \partial_i \phi^i_j + (\delta^s_r \phi^t_j - \delta^t_j \phi^s_r)\Gamma^r_{st}.$$

is a co-vector field.

Problem 3. We accept that Γ is a control. We introduce the optimal control problem (a least squares problem):

$$\min_{\Gamma} \frac{1}{2} \int_{\Omega_{0,K}} g^{jk}((\mathrm{div}_{\Gamma}\phi)_j(x) - \eta_j(x))((\mathrm{div}_{\Gamma}\phi)_k(x) - \eta_k(x))\, dx^1 \cdots dx^n$$

subject to a controlled Killing type evolution PDE

$$\eta_{ij}(x) + \eta_{ji}(x) = 0, \quad i, j = 1, \ldots, n.$$

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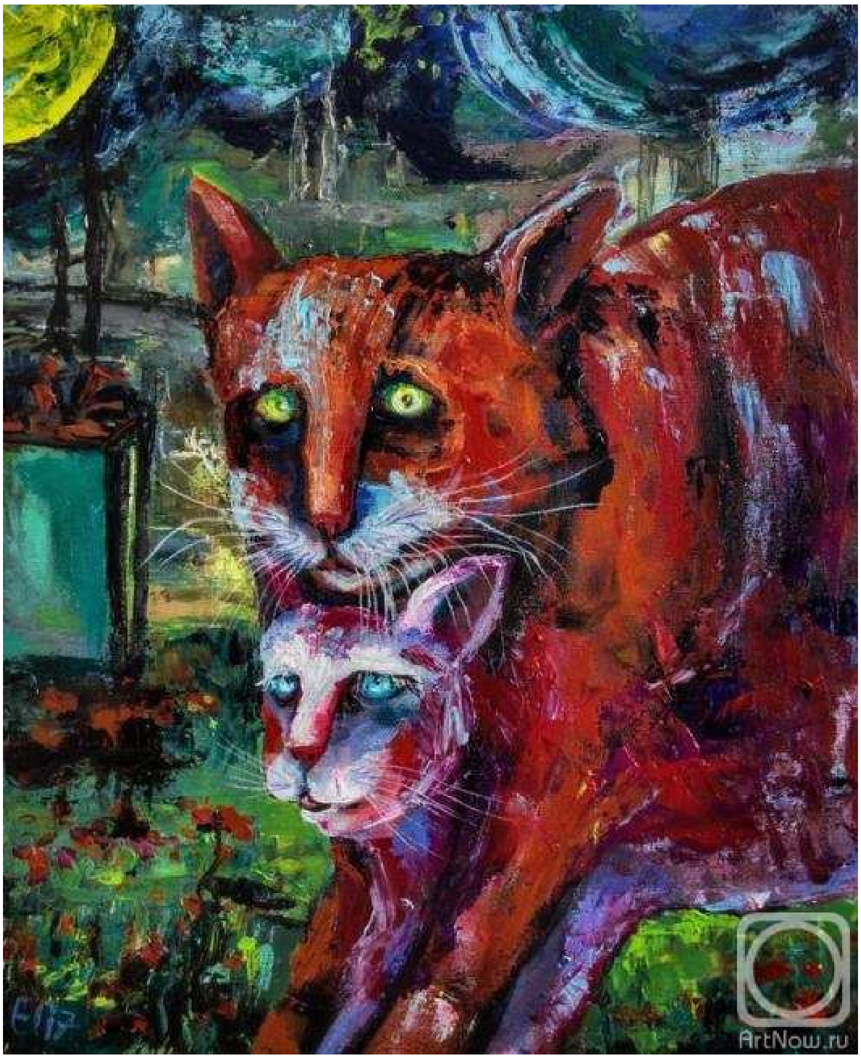
Submanifolds of generalized Kähler manifolds

Izu Vaisman (University of Haifa, Israel)

We present the class of generalized Kähler manifolds, first encountered by physicists in string theory, and of the more general class of generalized CRFK manifolds. The latter consists of manifolds M with an endomorphism $\mathcal{F}$ of $TM \oplus T^*M$ that is skew-symmetric for the pairing metric and satisfies $\mathcal{F}^3 + \mathcal{F} = 0$. This accounts for the letter F in the name. Moreover, $\mathcal{F}$ has involutive $\pm i$-eigenbundles, which accounts for CR (Cauchy–Riemann), and is compatible with an adequate positive definite metric G, and, in a certain sense, with the Riemannian connection of the projection of G to TM. This accounts for the letter K (Kähler). If $\mathcal{F}$ is an automorphism, the manifold is generalized Kähler.

Then, we discuss submanifolds of a generalized Kähler manifold that inherit an induced generalized CRFK structure.



On spaces with symmetric basic tensor and non-symmetric connection

Ana Velimirović (University of Niš, Serbia)

The spaces with a non-symmetric conection have an important role. Various cases of such connections are examined (semisymmetric, semimetric, metric).

Different concepts related to these connections will be examined.

On curvature functionals related to bending of knots

Ljubica Velimirović (University of Niš, Serbia)

We study infinitesimal bending of curves and specially of knots. Change of curvature functionals related to shape under small deformations are considered.

Some examples are visualized.

Immersed surfaces in the 4-ball bounded by a transverse knot

Marina Ville (University of Tours, France)

We consider a surface S generically immersed in the 4-ball $\mathbb{B}^4$ and bounded by a transverse link L in $\mathbb{S}^3$. Under some conditions at the boundary, we express the self-linking number $sl(L)$ (w.r.t. the contact structure) as

$$sl(L) = -\chi(S) + 2D_S + wind_+$$

where χ denotes the Euler characteristic, D_S is the number of crossing points and $wind_+$ counts the tangent planes to S which are Lagrangian and J-complex for some complex structure J on $\mathbb{R}^4$.

We will sketch the proof, discuss the case when the condition at the boundary is not satisfied, give examples and look at the relevance of the formula for minimal surfaces.

Improper integrals in extrinsic geometry of foliations

Paweł Walczak (University of Łódz, Poland)

We shall discuss a relation between two approaches (see [BL, LW]) to improper integration on Riemannian manifolds and show how to produce integral formulae for extrinsic geometric [W] (and related, [RW2]) quantities of foliations (and general, a priori nonintegrable, distributions) on closed manifolds defined outside the singularity set, that is the union of finitely many closed submanifolds of variable codimension ≥ 2. Such formulae are known since long time for foliations/distributions without singularities (see [RW1] and the bibliography therein) and provide, for instance, obstructions for the existence of foliations with leaves enjoying given geometric properties.

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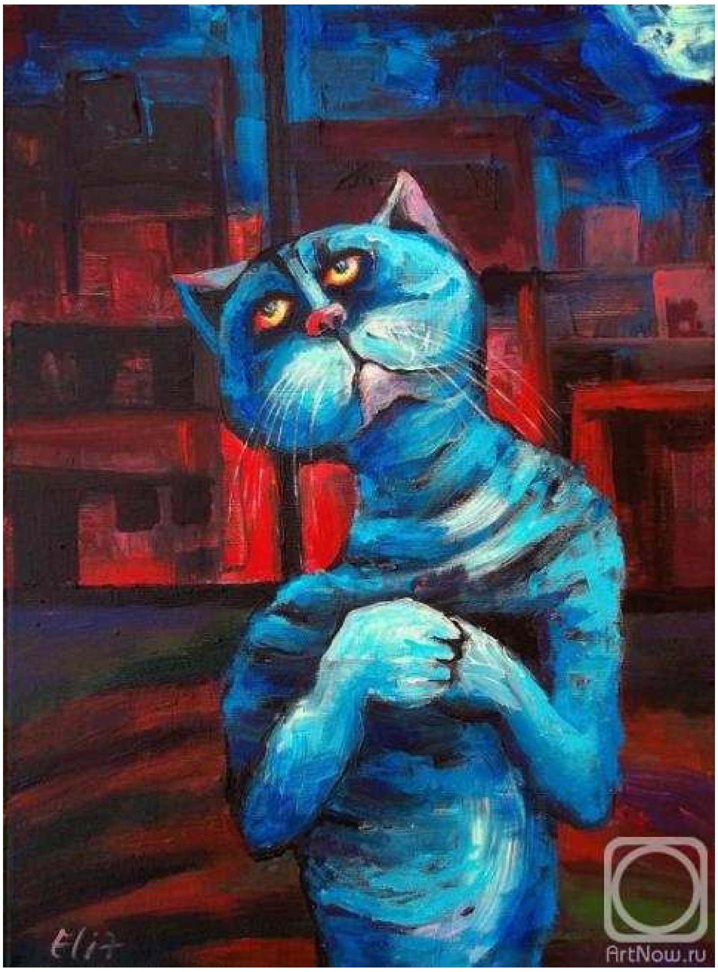


Presenting mathematics with beamer class

Zofia Walczak (University of Łódz, Poland)

TEXis pretty old system, invented by Donald Knuth. He started to work on it in late 70s and he needed about 10 years to complete. Along the way he had lots of help from people who are well known, like Hermann Zapf, Chuck Bigelow, Kris Holmes, Matthew Carter and Richard Southall. Since then the system was improved to be used for many different computer systems and languages.

Nowadays TEX/LATEXcan use and collaborate with many other programs. In my presentation I will show that it is possible to call the content of theorems and equations later in the presentation. And of course I shall explain how to do that.

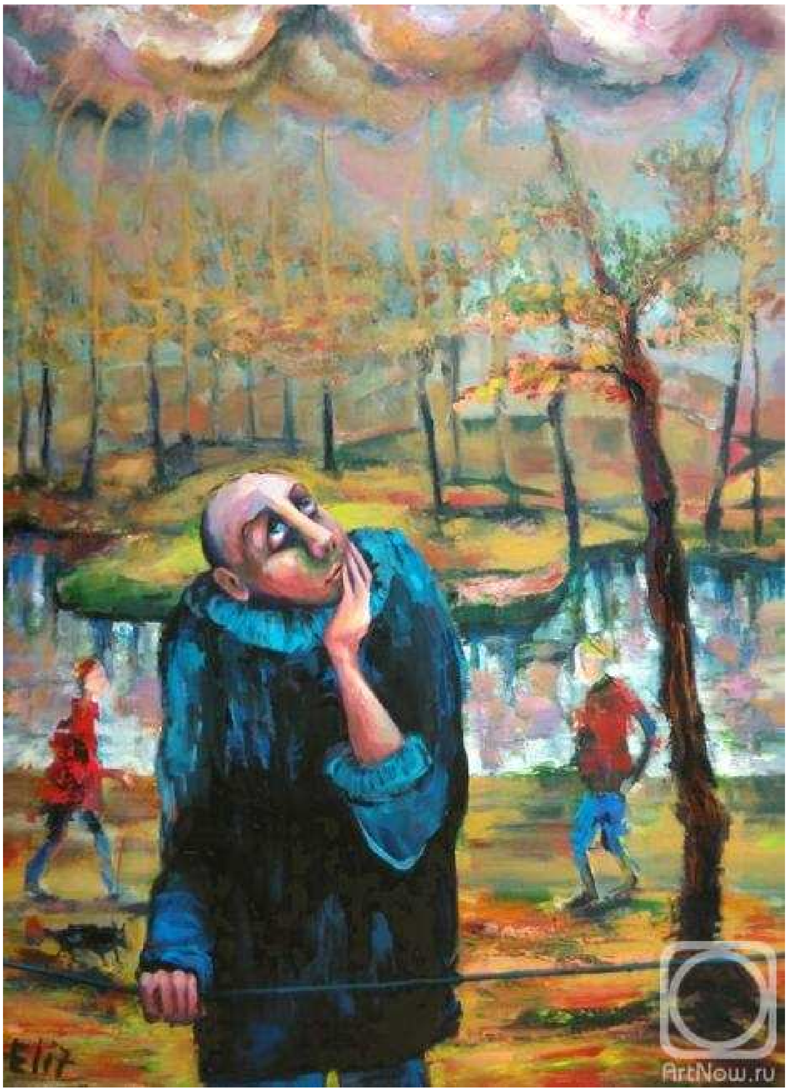
In addition, I will tell some words about Tikz – a very useful tool for creating graphics inside tex document.

Smooth parametrizations (of semi-algebraic and o-minimal sets) and their applications in dynamics, analysis, and geometry

Yosef Yomdin (Weizmann Institute of Science, Israel)

Smooth parametrization consists in a subdivision of a mathematical object under consideration into simple pieces, and then parametric representation of each piece, while keeping control of high order derivatives. Main examples for this talk are C^k or analytic parameterizations of semi-algebraic and o-minimal sets. We provide an overview of some results, open and recently solved problems on smooth parameterizations, and their applications in several apparently rather separated domains: Smooth Dynamics, Diophantine Geometry, and Analysis. The structure of the results, open problems, and conjectures in each of these domains shows in many cases a remarkable similarity, which we plan to stress. We consider a special case of smooth parametrization: "doubling coverings" (or "conformal invariant Whitney coverings"), and "Doubling chains". We present some new results on the complexity bounds for doubling coverings, doubling chains, and on the resulting bounds in Kobayashi metric and Doubling inequalities.

We plan also to present a short report on a remarkable progress, recently achieved in this (large) direction by two independent groups (G. Binyamini, D. Novikov, on one side, and R. Cluckers, J. Pila, A. Wilkie, on the other).

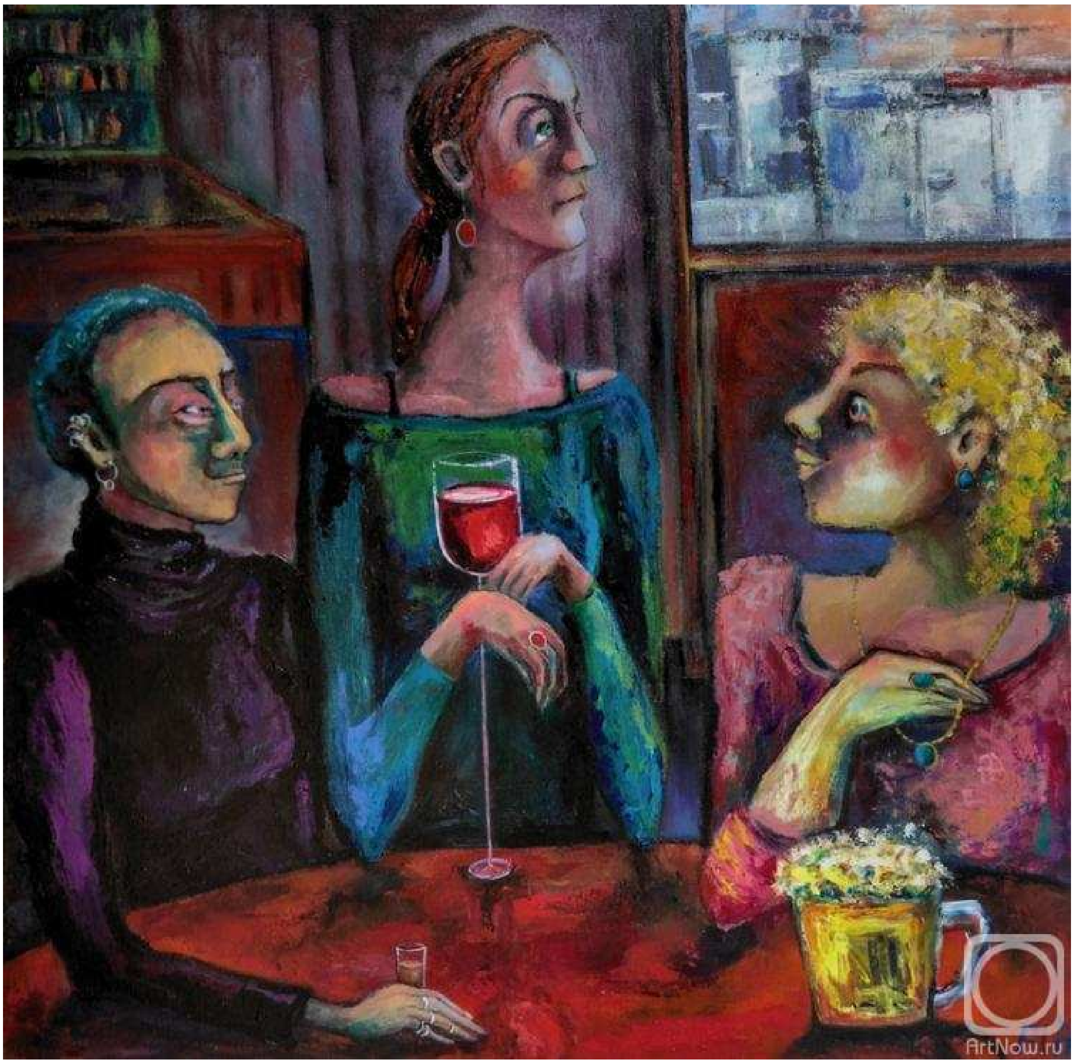


Variations of norm of the integrability tensor

Tomasz Zawadzki (University of Łódz, Poland)

We consider the norm of integrability tensor of a distribution orthogonal to a vector field as a functional on the space of Riemannian metrics. We show that contact metric structures are critical points of this action and can be characterized as critical points of a similar functional.

We also compute the second variation of this functional.



Abstracts

Section C

Energy and wave functions in the Schrodinger equation via Laplace transform for a quasi-harmonic potential

Diana Constantin (Astronomical Institute of Romanian Academy, Bucharest, Romania),
V.I.R. Niculescu (National Institute for Lasers, Plasma and Radiation Physics, Bucharest, Romania)

In the quantum frame, for $D = 3$ dimensional space, in the two body problem case, we approach the Schrodinger equation (SE) taking in account the potential:

$$V_q(r) = \Delta r + A/r + B/r^2$$

called by us quasi-harmonic potential with the centrifugal type term B/r^2, $\Delta, A, B > 0$ and $\Delta << 1$. We use Laplace transform method (LTM) and we find for the first time an analytic solution of the V_q potential problem. Namely, using directly and inverse Laplace transformations, we obtain the complete forms of the energy eigenvalues and wave functions. Furthermore, for this potential V_q, we make considerations about critical orbital quantum value l_c and we obtain a useful approximation of upper bound l_c^+ to l_c.

Key words: Schrodinger equation; Laplace transform; analytic eigenfunctions; quasi-harmonic potential.

Projection methods for solving singular differential game

Oleg Kelis (University of Haifa, Israel)

In this study we focus on a zero-sum linear-quadratic differential game. One of the main features of such a game is that the weight matrix of the minimizer's control cost in the cost functional is singular. Due to this singularity, the game cannot be solved either by applying the Isaacs Min-Max principle, [1], or the Bellman–Isaacs equation approach, [2]. In [3] such a game was analyzed with so-called regularization approach in the case where the weighting matrix of the minimizer's control cost equals zero. In [4], the game was studied and analyzed in which the weight matrix of the minimizer's control cost has appropriate diagonal singular form also using the regularization approach. In the present work we introduce a slightly more general case of the weight matrix of the minimizer's control cost than in [4]. This means that only a part of coordinates of the minimizer's control is singular, while the rest of coordinates are regular. As application we introduce a pursuit-evasion differential game and we propose two projection methods, the Arrow–Hurwicz–Uzawa and the Korpelevich methods, for solving this game. We present numerical illustrations demonstrating the iterative procedures performances.

This is joint work with Aviv Gibali (Ort Braude College of Engineering).

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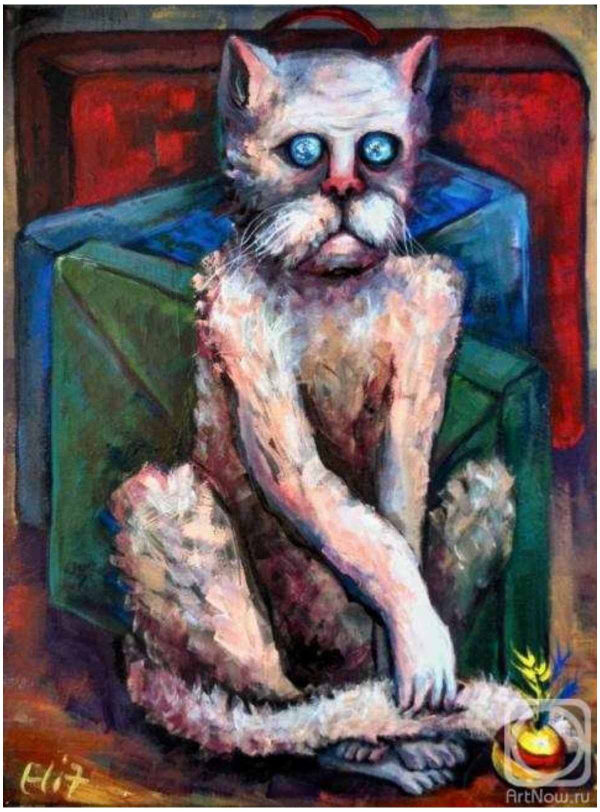

Einstein invariant metrics and symbolic computations

Nikonorov Yurii (Southern Mathematical Institute of Vladikavkaz Scientific Centre, Russia)
Nikonorova Yulia (Volgodonsk Engineering Technical Institute, Russia)

A Riemannian metric is *Einstein* if the Ricci curvature is a constant multiple of the metric. We know only some partial classifications of such metrics obtained under some additional requirements. Such requirements could be related either to the structure of the isometry group or to properties of the corresponding manifold. The standard reference book is [2]. By the well known variational principle, Einstein metrics on a given compact manifold are exactly critical points of the scalar curvature functional restricted to metrics of fixed volume [2]. The main technical tool to study homogeneous Einstein metrics is the method of Gröbner basis with computer-based calculations.

One of the most interesting problem in this direction is to classify homogeneous Einstein manifolds of low dimensions. It should be noted that all homogeneous Einstein manifolds of dim ≤ 5 and all compact homogeneous Einstein manifolds of dim $= 7$ are classified. Six-dimensional compact homogeneous was studied in [7], where the authors obtained the classification of all Einstein invariant metrics for all compact 6-dimensional (simply connected) homogeneous spaces with one exception: the Lie group $SU(2) \times SU(2) = \mathbb{S}^3 \times \mathbb{S}^3$. For this group, they classify all Einstein left-invariant metrics that have additional nondiscrete isometry groups. There are only two such metrics up to isometry and homothety: the standard metric g_{can} and the nearly Kähler metric g_{NK}.

This result was strengthened recently in [1]: any left-invariant Einstein metric g on $SU(2) \times SU(2)$ is homothetic ether to g_{can} or to g_{NK}, assuming that the isotropy group of g contains more that 2 elements. The authors of [1] used the variational principle in order to reduce the original problem to another one: to find all solutions of some special polynomial system. In particular, the authors have solved a special polynomial system with 10 equations and 10 variables in order to prove their main result.

Nevertheless, the case of a general left-invariant metrics on the group $SU(2) \times SU(2)$ is not currently fully investigated. It should be noted, that in this case the investigation reduces to solving a polynomial system with 14 unknowns.

Generalized Wallach spaces are a remarkable class of compact homogeneous spaces which were introduced in [5]. There are a lot of study on invariant Einstein metrics on generalized Wallach spaces, see a survey in [3]. It is interesting that the number of Einstein invariant metrics on a given generalized Wallach space is at least 1 and at most 4. Now we have the complete classification of invariant Einstein metrics on generalized Wallach spaces except $SO(k+l+m)/SO(k) \times SO(l) \times SO(m)$. The success is associated with the use of special techniques based on systems of symbolic calculations. Moreover, the spaces $SO(k+l+m)/SO(k) \times SO(l) \times SO(m)$ were studied in [3] with very special methods. In particular, the following results were obtained.

Theorem 1. *Suppose that $k \geq l \geq m \geq 1$ and $l \geq 2$. Then the number of invariant Einstein metrics on the space $G/H = SO(k+l+m)/SO(k) \times SO(l) \times SO(m)$ is 4 for $m > \sqrt{2k+2l-4}$ and 2 for $m < \sqrt{k+l}$, up to a homothety.*

Theorem 2. *Let q be any number from the set $\{2, 3, 4\}$. Then there are infinitely many homogeneous spaces $SO(k+l+m)/SO(k) \times SO(l) \times SO(m)$ that admit exactly q invariant Einstein metrics up to a homothety.*

The spaces $F^m/\mathrm{diag}(F)$ are called Ledger–Obata spaces, where F is a connected compact simple Lie group, $F^m = F \times \cdots \times F$ (m factors) and $\mathrm{diag}(F) = \{(X, \ldots, X) | X \in F\}$. The most interesting feature of invariant Einstein metrics on Ledger–Obata spaces $F^m/\mathrm{diag}(F)$

is the fact that the number of these metrics does not depend on the structure of the simple Lie group F. This result is given in [6], together with the result that $F^3/\mathrm{diag}(F)$ admits exactly two invariant Einstein metrics up to isometry and homothety. In [4], the authors obtained some new results. The main results of [4] are the following.

Theorem 3. *The Ledger–Obata space $F^4/\mathrm{diag}(F)$ admits exactly three invariant Einstein metrics up to isometry and homothety.*

Theorem 4. *Every Ledger–Obata space $F^{n+1}/\mathrm{diag}(F)$ admits at least $p(n)$ invariant Einstein metrics up to isometry and homothety, where $p(n)$ is the number of integer partitions of n. In particular, there are more than $\frac{1}{13n}\exp\left(\frac{5}{2}\sqrt{n}\right)$ invariant Einstein metrics for all n up to isometry and homothety.*

The approach of [4] is based on a parametrization of invariant metrics and special computations using Gröbner basis. It should be noted that the space of invariant metrics on $F^{n+1}/\mathrm{diag}(F)$ has dimension $n(n+1)/2$.

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The Yamabe type problem for foliated Riemann–Cartan manifolds

Vladimir Rovenski and Leonid Zelenko (University of Haifa, Israel)

The mixed scalar curvature is the simplest curvature invariant of a foliated Riemannian manifold. We explore the problem of prescribing the leafwise constant mixed scalar curvature of a foliated Riemann–Cartan (in particular, Riemannian) manifold by conformal change of the structure in tangent and normal to the leaves directions, see [1, 2]. Under certain geometrical assumptions and in two special cases: along a compact leaf and for a closed fibered manifold, we reduce the problem to solution of a nonlinear leafwise elliptic equation for the conformal factor. We are looking for its solutions that are stable stationary solutions of the associated parabolic equation. Our main tool is using of majorizing and minorizing nonlinear heat equations with constant coefficients and application of comparison theorems for solutions of Cauchy's problem for parabolic equations.

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Topological Molino's theory

Galicia M. F. Moreira and López J. A. Álvarez (Universidade de Santiago de Compostela, Spain)

Keywords: equicontinuous foliated space, equicontinuous pseudogroup, groupoid, germ, partial map, compact-open topology, local group, local action, growth.

Molino's description of Riemannian foliations on compact manifolds is generalized to the setting of compact equicontinuous foliated spaces, in the case where the leaves are dense. In particular, a structural local group is associated to such a foliated space.

As an application, we obtain a partial generalization of results by Carrière and Breuillard–Gelander, relating the structural local group to the growth of the leaves.

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Parameterized complexity of the matrix determinant and permanent

Singh Ranveer (Technion, Israel)

The determinant and the permanent of a square matrix are the classical problems of matrix theory. While determinant can be solved in polynomial time, computing permanent of a matrix is a "NP-hard problem" which can not be done in polynomial time unless, $P = NP$, [3, 4]. We can represent every square matrix $A = (a_{uv}) \in \mathbb{C}^{n\times n}$ as a digraph having n vertices. In the digraph, a block (or 2-connected component) is a maximally weakly connected subdigraph that has no cut-vertex. The determinant and the permanent of a matrix can be calculated in terms of the determinant and the permanent of some specifically induced subdigraphs of the blocks in the digraph. Interestingly, these induced subdigraphs are vertex-disjoint and they partition the digraph. Such partitions of the digraph are called the B-partitions, [2]. We will first discuss an algorithm to find the B-partitions. Next, we analyze the parameterized complexity of matrix determinant and permanent, where, the parameters are the sizes of the blocks and the number of cut-vertices of the digraph. We give a class of combinations of cut-vertices and block sizes for which the parametrized complexities beat the state of the art complexities of the determinant and the permanent. It is to be noted that the asymptotic complexity of determinant of a square matrix of order n is same as that of multiplication of two square matrices of order n, [1]. In general the complexity of multiplication of two matrices of order n is $O(n^\epsilon)$, where $2 \leq \epsilon \leq 3$. However, the fastest known method to compute the permanent of matrix of order n is Ryser's method, having complexity $O(2^n n^2)$. Let the digraph G have k blocks $B_1, \dots, B_k$. Let the sizes (number of vertices) of the blocks be $n_1, \dots, n_k$, and the number of cut-vertices in the blocks be $t_1, \dots, t_k$, respectively. Let Δ be the largest of the numbers of the cut-vertices in any block that is $\Gamma = \max\{t_1, \dots, t_k\}$, and Δ be the size of the largest block that is $\Delta = \max\{n_1, \dots, n_k\}$. The following are the parametrized complexities of the determinant, permanent: $O\big(\sum_{i=1}^k 2^{t_i} n_i^\epsilon\big)$, $O\big(\sum_{i=1}^k 2^{t_i} 2^{n_i} n_i^2\big)$, respectively. Then, for the following condition, the parametrized complexity beats the state of art complexities of matrix determinant $\Gamma = O\big(\log\big(\frac{1}{k}\big(\frac{n}{\Delta}\big)^\epsilon\big)\big)$. Similarly, for the following condition, the parametrized complexity beats the state of art complexity of the permanent, $\Gamma = O\big(\log\big(\frac{2^n n^2}{2^\Delta \Delta^2}\big)\big)$.

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About the Department of Mathematics

The Department of Mathematics (`http://sciences.haifa.ac.il/math/wp/`) at the University of Haifa was founded in the mid 70's. Today it is one of the seven mathematics departments in Israeli Universities. The faculty is engaged in world-class research in both pure and applied Mathematics. The research interests of our faculty include algebra, applied mathematics, combinatorics, differential equations, functional analysis, geometry and topology, logic, and theoretical computer science. The Center for Computational Mathematics and Scientific Computing (CMSC) was founded at the department in 2001. Since then it supports a large variety of research activities, conferences, special lecture series and workshops.

The department offers a variety of undergraduate and graduate programs, both in mathematics and in mathematics and computer science. Undergraduate students are offered also double major programs with other departments. In addition, the department provides courses in Mathematics for students in other university departments.

There is a number of colloquiums and research seminars at the Mathematical Department, where current results of the project will be presented. Traditionally, these seminars go on in a very informal and friendly manner so that a speaker can pose open questions and discuss difficulties of his study in order to get a good advise or recommendation from the audience. More than half of talks on the seminars are presented by invited speakers from other Israel universities as well as from overseas mathematical research centers.

The Mathematical Department at the University of Haifa routinely organizes international conferences and workshops in various mathematical fields with inviting world leaders of the field.

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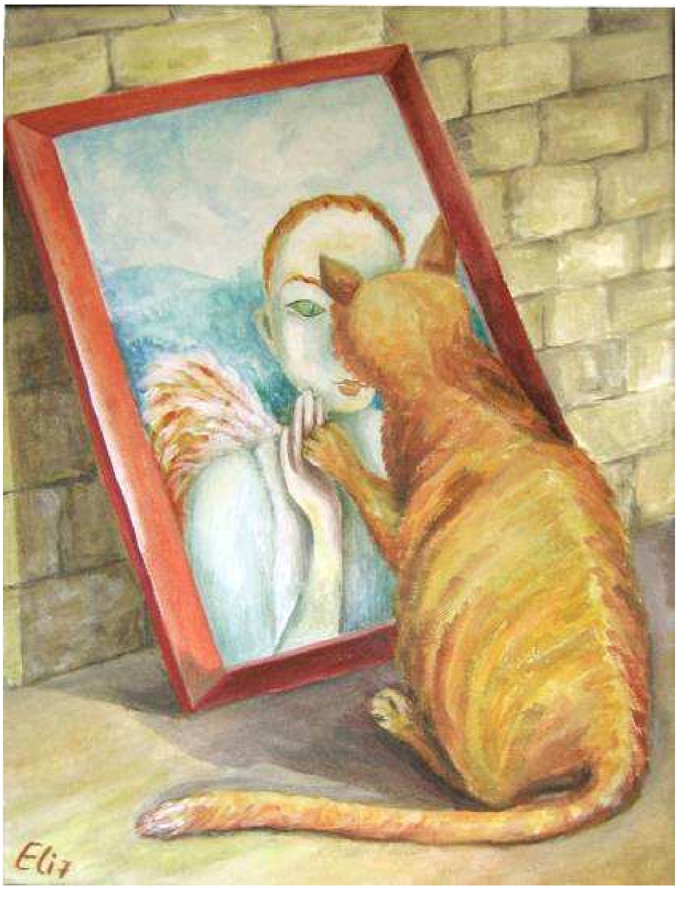
for their help in organizing the International Workshop.

Illustrations

The Organizing committee thanks the painter Nesis Elisheva, see homepage

`https://www.artmajeur.com/en/art-gallery/portfolio/nesis`

for accepting to illustrate the booklet by the series of original paintings, dedicated to the cats – the most mystic animals on Earth.



Useful Information

Workshop location
http://www.cri.haifa.ac.il/index.php/84-content/events/2018/525-the-3-rd-international-workshop-geometric-structures-and-interdisciplinary-applications
is in offices at the Haifa University Education and Sciences Building, 5th floor.

For registration: exiting the elevator (6th floor), turn right.

From Central Carmel (Haifa) to Haifa University (and opposite way): bus lines 37, 37א, 30.

From Central Carmel (Haifa) to the sea beach Hof Ha-Carmel (and opposite way): bus lines 132, 133 and 3.

From Haifa University to the sea beach Hof Ha-Carmel (and opposite way): bus lines 46 and 146.

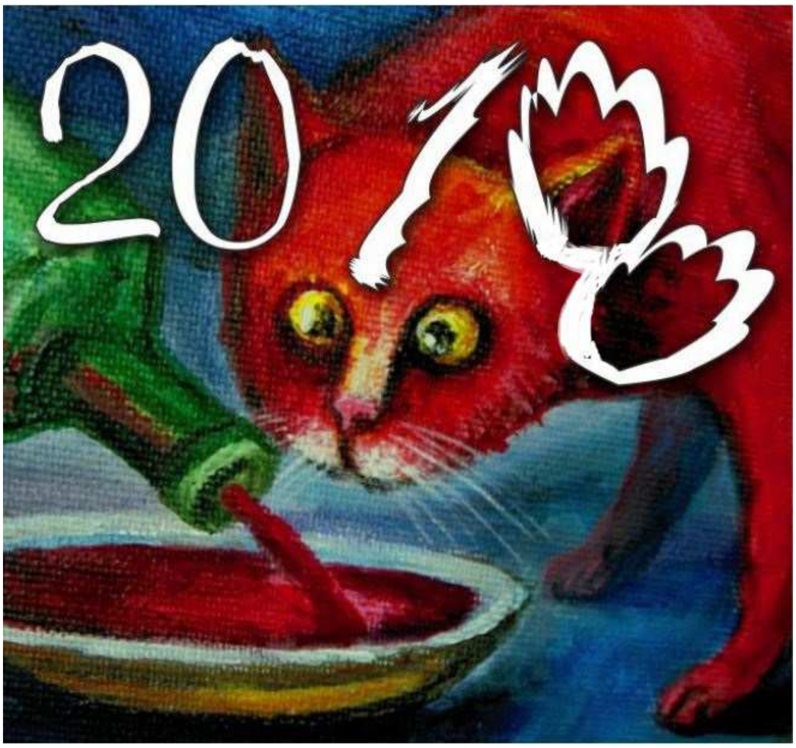
From Haifa University to Hadar (and opposite way): bus lines 24, 36, 37 and 37א.

Workshop Dinner

11 of May, 17:30 – 21:30.

The place: restaurant Atsmon, 32 Ha-Neviim Str., Hadar, Haifa.

The 65th anniversary of Professor Vladimir Rovenski
will be celebrated in this occasion.



NOTES