



Second International Workshop on Geometry and Symbolic Computations

University of Haifa
ISRAEL

Program and Book of Abstracts

Organizer: Vladimir Rovenski

15 – 18 May, 2013

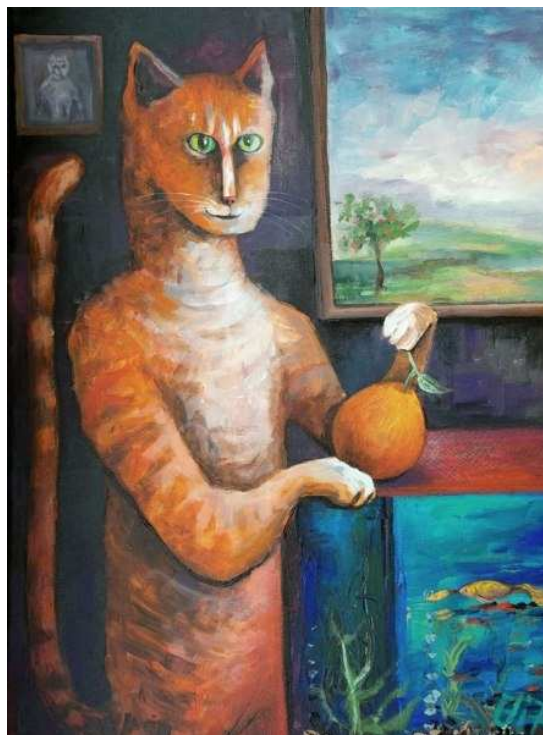


Figure 1: I'M FINE DARLING

Organizing committee

Prof. Vladimir Rovenski

Dr. Irina Albinsky



Figure 2: INVITATION TO ...

Scientific committee

Prof. Vladimir Rovenski (Israel)

Prof. Paweł Walczak (Poland)

Prof. Vladimir Golubyatnikov (Russia)



Figure 3: FLESH AND INFINITY

With financial support of

The Caesarea Edmond Benjamin de Rothschild
Foundation Institute for Interdisciplinary
Applications of Computer Science
(CRI) at the University of Haifa



Figure 4: ROULETTE

The Center for Computational Mathematics
and Scientific Computation (CCMSC)
at the University of Haifa

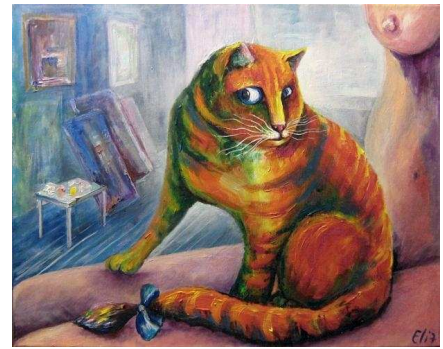


Figure 5: THE CAT OF A MODEL

The Faculty of Natural Sciences and
The Department of Mathematics
at the University of Haifa.



Figure 6: TAIL MEDITATION

Schedule

All talks will be held in room 665 – 6th floor of the Science & Education Building

14 May	15 May	16 May	17 May	18 May
	Excursion	09:00 – 9:20 Registration		Excursion
		09:20 – 09:35 Opening: I. Albinsky V. Rovenski	09:00 – 09:40 Y. Nikonorov	
		09:40 – 10:20 P. Walczak	09:45 – 10:25 V. Rovenski	
		10:30 – 11:10 Y. Burago	10:30 – 11:00 Coffee Break	
		11:10 – 11:40 Coffee Break	11:00 – 11:40 G. Hector	
		11:40 – 12:15 A. Yampolsky	11:45 – 12:25 B. Opozda	
		12:25 – 13:00 I. Gaissinski	12:30 – 13:10 A. Borisenko	
		13:00 – 14:30 Lunch Break	13:15 – 14:10 Lunch Break	
		14:30 – 15:00 Poster Session	14:10 – 14:50 V. Golubyatnikov	
		15:10 – 15:50 V. Gol'dshtein	14:55 – 15:15 P. Kopacz	
		16:00 – 16:30 Y. Yomdin	15:20 – 15:30 Closing	
		16:40 – 17:00 D. Batenkov		
19.00 – Touring (Haifa)	18.00 – Haifa Beach		17.00 – 21.00 Banquet	



Figure 7: TANGO-CAT

Preface

Dear Distinguished Professors, Participants, Students, Ladies and Gentlemen !

On behalf of the Organizing Committee of the International Workshop “*Geometry and Symbolic Computations*” I extend a warm welcome to all of you.

The workshop will be held from May 15, 2013 until May 18, 2013, at Haifa University, Israel. All lectures are held on May 16-17 Room 665, Education Building. Pre-workshop Informal Sessions on May 14 - 15, and Tours on May 14, 15 and 18.

This Workshop is dedicated to modeling (using symbolic calculations) in Differential Geometry and Its Applications (in computer sciences, tomography, mechanics, etc). The aims of the workshop: To create a forum for workers in pure, applied and computational geometry, including students, to promote discussion of modern state of art in Geometric modeling using symbolic computer programs (Maple®, Mathematica®, etc); presentation of new results.

The first International Workshop in this series, named “*Reconstruction of Geometrical Objects Using Symbolic Computations*”, was on September 2008, University of Haifa.

This workshop is supported by The Caesarea Edmond Benjamin de Rothschild Foundation Institute for Interdisciplinary Applications of Computer Science at the University of Haifa (CRI), The Center for Computational Mathematics and Scientific Computation (CCMSC) and the Mathematical Faculty at the University of Haifa.

This booklet presents abstracts, a list of all the activities of the workshop: lectures, posters and talks, presentations and other activities for workshop’s days.

Booklet is illustrated by paintings of my and Vladimir’s friend, painter Elisheva Nesis, and also contains some useful practical information, which I hope will help to make your stay in Haifa, Israel comfortable.

The organizers wish you a very fruitful and pleasant stay at Haifa and a very successful workshop.

With warm regards,

Irina Albinsky, Ph.D. Computer Sciences and Modeling Specialist

Secretary of Organizing Committee

Haifa, Israel. May 2013.



Figure 8: THE QUEEN and THE KNIGHT. CHESS OF LOVE

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Figure 9: BALLET OF 3 PARALLELS

Abstracts

Section A



Figure 10: TO BELONG

Sphericity of hypersurfaces with the normal curvature bounded from below

A. Borisenko (Sumy State University, Ukraine)

K. Drach (Kharkiv Karazin National University, Ukraine)

In our talk we will discuss the global behavior of hypersurfaces whose normal curvatures are bounded from below. It appears that such surfaces are, in some sense, close to the spheres of the corresponding spaces. Notably, let us consider an arbitrary circle on the Euclidean plane. Obviously, the angle between a ray from the center through any point on the circle and the outer normal at this point is identically zero. If to consider rays emanating not from the center but from another fixed point O inside the circle, the similar angle will not longer be identically zero. The same holds for arbitrary convex curves on the plane. However, the closer all of these angles are to zero, the closer a curve is to a circle and the point O – to its center. Thus, we can see that the values of the considered angles reveal the closeness of a curve to a circle. This is a motivation for us to study such angles in more general settings.

And it appears that the following theorem holds:

Theorem 1. *Let $M^{n+1}(c)$ be a complete simply connected Riemannian manifold of the constant sectional curvature c , Ω be a domain in it whose boundary $\partial\Omega$ is a C^2 -smooth hypersurface. Let $O \in \Omega$ be a point inside the domain, $h = \text{dist}(O, \partial\Omega)$ be the distance from O to the hypersurface and φ be the angle between a radial direction from the point O to a point on $\partial\Omega$ and the outer normal taken at this point.*

(1) *If $c = 0$, i. e. $M^{n+1}(c) = \mathbb{E}^{n+1}$ is the Euclidean space, and all normal curvatures of $\partial\Omega$ in any direction $k_n \geq k_0 > 0$, then*

$$\cos \varphi \geq \sqrt{2hk_0 - h^2k_0^2} \geq hk_0.$$

(2) *If $c = -k_1^2$, $k_1 > 0$, i.e. $M^{n+1}(c) = \mathbb{H}^{n+1}(-k_1^2)$ is the $(n+1)$ -dimensional Lobachevsky space, and all normal curvatures of $\partial\Omega$ in any direction $k_n \geq k_0 > k_1$, then*

$$\cos \varphi \geq (1 - \sinh^2(k_1(R - h)) / \sinh^2(k_1 R))^{1/2} \geq \sinh(k_1 h) / \sinh(k_1 R),$$

where $R = (1/k_1) \text{arccoth}(k_0/k_1)$ is the radius of a circle of the curvature k_0 on the two-dimensional Lobachevsky plane of the Gaussian curvature k_1^2 .

(3) *If $c = k_1^2$, $k_1 > 0$, i.e. $M^{n+1}(c) = S^{n+1}(k_1^2)$ is the $(n + 1)$ -dimensional sphere, and all normal curvatures of $\partial\Omega$ in any direction $k_n \geq k_0 \geq 0$, then*

$$\cos \varphi \geq (1 - \sin^2(k_1(R - h)) / \sin^2(k_1 R))^{1/2} \geq \sin(k_1 h) / \sin(k_1 R),$$

where $R = (1/k_1) \text{arccot}(k_0/k_1)$ is the radius of a circle of the curvature k_0 on the two-dimensional sphere of the Gaussian curvature k_1^2 .

Another way to measure the sphericity of hypersurfaces is to consider the width of a spherical layer which can enclose a hypersurface. It is quite clear, that the smaller this width is, the closer our surface is to a sphere.

Let us recall that a locally convex hypersurface $\partial\Omega \subset M^{n+1}(c)$ is λ -convex if at every point $P \in \partial\Omega$ there is a sphere S_P of the sectional curvature λ^2 passing through this point such that in the neighborhood of P the hypersurface lies on the convex side of S_P . The corresponding domain Ω is called a λ -convex domain. Note that $\partial\Omega$ can be non-regular.

We note that regular with the class C^k , $k \geq 2$, hypersurface $\partial\Omega$ is λ -convex if and only if all its normal curvatures at any point and in any direction satisfy $k_n \geq \lambda$. Thereby, the

notion of λ -convexity is the non-regular generalization of the fact that normal curvatures are bounded from below by λ .

Taking into account all the definitions above, for the width of a spherical layer the following theorem holds:

Theorem 2. *Let $\partial\Omega$ be a complete hypersurface in a complete simply connected $(n+1)$ -dimensional Riemannian manifold $M^{n+1}(c)$ of the constant sectional curvature c .*

(1) *Suppose that the ambient space is the Euclidean space $\mathbb{E}^{n+1}$. If $\partial\Omega$ is a k_0 -convex hypersurface, $k_0 > 0$, then $\partial\Omega$ can be enclosed in a spherical layer of the width*

$$d \leq (\sqrt{2} - 1)/k_0.$$

(2) *Suppose that the ambient space $M^{n+1}(c) = S^{n+1}(k_1^2)$ is $(n+1)$ -dimensional sphere, $k_1 > 0$. If $\partial\Omega$ is a k_0 -convex hypersurface, $k_0 > 0$, then $\partial\Omega$ can be enclosed in a spherical layer of the width*

$$d \leq \frac{2}{k_1} \arccos \sqrt{\cos k_1 R} - R,$$

where R is the radius of a circle of the curvature k_0 on the two-dimensional sphere of the Gaussian curvature k_1^2 .

(3) *Suppose that the ambient space $M^{n+1}(c) = \mathbb{H}^{n+1}(-k_1^2)$, $k_1 > 0$, is the Lobachevsky space. If $\partial\Omega$ is a k_0 -convex hypersurface, $k_0 > k_1$, then $\partial\Omega$ lies in the spherical layer of the width*

$$d \leq \frac{2}{k_1} \operatorname{arccosh} \sqrt{\cosh k_1 R} - R,$$

where R is the radius of a circle of the curvature k_0 on the (two-dimensional) Lobachevsky plane of the Gaussian curvature $-k_1^2$.

In the talk we will also discuss the generalizations of **Theorems 1** and **2** for hypersurfaces lying in Riemannian manifolds of the constant-sign sectional curvature.



Figure 11: JEWISH NEW YEAR. NEW GAME

Smoothable Alexandrov spaces after Petrunin and Lebedeva

Y. Burago (St. Petersburg Department of Steklov Institute of Math., Russia)

An Alexandrov space (shortly AS) X is *smoothable* if it is the Gromov–Hausdorff limit of a non-collapsing sequence of Riemannian manifolds M^n of fixed dimension n and with section curvatures uniformly bounded below. Non-collapsing means that volumes $\text{Vol } M^n \geq \delta$ for some fixed $\delta > 0$. It is well known (Perel'man) that such AS are topological manifolds, possibly with boundary, in particular not all AS allow such approximations. Nina Lebedeva and Anton Petrunin proved that curvature tensors of M_i^n ($i \in \mathbb{N}$) considered as tensor (signed) measures converge weakly. The limit tensor measure can be considered as curvature of X . This result was announced five years ago and never published; even proofs have not been written out in the whole since they contain very technically complicated parts.

Though the geometrical meaning of this curvature (i.e., how seriously it is responsible for geometry) is not cleared up yet, the significance of the result is obvious since it gives a new approach for solving some old problems in the theory of AS.

References

- [1] Burago D., Burago Yu., and Ivanov S.: *A course in metric geometry*. Graduate Studies in Mathematics, 33, AMS, Providence, RI, 2001, Chapter 10.
- [2] Petrunin A.: *An upper bound for the curvature integral*, St. Petersburg Math. J. 20 (2009), 255–265.
- [3] Perel'man G.: *DC-Structure on Alexandrov Space* (preliminary version), preprint, 2003.



Figure 12: JERUSALEM OF GOLD

Laminations which are not embeddable in codimension one

G. Hector (Université Claude Bernard Lyon 1, France)

A *codimension p foliation* $\mathcal{F}$ on a m -dimensional (connected) manifold M is a decomposition of M into submanifolds of dimension $n = m - p$ which looks locally like a product $\mathbb{R}^n \times \mathbb{R}^p$; the n -dimensional submanifolds are the *leaves* of $\mathcal{F}$. A subset $\mathcal{M} \subset M$ is $\mathcal{F}$ -*saturated* if it is a union of leaves; it is a *minimal set* for $\mathcal{F}$ if it is closed and any leaf L contained in $\mathcal{M}$ is dense in $\mathcal{M}$. These sets are important because they are the basic pieces for the reconstruction of the foliated manifold $(M, \mathcal{F})$: one shows that if M is compact, the closure $\bar{L}$ of any leaf $L \in \mathcal{F}$ contains a minimal set.

For a compact foliated manifold $(M, \mathcal{F})$ there are different types of minimal sets. For example,

- i) a compact leaf,
- ii) the whole manifold M if all leaves of $\mathcal{F}$ are dense in M , in which case we say that $\mathcal{F}$ itself is *minimal*,
- iii) a compact $\mathcal{F}$ -saturated submanifold N such that the foliation induced by $\mathcal{F}$ on N is minimal.

This list is not exhaustive as it appears already when considering 1-dimensional foliations (defined by integration of a vector field). Indeed the most interesting minimal sets are the so-called *exceptional minimal sets*: they are $\mathcal{F}$ -saturated compact topological spaces which look locally like a product $\mathbb{R}^n \times \mathbb{K}$ where $\mathbb{K}$ is the usual Cantor set. Such foliated sets will also be called (*minimal*) *laminations* or *Matchbox manifolds* of dimension n . They are *measured* if they admit a transverse invariant measure. These sets are of interest by itself.

We now focus on codimension one foliations and their minimal sets. It is a standard result that all leaves of a measured minimal foliation are homeomorphic. Moreover, they have one or two ends; our main result will be to prove a similar result for measured exceptional minimal sets.

Theorem 0.1. *Let $\mathcal{M}$ be a measured exceptional minimal set of a compact codimension one foliation $(M, \mathcal{F})$. Then all leaves in $\mathcal{M}$ have simultaneously one or two ends. Moreover, if $\mathcal{F}$ is of dimension 2 all these leaves are homeomorphic.*

This result relies heavily on a more general result stating that, in any case, the leaves of a minimal matchbox manifold have generically one, two or a Cantor set of ends (see [3] and [4]). Now there exists, for any n , minimal measured matchbox manifold $(\mathcal{M}_*, \mathcal{L}_*)$ of dimension n (compare [1] and [2]) such that

- i) the leaves of $\mathcal{L}_*$ have generically one end,
- ii) there are leaves with two ends or more,
- iii) $(\mathcal{M}_*, \mathcal{L}_*)$ is homeomorphic to an exceptional minimal set of a codimension 2 foliation on a $(n + 2)$ -manifold.

Corollary 0.2. *The matchbox manifold $(\mathcal{M}_*, \mathcal{L}_*)$ is not homeomorphic to a minimal set of any codimension 1 foliation on a compact $(n + 1)$ -manifold. In other words, $(\mathcal{M}_*, \mathcal{L}_*)$ cannot be embedded in any $(n + 1)$ -dimensional foliated manifold.*

According to Whitney's celebrated embedding theorem, we know that any compact manifold embeds in some Euclidean space $\mathbb{R}^q$; the same holds for compact matchbox manifolds. But for the examples above we get the following non embeddability result:

Theorem 0.3. *The n -dimensional matchbox manifold $(\mathcal{M}_*, \mathcal{L}_*)$ does not embed into $\mathbb{R}^{n+1}$.*

References

- [1] Alcalde Cuesta F., Lozano Rojo A. and Macho Stadler M.: *Dynamique transverse de la lamination de Ghys-Kenyon*, Astérisque, 323 (2009), 1–16.
- [2] Blanc E.: *Propriétés génériques des laminations*, Thèse, Lyon (2001).
- [3] Cantwell J. and Conlon L.: *Generic leaves*, Comment. Math. Helv., 73(2) (1998), 306–336.
- [4] Ghys E.: *Topologie des feuilles génériques*, Ann. of Math., 141(2) (1995), 387–422.
- [5] Hector G. and Hirsch U.: *Introduction to the geometry of Foliations, A and B*, (1981) and (1983), Vieweg and Sohn, Braunschweig/Wiesbaden.



Figure 13: THE ANGEL OF FEBRUARY

Conformal invariants

P. Walczak (University of Lodz, Poland)

I will discuss the results joint with Adam Bartoszek and Szymon M. Walczak concerning the problem of osculation (i. e., best tangency in a sense) of a Dupin cyclid (i. e., a conformal image of the torus, cylinder or cone of revolution) with a generic surface. Examples will be provided. The topic is related to the classical differential and conformal geometry as well as to the computer graphics.

References

- [1] A. Bartoszek, P. Walczak, Sz. Walczak: *Dupin cyclides osculating surfaces*, preprint, 2012.
- [2] A. Bartoszek, R. Langevin, P. Walczak: *Special canal surfaces of S^3* , Bull. Braz. Math. Soc. 42, (2011), 301–320.



Figure 14: FAIRY-TALES FOR DARLING

Minimal and totally geodesic unit sections of unit sphere bundles

A. Yampolsky (Kharkiv Karazin National University, Ukraine)

We consider a real vector bundle $\mathcal{E}$ of rank p and a unit sphere bundle $\mathcal{E}_1$ over the Riemannian M^n with the Sasaki-type metric. A unit section of $\mathcal{E}_1$ gives rise to a submanifold in $\mathcal{E}_1$. We give some examples of local minimal unit sections and present a complete description of local totally geodesic unit sections of $\mathcal{E}_1$ in the simplest non-trivial case $p = 2$ and $n = 2$.

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- [1] Borisenko A., and Yampolsky A.: *Riemannian geometry of bundles*. Russian Math. Surveys, 1991, 46/6, 55–106.
- [2] Yampolsky A.: *On the mean curvature of a unit vector field*. Math. Publ. Debrecen, 2002, 60/1-2, 131–155.
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- [4] Yampolsky A.: *On special types of minimal and totally geodesic unit vector fields*. 7-th Int. Conf. on Geometry, Integrability and Quantization, 2005, Varna, Bulgaria, 290–304.
- [5] Yampolsky A.: *Full Description of Totally Geodesic Unit Vector Fields on 2-dimensional Riemannian Manifold*. Matematicheskaya Fizika, Analiz, Geometriya 2004, 11/3, 355–365.
- [6] Yampolsky A.: *On intrinsic geometry of a unit vector field*. Comment. Math. Univ. Carol., 2002, 43/2, 131–155.



Figure 15: ANGEL HAS FLOWN

On the Ricci flow on some generalized Wallach spaces

N. Abiev (Taraz State University, Kazakhstan)

A. Arvanitoyeorgos (University of Patras, Greece)

Y. Nikonorov (South Math. Institute of Vladikavkaz Sci. Centre of RAS, Russia)

P. Siasos (University of Patras, Greece)

This talk is devoted to the study of the normalized Ricci flow for invariant Riemannian metrics on generalized Wallach spaces (see [3, pp. 6346–6347] and [4], [6] for the definition and detailed discussions). The equation

$$\frac{\partial}{\partial t} \mathbf{g}(t) = -2 \operatorname{Ric}_{\mathbf{g}} + \frac{2}{n} \mathbf{g}(t) S_{\mathbf{g}}$$

for the normalized Ricci flow on such (n -dimensional) spaces reduces to the system of ODE of the following type

$$\frac{dx_1}{dt} = f(x_1, x_2, x_3), \quad \frac{dx_2}{dt} = g(x_1, x_2, x_3), \quad \frac{dx_3}{dt} = h(x_1, x_2, x_3), \quad (1)$$

where $x_i = x_i(t) > 0$, $i = 1, 2, 3$, are the parameters of invariant metrics,

$$f(x_1, x_2, x_3) = -1 - a_1 x_1 \left(\frac{x_1}{x_2 x_3} - \frac{x_2}{x_1 x_3} - \frac{x_3}{x_1 x_2} \right) + x_1 B,$$

$$g(x_1, x_2, x_3) = -1 - a_2 x_2 \left(\frac{x_2}{x_1 x_3} - \frac{x_3}{x_1 x_2} - \frac{x_1}{x_2 x_3} \right) + x_2 B,$$

$$h(x_1, x_2, x_3) = -1 - a_3 x_3 \left(\frac{x_3}{x_1 x_2} - \frac{x_1}{x_2 x_3} - \frac{x_2}{x_1 x_3} \right) + x_3 B,$$

$$B = \left(\frac{1}{a_1 x_1} + \frac{1}{a_2 x_2} + \frac{1}{a_3 x_3} - \left(\frac{x_1}{x_2 x_3} + \frac{x_2}{x_1 x_3} + \frac{x_3}{x_1 x_2} \right) \right) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right)^{-1}$$

and a_i , $i = 1, 2, 3$, are some real numbers from the interval $(0, 1/2]$. It is easy to check that the volume $V = x_1^{1/a_1} x_2^{1/a_2} x_3^{1/a_3}$ is the first integral of the system (1). Therefore, we can reduce it to the system of two differential equations on the surface $V \equiv 1$ of the type

$$\frac{dx_1(t)}{dt} = \tilde{f}(x_1, x_2), \quad \frac{dx_2(t)}{dt} = \tilde{g}(x_1, x_2), \quad (2)$$

where

$$\tilde{f}(x_1, x_2) = f(x_1, x_2, \varphi(x_1, x_2)), \quad \tilde{g}(x_1, x_2) = g(x_1, x_2, \varphi(x_1, x_2)), \quad \varphi(x_1, x_2) = x_1^{-\frac{a_3}{a_1}} x_2^{-\frac{a_3}{a_2}}.$$

It is easy to see that the singular points of the system (1) are exactly Einstein invariant metrics on the generalized Wallach space under consideration. It is known [4], that every generalized Wallach space admits at least one Einstein invariant metric. Later in [5] a detailed study of invariant Einstein metrics was developed for all generalized Wallach spaces. In particular, it was proved that there are at most four Einstein metrics (up to homothety) for every such space.

It should be noted that (x_1^0, x_2^0, x_3^0) is a singular point of the system (1) with $V = 1$ if and only if (x_1^0, x_2^0) is a singular point of (2). It is our interest to determine the types of singularity of such points, and our investigation concern this problem for some special values of the parameters a_1, a_2 , and a_3 . The main result in this direction is the theorem

below, that gives a qualitative answer for almost all points (in measure theoretic sense) $(a_1, a_2, a_3) \in (0, 1/2] \times (0, 1/2] \times (0, 1/2]$. Note that latter inclusion is fulfilled for any triple (a_1, a_2, a_3) corresponding to some generalized Wallach. However we are interesting in the behavior of the dynamical system (2) for all values $a_i \in (0, 1/2]$ despite the fact that some triples do not correspond to “real” generalized Wallach spaces.

Consider a real algebraic surface Ω in $\mathbb{R}^3$ that is determined by the equation $Q(a_1, a_2, a_3) = 0$, where

$$\begin{aligned} Q(a_1, a_2, a_3) = & (2s_1 + 4s_3 - 1)(64s_1^5 - 64s_1^4 + 8s_1^3 + 12s_1^2 - 6s_1 + 1 \\ & + 240s_3s_1^2 - 240s_3s_1 - 1536s_3^2s_1 - 4096s_3^3 + 60s_3 + 768s_3^2) \\ & - 8s_1(2s_1 + 4s_3 - 1)(2s_1 - 32s_3 - 1)(10s_1 + 32s_3 - 5)s_2 \\ & - 16s_1^2(13 - 52s_1 + 640s_3s_1 + 1024s_3^2 - 320s_3 + 52s_1^2)s_2^2 \\ & + 64(2s_1 - 1)(2s_1 - 32s_3 - 1)s_2^3 + 2048s_1(2s_1 - 1)s_2^4, \end{aligned}$$

and

$$s_1 = a_1 + a_2 + a_3, \quad s_2 = a_1a_2 + a_1a_3 + a_2a_3, \quad s_3 = a_1a_2a_3.$$

Note that the polynomial Q has total degree 12.

Consider a part of the surface Ω in the cube $(0, 1/2)^3$ (using Maple[®] or Mathematica[®]). On can show that the set $(0, 1/2)^3 \cap \Omega$ is connected. There are three curves of degenerate points on Ω : one of them has parametric representation

$$a_1 = -\frac{1}{2} \frac{16t^3 - 4t + 1}{8t^2 - 1}, \quad a_2 = a_3 = t,$$

and the others are defined by permutations of a_i . These curves have a common point $(1/4, 1/4, 1/4)$ (for $t = 1/4$) which is a singular point of Ω of degree 3. The type of this point is *elliptic umbilic* in the sense of G. Darboux (see [2, pp. 448–464]) or of type D_4^- in other terminology (see e.g. [1, Ch. III, Sect. 21.3, 22.3]).

The set $(0, 1/2)^3 \setminus \Omega$ has exactly three connected components. Denote by O_1 , O_2 , and O_3 the components containing the points $(1/6, 1/6, 1/6)$, $(7/15, 7/15, 7/15)$, and $(1/6, 1/4, 1/3)$ respectively.

We proved that for all points $(a_1, a_2, a_3) \in (0, 1/2)^3 \setminus \Omega = O_1 \cup O_2 \cup O_3$ system (2) has only *non degenerate singular points*. The number of these points and corresponding types is the same on each component O_i (under some suitable identification for various values of parameters a_1, a_2, a_3). Therefore, it suffices to check *only one point in the set O_i* for a given $i \in \{1, 2, 3\}$.

Our main result is the following theorem which clarifies the above observation and provides a general result about the types of the non degenerate singular points of the system (2).

Theorem. *For $(a_1, a_2, a_3) \in O_i$ the following possibilities for singular points of the system (2) can occur:*

- (i) *If $i = 1$ or $i = 2$ then there is one singular point that is a node and three singular points that are saddles;*
- (ii) *If $i = 3$ then there are two singular points that are saddles.*

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Figure 16: HONEY HEAVEN OF JEWISH NEW YEAR

Flows of metrics on foliations with visualizing for surfaces

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A flow of metrics, g_t , on a manifold is a solution of evolution equation $\partial_t g = S(g)$, where a geometric functional $S(g)$ is a symmetric $(0, 2)$ -tensor usually related to some kind of curvature. The mixed sectional curvature of a foliated manifold $(M, \mathcal{F})$ regulates the deviation of leaves along the leaf geodesics. Let $\{\varepsilon_\alpha, e_i\}_{\alpha \leq p, i \leq n}$ be a local orthonormal frame on TM adapted to $T\mathcal{F}$ and $\mathcal{D} := T\mathcal{F}^\perp$. The *mixed scalar curvature* is defined by $\text{Sc}_{\text{mix}}(g) = \sum_{i=1}^n \sum_{\alpha=1}^p R(\varepsilon_\alpha, e_i, \varepsilon_\alpha, e_i)$, where R is the Riemannian curvature. For a surface (M^2, g) (i.e., $n = p = 1$) we have $\text{Sc}_{\text{mix}} = K$ – the gaussian curvature.

We introduce and study the flow of metrics on a foliation, whose velocity along the orthogonal distribution $\mathcal{D}$ is proportional to the *mixed scalar curvature*:

$$\partial_t g = -2(\text{Sc}_{\text{mix}}(g) - \Phi) \hat{g}. \quad (1)$$

Here $\Phi : M \rightarrow \mathbb{R}$ is a leaf-wise constant function. The $\mathcal{D}$ -truncated metric tensor $\hat{g}$ is given by $\hat{g}(X_1, X_2) = g(X_1, X_2)$ and $\hat{g}(Y, \cdot) = 0$ for $X_i \in \mathcal{D}$, $Y \in T\mathcal{F}$. Suppose that the leaves of $\mathcal{F}$ are compact minimal submanifolds. The flow (1) is used to examine the question: *Which foliations admit a metric with a given property of Sc_{mix} (e.g., positive or negative)?*

Let $h_{\mathcal{F}}$, h be the second fundamental forms and $H_{\mathcal{F}}$, H the mean curvature vectors of $T\mathcal{F}$ and the distribution $\mathcal{D}$, respectively. Also denote T the integrability tensor of $\mathcal{D}$. Then [3]:

$$\text{Sc}_{\text{mix}}(g) = \text{div}(H + H_{\mathcal{F}}) + \|H\|^2 + \|T\|^2 - \|h\|^2 + \|H_{\mathcal{F}}\|^2 - \|h_{\mathcal{F}}\|^2. \quad (2)$$

We observe that (1) preserves total geodesy (i.e. $h_{\mathcal{F}} = 0$) and *harmonicity* (i.e. $H_{\mathcal{F}} = 0$) of foliations and yields the Burgers type equation

$$\partial_t H + \nabla^{\mathcal{F}} g(H, H) = n \nabla^{\mathcal{F}} (\text{Div}_{\mathcal{F}} H) + n \nabla^{\mathcal{F}} (\|T\|_g^2 - \|h_{\mathcal{F}}\|_g^2 - n \beta_{\mathcal{D}}). \quad (3)$$

The time-independent function $\beta_{\mathcal{D}} := n^{-2}(n \|h\|^2 - \|H\|^2) \geq 0$ is a measure of “*non-umbilicity*” of $\mathcal{D}^n$. (For $\dim \mathcal{F} = p = 1$, we have $\beta_{\mathcal{D}} = n^{-2} \sum_{i < j} (k_i - k_j)^2$, where k_i are the principal curvatures of $\mathcal{D}$). Suppose that $H_0 = -n \nabla^{\mathcal{F}} (\log u_0)$ (leaf-wise conservative) for a function $u_0 > 0$. If $\Psi := u_0^4 (\|T\|_{g_0}^2 - \|h_{\mathcal{F}}\|_{g_0}^2) > 0$ then solution is represented as $u = \Psi^{1/4} (\|T\|_{g_t}^2 - \|h_{\mathcal{F}}\|_{g_t}^2)^{-1/4}$ and the non-linear *Schrödinger heat equation* holds:

$$(1/n) \partial_t u = \Delta_{\mathcal{F}} u + (\beta_{\mathcal{D}} + \Phi/n) u - (\Psi/n) u^{-3}, \quad u(\cdot, 0) = u_0. \quad (4)$$

If $\Psi \equiv 0$ then (3) reads as a forced Burgers equation

$$\partial_t H + \nabla^{\mathcal{F}} g(H, H) = n \nabla^{\mathcal{F}} (\text{Div}_{\mathcal{F}} H) - n^2 \nabla^{\mathcal{F}} \beta_{\mathcal{D}} \quad (5)$$

and the leaf-wise potential function for H may be chosen as a solution of the linear PDE $(1/n) \partial_t u = \Delta_{\mathcal{F}} u + \beta_{\mathcal{D}} u$, $u(\cdot, 0) = u_0$. Let $\lambda_0 \leq 0$ be the first eigenvalue of Schrödinger operator $\mathcal{H}(u) = -\Delta_{\mathcal{F}} u - \beta_{\mathcal{D}} u$, and $e_0 > 0$ the corresponding unit L_2 -norm eigenfunction. Under certain conditions (on any leaf F)

$$\Phi > -n \beta_{\mathcal{D}}, \quad |n \lambda_0 + \Phi| \geq \max_F (\|T\|_{g_0}^2 - \|h_{\mathcal{F}}\|_{g_0}^2) \left(\max_F (u_0/e_0) / \min_F (u_0/e_0) \right)^4 \quad (6)$$

the asymptotic behavior of solutions to (4) is the same as of the linear equation, when the Burgers equation (5) has a single-point global attractor: $H_t \rightarrow -n \nabla^{\mathcal{F}} (\log e_0)$ as $t \rightarrow$

∞ . Applying the scalar maximum principle for parabolic PDEs, we show that there exists a positive solution $\tilde{u}$ of the auxiliary linear PDE $(1/n) \partial_t \tilde{u} = \Delta_{\mathcal{F}} \tilde{u} + (\beta_{\mathcal{D}} + \lambda_0) \tilde{u}$ such that for any $\alpha \in (0, \min\{\lambda_1 - \lambda_0, 4|\lambda_0|\})$ and any $k \in \mathbb{N}$

(i) $u = e^{-\lambda_0 t}(\tilde{u} + \theta(x, t))$ where $\|\theta(\cdot, t)\|_{C^k} = O(e^{-\alpha t})$ as $t \rightarrow \infty$;

(ii) $\nabla^{\mathcal{F}}(\log u) = \nabla^{\mathcal{F}}(\log e_0) + \theta_1(x, t)$ where $\|\theta_1(\cdot, t)\|_{C^k} = O(e^{-\alpha t})$ as $t \rightarrow \infty$.

In this case, the flow (1) admits a unique global solution g_t ($t \geq 0$), whose Sc_{mix} converges exponentially to a leaf-wise constant $n\lambda_0$. The metrics are smooth on M when all leaves are compact and have finite holonomy group. Using rescaling along $\mathcal{D}$, one may provide convergence to metrics with Sc_{mix} as positive so negative:

Let (M, g) be endowed with a harmonic compact foliation $\mathcal{F}$. Suppose that $\|T\|_g^2 > \|h_{\mathcal{F}}\|_g^2$ and $H = -n\nabla^{\mathcal{F}}(\log u_0)$ for a function $u_0 > 0$.

If $\lambda_0 < 0$ then there exists $\mathcal{D}$ -conformal to g metric $\bar{g}$ with $\text{Sc}_{\text{mix}}(\bar{g}) < 0$.

If $\lambda_0 > -\frac{1}{n}(\frac{u_0}{\bar{u}_0 e_0})^4(\|T\|_g^2 - \|h_{\mathcal{F}}\|_g^2)$ then there is $\mathcal{D}$ -conformal to g metric $\bar{g}$ with $\text{Sc}_{\text{mix}}(\bar{g}) > 0$.

For a surface $M^2 \subset \mathbb{R}^3$ of gaussian curvature K , the PDE (1) reads as $\partial_t g = -2(K(g) - \Phi)\hat{g}$. For a surface of revolution $[\rho(x) \cos \theta, \rho(x) \sin \theta, h(x)]$ ($0 \leq x \leq l$, $|\theta| \leq \pi$) with $(\rho')^2 + (h')^2 = 1$, this yields the heat equation $\partial_t \tilde{\rho} = \tilde{\rho}_{,xx}$ for $\tilde{\rho} = e^{-\Phi t} \rho$. If $\Phi < (\pi/l)^2$ then M_t converge to a flat surface, and if $\Phi = (\pi/l)^2$ then $\lim_{t \rightarrow \infty} \rho(t) = A \sin(\pi x/l)$, and M_t converge to a surface of constant $K = \Phi$ (a sphere of radius l/π when $A = l/\pi$). We demonstrate this with Maple®.

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Figure 17: SCIENTIST CAT

Bochner's technique on statistical manifolds

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By a statistical structure we mean a Riemannian manifold (M, g) equipped with a torsion-free connection ∇ such that the cubic form ∇g is symmetric. Among statistical structures the following cases play an important role:

- 1) There is a global volume form ν (possibly different than the volume form ν_g determined by g) such that $\nabla \nu = 0$.
- 2) $\nabla \nu_g = 0$.
- 3) The curvature tensors for ∇ and its conjugate coincide.

Extensions of the following topics to the above cases of statistical structures are presented: Bochner's theorems on the Ricci tensor and harmonic forms or Killing vector fields, Lichnerowicz's theorems on eigenvalues of suitable Laplacians, Hodge theorems, Bochner-Weitzenböck formulas. A concept of the sectional curvature for statistical structures is introduced. Using a new curvature tensor we propose some versions of Tachibana's and Myer-Gallot's theorems.

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Figure 18: AMONG THE MOONS

Sobolev inequalities for differential forms and their applications

V. Gol'dshtein (Ben Gurion University of the Negev, Israel)

We study Sobolev type classes of p integrable differential forms with q integrable exterior differentials on Riemannian manifolds. We call cohomology of these Banach complexes as $L_{q,p}$ -cohomology. These cohomology are Lipschitz invariants and for some combination of p and q are (quasi)conformal invariants. Vanishing of $L_{q,p}$ -cohomology is equivalent to existence of Sobolev inequality for corresponding classes of differential forms. This relation is correct for the classical Sobolev inequality also and corresponds to 1-dimensional $L_{q,p}$ -cohomology.

We show some applications of this machinery to manifolds with pinched curvature and to quasi-linear equations.



Figure 19: Hm-m-m ...

Multi-weighted parabolic systems in Sobolev spaces

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Advisors: Prof. Alexander Kozhevnikov, Prof. Vladimir Rovenski

We consider the following system of differential equations:

$$\mathcal{A}(D; \partial_t) = \partial_t \mathbf{u} + A\mathbf{u} = \mathbf{f} \quad \text{in } Q := \Omega \times [0, T], \quad T < \infty,$$

where Ω is a bounded connected open set in $\mathbb{R}^n$, $\mathcal{A}$ is a multi-order operator elliptic in the sense of Douglis-Nirenberg with different diagonal orders [1]. A parameter-ellipticity condition for these type of operators was invented by Kozhevnikov [1] for operators over a compact boundaryless n -manifold and later [2] in the case of boundary value problems.

Using method [2] and some results by Lions-Magenes [3], we study solvability of the parabolic multi-order initial-boundary value problems in appropriate anisotropic Sobolev-type spaces.

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Figure 20: HALLOWEEN CAT

Euclidean hypersurfaces with a totally geodesic foliation of codimension one

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A Riemannian manifold M^n that carries a totally geodesic foliation of codimension one is, at least locally, isometric to a product manifold $F^{n-1} \times \mathbb{R}$, with a twisted product metric $dx^2 + \rho^2 dt^2$, where dx^2 is a fixed metric on F^{n-1} and $\rho \in C^\infty(M)$.

We classify the complete hypersurfaces of Euclidean space that carry a totally geodesic foliation of codimension one. We discuss several families of solutions to this problem.

Flat hypersurfaces. Locally, any such hypersurface is an example. It is known that complete flat hypersurfaces of Euclidean space are cylinders over curves.

Surface-like hypersurfaces. Let $f_0 : L^2 \rightarrow \mathbb{R}^3$ be a surface, and D_0 the one-dimensional distribution spanned by the tangent directions to a one-parameter family of geodesics of L^2 . Set $M^n = L^2 \times \mathbb{R}^{n-2}$ and define $f : M^n \rightarrow \mathbb{R}^{n+1}$ by $f = f_0 \times id$, where $id : \mathbb{R}^{n-2} \rightarrow \mathbb{R}^{n-2}$ is the identity map. Then $D_0 \oplus \mathbb{R}^{n-2}$ is a totally geodesic distribution on M^n of codimension one.

Ruled hypersurfaces. Non-flat ruled hypersurfaces $f : M^n \rightarrow \mathbb{R}^{n+1}$ carry a one codimensional smooth foliation of (open subsets of) affine subspaces of $\mathbb{R}^{n+1}$, the rulings of f . Thus, the rulings are totally geodesic in $\mathbb{R}^{n+1}$, hence also in M^n .

Partial tubes, [1]. Let $\gamma : I \subset \mathbb{R} \rightarrow \mathbb{R}^{n+1}$ be a unit speed curve. Consider a hypersurface F^{n-1} of the (affine) normal space $N_\gamma I(s_0)$ to γ at some point $s_0 \in I$ and parallel transport F^{n-1} along γ with respect to the normal connection. Then, if F^{n-1} lies in a suitable open subset of $N_\gamma I(s_0)$, this generates an n -dimensional hypersurface M^n of $\mathbb{R}^{n+1}$, which is complete if F^{n-1} is complete and $I = \mathbb{R}$. It is called the partial tube over with fiber F^{n-1} . The parallel translates of F^{n-1} give rise to a one-codimensional totally geodesic foliation of M^n .

In dimension three (i.e., $n = 3$) any envelope of a one-parameter family of flat hypersurfaces falls into one of those classes of examples. A hypersurface $f : M^n \rightarrow \mathbb{R}^{n+1}$ is the *envelope of a one-parameter family* of hypersurfaces $f_t : M^n \rightarrow \mathbb{R}^{n+1}$ if there exists a distribution D of rank $n - 1$ on M^n such that f and f_t coincide and have the same Gauss map on the leaf F_t of D , where t denotes the parameter along an orthogonal trajectory of D .

The classes of hypersurfaces just described are clearly not disjoint. For instance, the class of flat hypersurfaces is precisely the intersection of the classes of ruled hypersurfaces and partial tubes over curves. It is easy to construct examples of hypersurfaces (even complete ones) where different types of hypersurfaces are smoothly attached.

For an oriented hypersurface $M^n \subset \mathbb{R}^{n+1}$ we denote by A its shape operator with respect to the Gauss map N and by $\Delta = \ker A$ its *relative nullity* distribution. If $D \subset TM$ is a curvature invariant distribution of rank k , then one of the following cases holds point-wise:

$$(i) \ A(D) \subset D^\perp, \quad (ii) \ A(D) \subset D, \quad (iii) \ \text{rank}(D \cap \Delta) = k - 1.$$

Lemma 1 *If (iii) holds with $k = n - 1$, then there exist locally a smooth orthonormal frame $\{Y, X, T_1, \dots, T_{n-2}\}$ and smooth functions β, μ, ρ and λ_j ($1 \leq j \leq n - 2$), such that*

$$\begin{aligned} AY &= \beta Y + \mu X, & AX &= \mu Y + \rho X, & AT^j &= 0, \\ \nabla_{T_j} T_i &= \nabla_{T_j} X = \nabla_{T_j} Y = \nabla_X Y = 0, & \nabla_X T_j &= \lambda_j X & (1 \leq i, j \leq n - 2). \end{aligned}$$

Moreover, the Gauss equations yield

$$\begin{aligned} X \langle \nabla_Y Y, T_i \rangle &= \langle \nabla_Y Y, X \rangle (\langle \nabla_Y Y, T_i \rangle - \langle \nabla_X X, T_i \rangle) \\ T_i \langle \nabla_X X, T_j \rangle &= \langle \nabla_X X, T_i \rangle \langle \nabla_X X, T_j \rangle & T_i \langle \nabla_Y Y, T_j \rangle &= \langle \nabla_Y Y, T_i \rangle \langle \nabla_Y Y, T_j \rangle, \end{aligned}$$

whereas the Codazzi equations yield

$$T_i(\rho) = \rho \langle \nabla_X X, T_i \rangle, \quad T_i(\mu) = \mu \langle \nabla_X X, T_i \rangle, \quad T_i(\mu) = \rho \langle \nabla_Y X, T_i \rangle + \mu \langle \nabla_Y Y, T_i \rangle.$$

The symbolic computations were verified with Maple[®].

Theorem 1 *Let $f : M^n \rightarrow \mathbb{R}^{n+1}$, $n \geq 3$, be an isometric immersion of a nowhere at complete connected Riemannian manifold that carries a totally geodesic foliation of codimension one. Assume that M^n does not contain an open subset isometric to a product $L^2 \times \mathbb{R}^{n-2}$ where f splits as $f = f_1 \times \text{id}$, with $f_1 : L^2 \rightarrow \mathbb{R}^3$ an isometric immersion and $\text{id} : \mathbb{R}^{n-2} \rightarrow \mathbb{R}^{n-2}$ the identity map. Then f is either ruled or a partial tube over a curve.*

Corollary 2 *Let $f : M^n \rightarrow \mathbb{R}^{n+1}$, $n \geq 3$, be an isometric immersion of a complete Riemannian manifold with positive Ricci curvature that carries a totally geodesic foliation of codimension one. Then f is a partial tube over a curve.*

Next, we consider the more general local version of the problem.

Theorem 2 *Let $f : M^n \rightarrow \mathbb{R}^{n+1}$, $n \geq 3$, be an isometric immersion of a Riemannian manifold that carries a totally geodesic foliation of codimension one. Then, there is an open dense subset of M^n where f is locally either surface-like, ruled, a partial tube over a curve or an envelope of a one-parameter family of at hypersurfaces.*

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Figure 21: THE ANGEL OF MARCH

Abstracts

Section B



Figure 22: SEARCH OF THE KEY

Geometry of phase portraits of some nonlinear dynamical systems

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1. We study phase portraits of nonlinear dissipative dynamical systems of chemical kinetics

$$\frac{dx_j}{dt} = f_j(x_{j-1}) - m_j x_j, \quad m_j > 0, \quad j = 1, 2, \dots, n \quad (1)$$

considered as models of gene networks functioning regulated by negative feedbacks, see [1]. Here all variables are non-negative and all functions are smooth, positive, and monotonically decreasing. Note that divergence of the vector field determined by the right-hand sides of the system (1) is strictly negative: $\text{div} \equiv -\sum_{j=1}^n m_j$, and hence, the volume of each bounded domain in its phase portrait decreases exponentially when $t \rightarrow \infty$. However, the limit set of this system can be much more complicated than just a singleton, as it is shown below.

In the case of odd dimensions $n = 2k + 1$, we construct in the positive octant $\mathbb{R}_+^{2k+1}$ an invariant parallelepiped Q of the system (1) and show that this system has a unique stationary point S_0 in the interior of Q .

We prove that if this point is hyperbolic, then there exists non-convex polyhedron $Q_* \subset Q$ which is star-shaped with respect to S_0 , is composed by $4k + 2$ triangle prisms with disjoint interiors, and contains at least one cycle of the system (1). This point S_0 is a vertex of Q_* . We obtain also some sufficient conditions of existence of stable cycles there. The case $n = 3$ was studied in [2,3].

In the symmetric dimensionless case, when all the functions f_j coincide, and $m_j = 1$ for all j , we show the following

Theorem. *If the dimension $n = 2k + 1$ is not a prime number, and if the stationary point S_0 is hyperbolic, then the system (1) has several cycles in Q .*

Each of these cycles c_i corresponds to an odd multiplier $2l_i + 1$ of $2k + 1$, and there exist an invariant $(2l_i + 1)$ -dimensional plane, and a polyhedral domain Q_i containing the cycle c_i .

Interiors of domains Q_i containing different cycles are disjoint.

Similar considerations can be done in the case when the functions f_i are unimodal, cf. [2]. This corresponds to simplest combinations of positive and negative feedbacks in gene networks.

2. For arbitrary dimensions, we study also symmetric systems (1) with piece-wise constant functions f_j which correspond to threshold negative feedbacks in the chemical kinetics. In this case we construct 2-dimensional invariant surfaces of these systems which are contained in star shaped non-convex polyhedra similar to Q_* , and in turn, contain cycles of corresponding dynamical system.

If $n = 2k$, then the dynamical systems of the type (1) have three stationary points, two of them are stable, and their attracting basins are separated by an invariant hyper-surface $P^{2k-1} \subset Q \subset \mathbb{R}_+^{2k}$. This hyper-surface contains the diagonal $\Delta := \{x_1 = x_2 = \dots = x_{2k}\}$ of Q and the third (unstable) stationary point $S_0 \in \Delta$. In this case, we construct $k - 1$ invariant 2-dimensional surfaces $P_i^2 \subset Q$, each of them contains at least one cycle c_i of the system (1), and all these surfaces are contained in P^{2k-1} .

If $n = 2k + 1$, then there are k invariant 2-dimensional surfaces P_i^2 in the phase portrait of the dynamical system (1), and each of these surfaces contains at least one cycle c_i . Three-dimensional dynamical systems of this type were studied in [3].

All geometrical propositions described above are illustrated by numerical experiments.

In the general case of asymmetric forms of the system (1) with different smooth functions f_i , all these constructions seem to be much more complicated.

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Figure 23: WINGS GUARD

Geometry of submanifolds with Maple[®]

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We present some Maple[®] procedures that allow us to calculate extrinsic characteristics of a submanifold in Euclidean space. Namely, the normal curvature tensor, the Gaussian torsion of a surface in E^4 , the parameters of the normal curvature ellipse.

In addition, we give plotting procedures for the normal curvature ellipse for $F^2 \subset E^4$ and the normal curvature indicatrix for $F^3 \subset E^6$.

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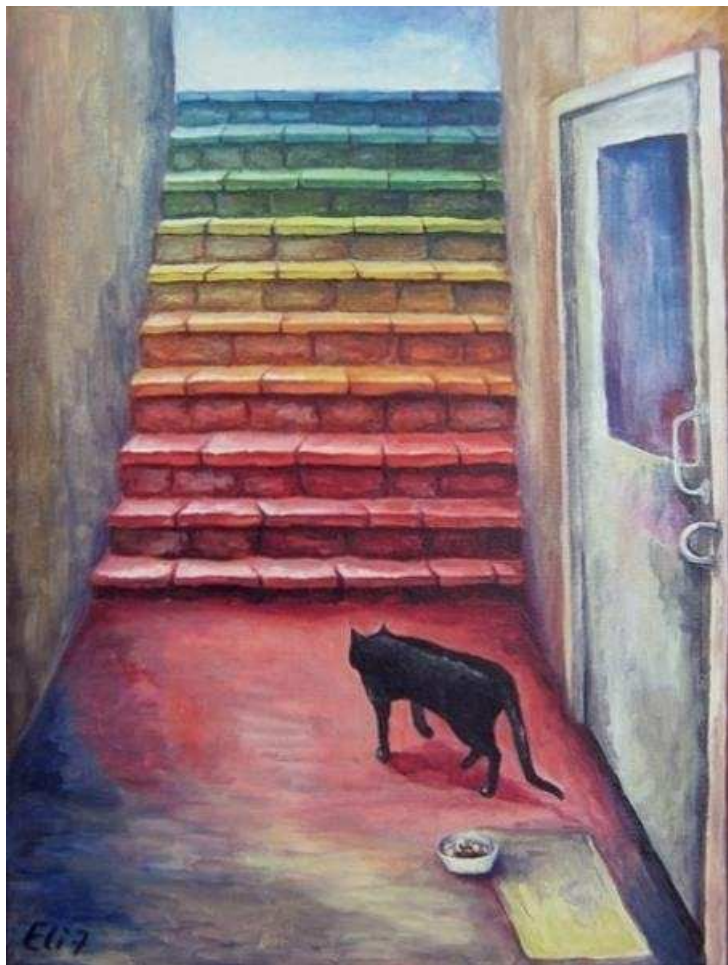


Figure 24: TO THE NEXT LEVEL

Rotational liquid film interacted with ambient gaseous media

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Annular jets of an incompressible liquid moving in a gas at rest are of interest for applications.

The experimental investigations of annular liquid jets show existing *tulip* and *bubble* jet shapes, and also predict the existence of periodic shape. However, sufficient simplifications of mathematical models of the flow details were made: the effects of the forces of surface tension of the longitudinal motion and the variability of the tangential velocity component of the centrifugal forces in the field were neglected.

In the present work, the equations described the flow of rotational annular jets of viscous liquid in an undisturbed medium with allowance of the above mentioned effects.

In the case of axisymmetric stable flow, the motion equations for quantities averaged over the thickness of the film are described (using the thin film dynamics [1, 2]) as

$$\begin{aligned}\frac{dV_\tau}{ds} &= \frac{V_\theta^2}{V_\tau R} \sin \psi + \text{Re}^{-1} \left[\frac{d}{ds} \left(\frac{\sigma_{\tau\tau}}{V_\tau} \right) - \frac{\sigma_{\theta\theta}}{V_\tau R} \sin \psi \right] + \text{Fr}^{-1} \frac{\cos \psi}{V_\tau}, \\ \frac{d\psi}{ds} &= \left(V_\tau - \text{We}^{-1} R - \text{Re}^{-1} \frac{\sigma_{\tau\tau}}{V_\tau} \right)^{-1} \times \\ &\quad \times \left[\left(\frac{V_\theta^2}{V_\tau R} - \text{We}^{-1} - \text{Re}^{-1} \frac{\sigma_{\theta\theta}}{V_\tau R} \right) \cos \psi - \text{Fr}^{-1} \frac{\sin \psi}{V_\tau} - \frac{\text{Eu}}{\text{We}} R \right], \\ \frac{dV_\theta}{ds} + \frac{V_\theta}{R} \sin \psi &= \text{Re}^{-1} \left[\frac{d}{ds} \left(\frac{\sigma_{\tau\theta}}{V_\tau} \right) - \frac{\sigma_{\theta\theta}}{V_\tau R} \sin \psi \right], \\ \frac{dR}{ds} &= \sin \psi, \quad \frac{dx}{ds} = \cos \psi\end{aligned}$$

with initial conditions

$$x|_{s=0} = 0, \quad R|_{s=0} = 1, \quad \psi|_{s=0} = \psi_0, \quad V_\tau|_{s=0} = 1, \quad V_\theta|_{s=0} = \frac{\Omega R_0}{V_{\tau 0}} = \frac{V_{\theta 0}}{V_{\tau 0}} \equiv V_\Omega.$$

Here, V_τ and V_θ are the longitudinal and rotational components of the fluid velocity vector; the index 0 denotes the values of the quantities on the nozzle exit; s is the coordinate reckoned along the generator of the middle surface of the film, ψ the angle between the tangent τ and the x -axis (of symmetry of the film), p_o and p_i are outer and inner ambient media static pressures, $d\psi/ds$ is the curvature of the generator of the middle surface of the film, g the acceleration due to the force of gravity directed along the x -axis,

$$\sigma_{\tau\tau} = 2 \left(2 \frac{dV_\tau}{ds} + \frac{V_\tau \sin \psi}{R} \right), \quad \sigma_{\tau\theta} = \frac{dV_\theta}{ds} - \frac{V_\theta \sin \psi}{R}, \quad \sigma_{\theta\theta} = 2 \left(\frac{dV_\tau}{ds} + \frac{2V_\tau \sin \psi}{R} \right)$$

are the components of the stress tensor in the coordinate system associated with the middle surface of the film; $\sigma_\star$ the surface tension coefficient, p the liquid density, μ the liquid viscosity, $\text{We} = \frac{\rho h_0 V_{\tau 0}^2}{2\sigma_\star}$, $\text{Re} = \frac{\rho R_0 V_{\tau 0}}{\mu}$, $\text{Fr} = \frac{V_{\tau 0}^2}{g R_0}$, $\text{Eu} = \frac{(p_i - p_o) R_0}{2\sigma_\star}$ are Weber, Reynolds, Froude, and Euler numbers.

The pressure difference outside and within the jet were obtained and analyzed. The calculations show the dependence of the jet shape on the relative contributions of the initial rotation rate, viscosity, surface tension, the gravity forces, and the pressure difference. An exact solution to the problem of the motion of a thin cylindrical shell due to different internal and external pressures is obtained. To compute an annular jet of viscous liquid, the

method of successive approximation (of the perturbation theory, [3, 4]) was used. Analysis of non-linear instabilities of the Rayleigh-Taylor type in meridional cross-section was carried out. Using the motion equation, the perturbed solution was found analytically, see [5],

$$Z(\varphi, t) = R(t) [\exp(i\varphi) - \lambda(t)k^{-1} \exp(ik\varphi)]$$

$$\lambda(\tau) = \lambda(0)/\sqrt{|k|} \times \begin{cases} \frac{1}{\cos \sqrt{a}\tau + b \sin \sqrt{a}\tau} \begin{cases} \cos \sqrt{ak}\tau (\sqrt{k} + b \tan \sqrt{ak}\tau), & k > 0 \\ \cosh \sqrt{a|k|}\tau (\sqrt{|k|} + b \tanh \sqrt{a|k|}\tau), & k < 0 \end{cases} & \Delta p > 0, \\ \sqrt{|k|}, & \Delta p = 0, \\ \frac{1}{\cosh \sqrt{|a|}\tau + b \sinh \sqrt{|a|}\tau} \begin{cases} \cosh \sqrt{|ak|}\tau (\sqrt{|k|} + b \tanh \sqrt{|a|k|}\tau), & k > 0 \\ \cos \sqrt{|ak|}\tau (\sqrt{|k|} + b \tan \sqrt{|ak|}\tau), & k < 0 \end{cases} & \Delta p < 0, \end{cases}$$

where $\tau = t/t_0$ ($t_0 = R_0/V_{\tau 0}$) is dimensionless time, k is the wave mode, $\Delta p = p_i - p_o$, and parameters $a = \frac{1}{2\pi} \text{Eu}/\text{We}$, $b = V_\Omega \sqrt{2\pi \text{We}/\text{Eu}}$ depend on Weber and Euler numbers.

When $|\lambda(t)| \geq 1$, the curve $Z(\varphi, t)$ becomes self-intersecting that leads destruction of the film. It is shown that the instabilities, which appear due to pressure drop, cannot be stabilized by rotation.

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Figure 25: WAITING

Zermelo problem - based trajectory modeling in navigation

P. Kopacz (Gdynia Maritime University, Poland)

Advisor: Prof. Robert Wolak (Institute of Mathematics of Jagiellonian University, Poland)

We research the Zermelo's problem of navigation in Riemannian space for $\dim(\mathbb{R} \times M) = 3$ with respect to navigational passage planning. We construct the model in which the sea is represented by a fibered manifold $\pi : \mathbb{R} \times M \rightarrow \mathbb{R}$ where π is the first canonical projection. The wind distribution is represented by the vector field under the force representing the action of the perturbation on the manifold M . We perturb the sea by a time-dependent vector field on M , i.e. a projectable vector field on $\mathbb{R} \times TM$. We obtain the general form of the system of differential equations providing the solution of the Zermelo problem after perturbation followed by the corresponding simulation for $\dim(\mathbb{R} \times M) = 3$.

Keywords: Zermelo navigation; Riemannian manifold; Euler-Lagrange equations; passage planning.

References

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Figure 26: LOST IN INDIA

Fourier Sampling of Piecewise-Smooth Functions, Turan-Nazarov Inequality, and Taylor domination

Y. Yomdin (Weizmann Institute of Science, Israel) (joint with D. Batenkov)

A periodic C^d smooth function f can be reconstructed from its first N Fourier coefficients with an error of order $1/N^d$. However, for f only piecewise C^d smooth the classical Fourier approximation has an error of order $1/N$, no matter how large d is. There is a long-standing open problem (Eckhoff Conjecture) concerning a possibility to gain the “smooth” accuracy rate $1/N^d$ via a non-linear manipulations with the first N Fourier coefficients of any piecewise C^d smooth function f . This problem was recently solved by D. Batenkov, via “Algebraic Sampling” approach. The key point was a proper choice of the samples among the first N Fourier coefficients of f .

On the other hand, the problem of estimating robustness of the Fourier sampling on a given sampling set S is addressed by a “discrete” version of the well-known Turan-Nazarov inequality for exponential polynomials. It turns out to give rather accurate estimates and challenging predictions.

Yet another important connection is provided by the fact that the discrete Turan-Nazarov inequality (in fact, the “Turan lemma” which is the starting point in this research line) has another interpretation. This is, in particular, a result on Taylor coefficients for rational functions, which can be extended to wider classes, and which provides important analytic information.

We shall discuss the above results and connections, as well as some related open questions.

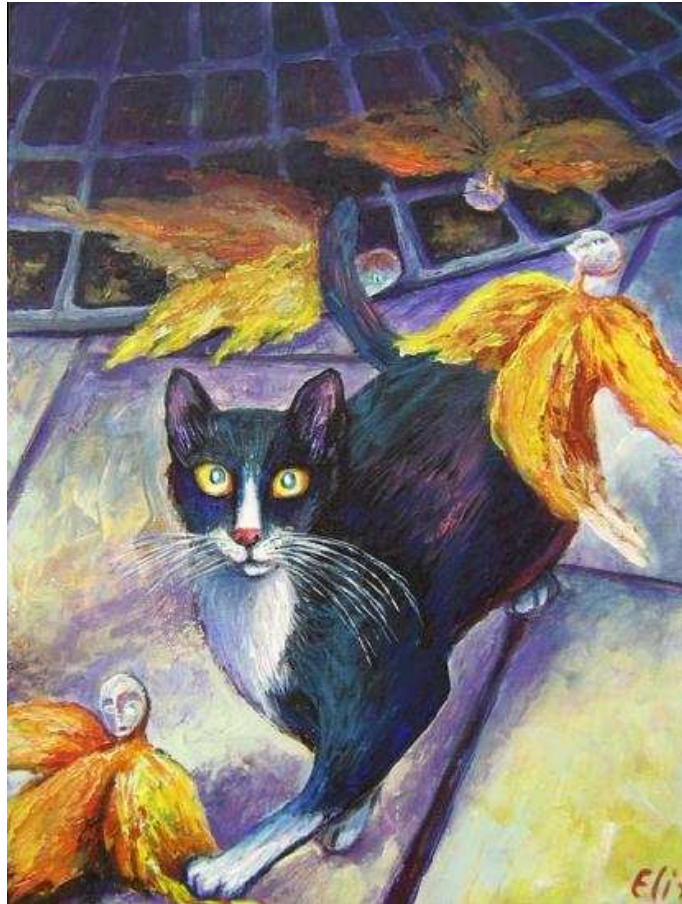


Figure 27: THE LEAF FALL

Algebraic reconstruction of geometric models from integral measurements

D. Batenkov (Weizmann Institute of Science, Israel)

Algebraic Sampling, an active area of research in Signal Processing, aims at reconstructing a finite-parametric model from a small number of integral measurements. The basic idea is to substitute the unknown “symbolic” parameters (which usually reflect the geometry of the model) into the measurement equations and subsequently solve the resulting system of explicit algebraic equations. In many cases, these algebraic systems have the well-known “Prony-type” structure. As a result, questions of existence, uniqueness and stability of solutions of the original reconstruction problems can be investigated in a unified framework. To illustrate the power of the algebraic sampling approach, we present some general results on Prony-type systems, as well as consider two particular realizations - piecewise D -finite reconstruction in one and two dimensions, and reconstruction of arbitrary piecewise-smooth functions from their Fourier coefficients.

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Figure 28: RIDING THE TAIL

The solutions of some extremal problems using Maple®

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The very natural functionals on a set of convex bodies in the k -dimensional Euclidean space are *quermass integrals* (*Minkovski functionals*) W_i , $i = 0, 1, \dots, k$ (see, for example, [1, section 6.1.6]). Let A be a convex body in 3-dimensional Euclidean space $\mathbb{E}^3$. For the convex body A the following quermass integrals are true: $W_0(A) = V(A)$ is the volume, $W_1(A) = F(A)/3$, $W_2(A) = M(A)/3$, $W_3(A) = \text{const} = 4\pi/3$, where $F(A)$ is the surface area, $M(A)$ is the integral mean curvature.

Let us remind values of quermass integrals for a rectangular parallelepiped $P = ABCDA'B'C'D'$ in $\mathbb{E}^3$ with edge lengths $|AB| = a$, $|AD| = b$, $|AA'| = c$, where $0 \leq a \leq b \leq c$. Well-known formulas for volume $V(P) = W_0(P) = abc$ and the area of a surface $F(P) = 2(ab + ac + bc) = 3W_1(P)$ are supplemented by the formulas $W_3(P) = 4\pi/3$ and $M(P) = \pi(a + b + c) = 3W_2(P)$ [4].

For the parallelepiped P we will denote a surface of this parallelepiped by $\partial(P)$ (its boundary in a natural topology of 3-dimensional Euclidean space). Let $d(M, N)$ be a geodesic (intrinsic) distance between points $M \in \partial(P)$ and $N \in \partial(P)$, i. e. minimal length of polygonal lines, connecting the points M and N , in $\partial(P)$. By $D(P)$ we will denote a geodesic (intrinsic) diameter of the parallelepiped P (more precisely, of the surfaces of a parallelepiped) is the maximal intrinsic distance between pair of points on the surfaces of a parallelepiped.

An interesting problem of finding extremal values of quermass integrals (excepting a trivial case of a constant $W_3 = 4\pi/3$) for rectangular parallelepiped P with a given intrinsic diameter. For convenience we will also consider *degenerate parallelepipeds* with $a = 0$.

The maximal surface area was found in [2] Yu. G. Nikonorov and Yu. V. Nikonorova (a minimal surface area, obviously, is equal to 0 and it is attained exactly by a degenerate parallelepiped with $a = b = 0$), it is attained by a parallelepiped with the relation $a : b : c = 1 : 1 : \sqrt{2}$ for edge lengths. In particular, for any parallelepiped P the inequality $ab + ac + bc \leq \frac{1+2\sqrt{2}}{6} (D(P))^2$ is true.

Applications of the mathematical modeling allow to test the different assumptions that arise in solving geometric problems. Experimental data can either confirm the hypotheses, or suggest some solutions, and then it is possible to receive the theoretical justification of the obtained numerical results. Besides, the numerical solution can be obtained for the problems that do not have an analytical solution.

The extreme values of the unexplored quermass integrals for a rectangular parallelepiped P were obtained using approximate computations in the Maple®.

The following theorems were proved:

Theorem 1 *Among all rectangular parallelepipeds with given intrinsic diameter $D(P)$ the maximal integral mean curvature is attained by a parallelepiped with the relation $a : b : c = 0 : 1 : 1$ for edge lengths, and the minimal integral mean curvature is attained by a parallelepiped with the relation $a : b : c = 0 : 0 : 1$ for edge lengths. In other words, for any rectangular parallelepiped P the following inequality holds:*

$$\pi D(P) \leq M(P) \leq \pi \sqrt{2} D(P).$$

Theorem 2 *Among all rectangular parallelepipeds with given intrinsic diameter $D(P)$ the maximal volume is attained by a parallelepiped with the relation $a : b : c = 1 : 1 : \sqrt{2}$ for edge lengths (the minimal volume is equal to 0 and is attained in accuracy by degenerate*

parallelepipeds). In other words, for any rectangular parallelepiped P the following inequality holds:

$$0 \leq V(P) \leq \frac{1}{6\sqrt{3}} (D(P))^3.$$

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Figure 29: THE LAST ONE

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Figure 30: THE EIGHT EYES OF HANUKKAH

About the Department of Mathematics

The Department of Mathematics (<http://math.haifa.ac.il/>) at the University of Haifa was founded in the mid 70's. Today it is one of the seven mathematics departments in Israeli Universities. The faculty is engaged in world-class research in both pure and applied Mathematics. The research interests of our faculty include algebra, applied mathematics, combinatorics, differential equations, functional analysis, geometry and topology, logic, and theoretical computer science. The Center for Computational Mathematics and Scientific Computing (CCMSC) was founded at the department in 2001. Since then it supports a large variety of research activities, conferences, special lecture series, workshops, etc.

The department offers a variety of undergraduate and graduate programs, both in mathematics and in mathematics and computer science. Undergraduate students are offered also double major programs with other departments. In addition, the department provides courses in Mathematics for students in other university departments.

There is a number of colloquiums and research seminars at the Mathematical Department, where current results of the project will be presented. Traditionally, these seminars go on in a very informal and friendly manner so that a speaker can pose open questions and discuss difficulties of his study in order to get a good advise or recommendation from the audience. More than half of talks on the seminars are presented by invited speakers from other Israel universities as well as from overseas mathematical research centers.

The Mathematical Department at the University of Haifa routinely organizes international conferences and workshops in various mathematical fields with inviting world leaders of the field.



Figure 31: SUN-BALL

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Many thanks to colleagues from University of Haifa:

Prof. David Blanc

Prof. Kobi Peterzil

Director of CRI: Prof. Martin Golumbic

Workshop Coordinator: Ms. Danielle Friedlander

Technical Consultant: Mr. Hananel Hazan

for their help in organizing the International Workshop.



Figure 32: THE ANGEL OF MAY

Illustrations

The Organizing committee thanks the painter Nesis Elisheva, see homepage

<http://www.artmajeur.com/en/artist/nesis/portfolio?page=1>

for accepting to illustrate the booklet by the series of original paintings, dedicated to the cats – the most mystic animals on Earth.

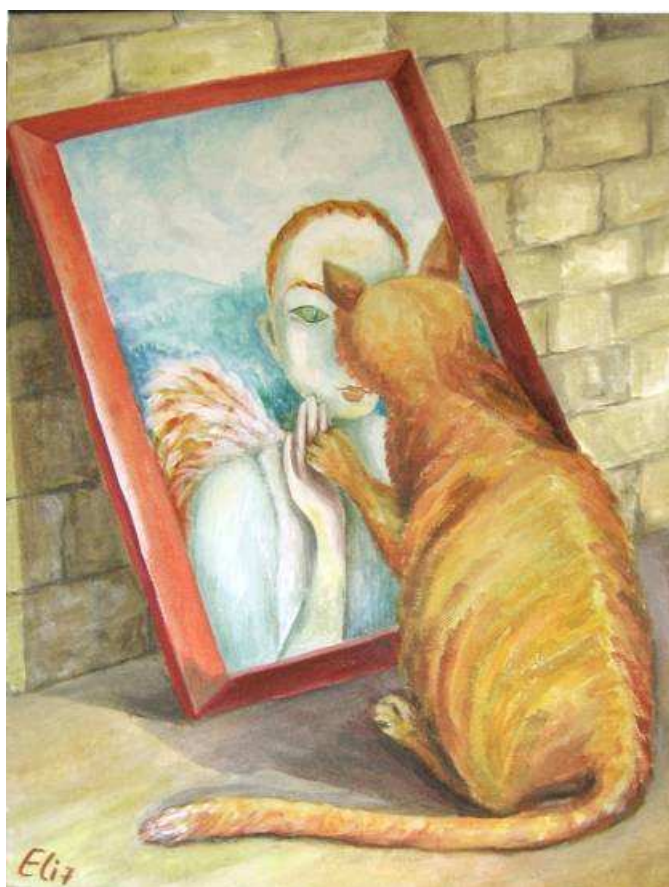


Figure 33: TRUE MIRROR

Useful Information

Workshop location <http://www.cri.haifa.ac.il/crievents/2013>
is in CRI offices at the Haifa University Education and Sciences Building, 6th floor. Exiting the elevator (6th floor), turn right.

From Haifa Central Carmel (near Hotel Nof) to Haifa University (and opposite way): bus lines 37, 37N and 30.

From Haifa Central Carmel (near Hotel Nof) to the sea beach Hof Ha-Carmel (and opposite way): bus lines 131, 132 and 133.

From Haifa University to the sea beach Hof Ha-Carmel (and opposite way): bus lines 46 and 146.



Figure 34: FISHING

Banquet

17 of May, 17:00 – 21:00

The place will be announced.

The 60th anniversary of Vladimir Rovenski will be celebrated in this occasion.



Figure 35: HAPPY NEW YEAR TO COLLEAGUES!

NOTES



Figure 36: FAIR WIND CATCHING

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